Rules for exponents

Example

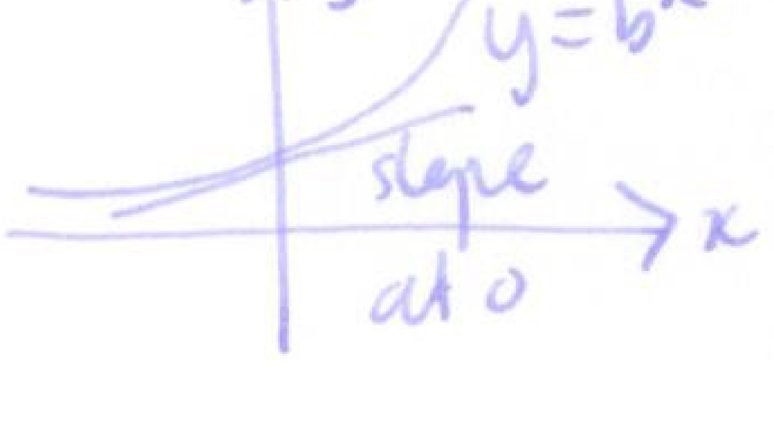
- $b^0 = 1$ $\qquad\qquad 2^0 = 1$
- $b^x b^y = b^{x+y}$ $\qquad\qquad 2^4 \cdot 2^7 = 2^{4+7} = 2^{11}$
- $b^x / b^y = b^{x-y}$ $\qquad\qquad 2^4 / 2^7 = 2^{4-7} = 2^{-3}$
- $b^{-x} = \frac{1}{b^x}$ $\qquad\qquad 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
- $(b^x)^y = b^{xy}$ $\qquad\qquad (2^3)^2 = 2^6$
- $b^{1/n} = \sqrt[n]{b}$ $\qquad\qquad 2^{1/2} = \sqrt{2}$

~~Solving equations involving exponentials:~~

~~example:~~

The number $e \approx 2.718\ldots$

e is special because:

- 

$y = e^x$ is the unique exponential function with slope 1 at $x=0$.

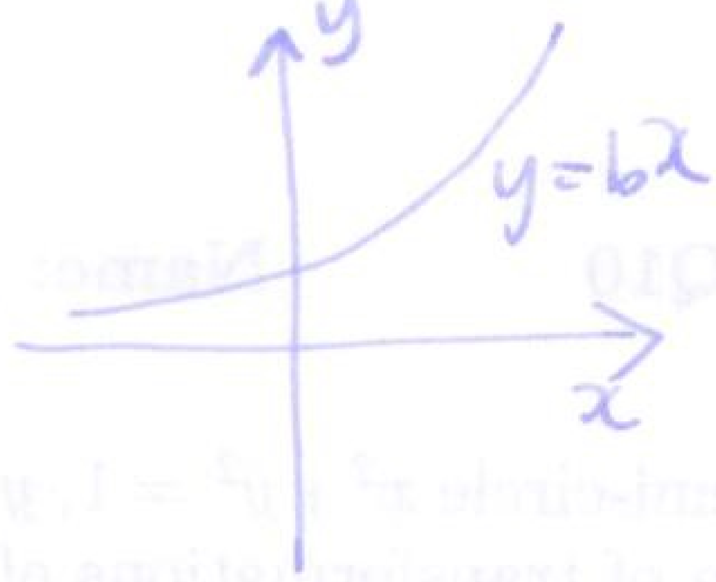
Logarithms

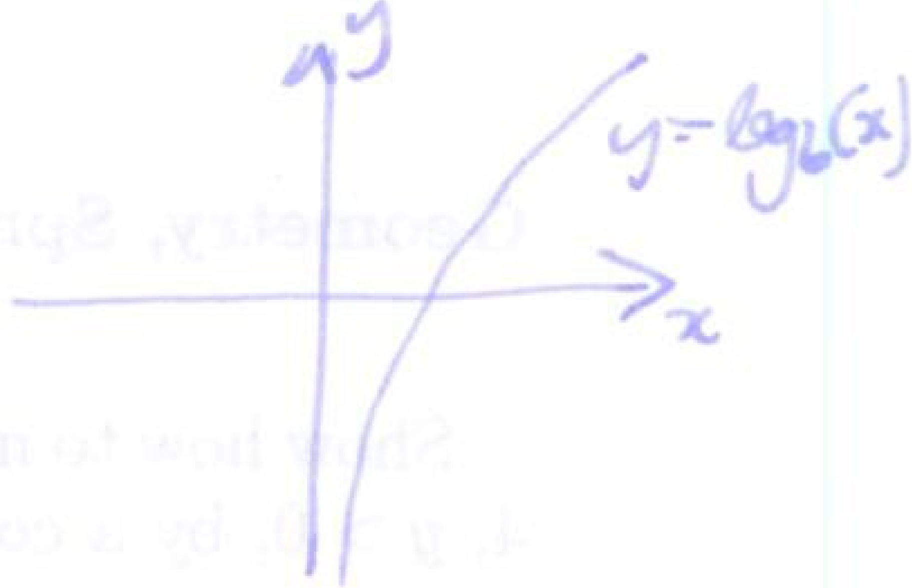
The logarithm is the inverse of the exponential function.

$f(x) = b^x \qquad\qquad f^{-1}(x) = \log_b(x)$

<u>note</u>: b^x is one-to-one so inverse exists.

	domain ~~range~~	range ~~domain~~
b^x	$(-\infty,\infty)$	$(0,\infty)$
$\log_b(x)$	$(0,\infty)$	$(-\infty,\infty)$





<u>recall</u> $f(f^{-1}(x)) = x = f^{-1}(f(x))$

so $b^{\log_b(x)} = x = \log_b(b^x)$.

"$\log_b(x)$ is the number to which b must be raised to get x".

$2^3 = 8$ $\quad \log_2(8) = 3$.

<u>Rules for logarithms</u> <u>Example</u>

- $\log_b(1) = 0$
- $\log_b(b) = 1$
- $\log_b(xy) = \log_b(x) + \log_b(y)$ $\qquad \log_3(5.7) = \log_3(5) + \log_3(7)$
- $\log_b(x/y) = \log_b(x) - \log_b(y)$ $\qquad \log_2(3/2) = \log_2(3) - \log_2(2)$
- $\log_b\left(\frac{1}{x}\right) = -\log_b(x)$ $\qquad \log_2(1/8) = -\log_2(8) = -3$.
- $\log_b(x^n) = n\log(x)$ $\qquad \log_{10}(10^4) = 4\log_{10}(10) = 4$

change of base: $\log_b x = \dfrac{\log_a(x)}{\log_a(b)}$ for any a

in particular: $\log_b(x) = \dfrac{\ln(x)}{\ln(b)}$