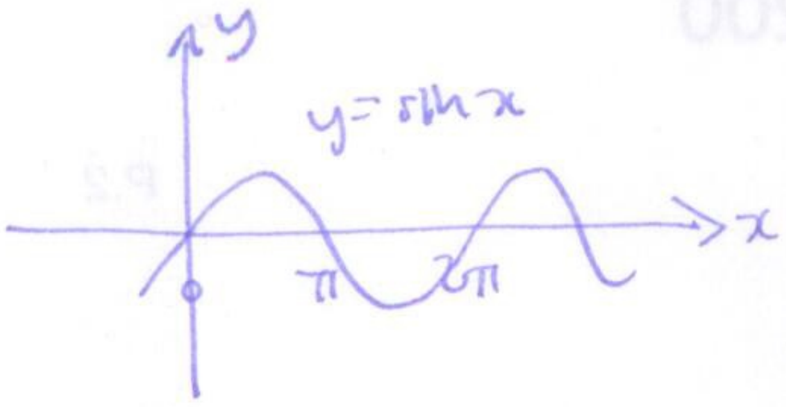
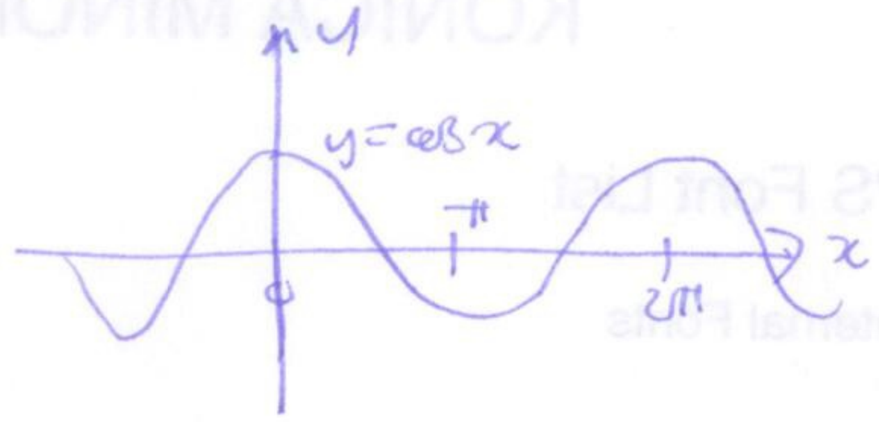
The graphs of $\sin x$ and $\cos x$ are <u>periodic</u> with period 2π





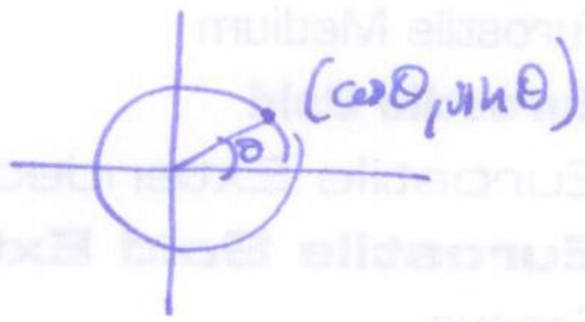
In general: a function $f(x)$ is periodic with period T if $f(x+T) = f(x)$ for all x, and T is the smallest such number which works.

Other trig functions

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \operatorname{cosec}(x) = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Trig identities

Pythagorean: $\cos^2\theta + \sin^2\theta = 1$



gives: $1 + \tan^2\theta = \frac{1}{\cos^2\theta} = \sec^2\theta$

$$\cot^2\theta + 1 = \frac{1}{\sin^2\theta} = \operatorname{cosec}^2\theta$$

Double angle: $\sin 2\theta = 2\sin\theta\cos\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

Addition: $\sin(\theta+\phi) = \sin\theta\cos\phi + \sin\phi\cos\theta$

$$\cos(\theta+\phi) = \cos\theta\sin\phi - \sin\theta\sin\phi$$

special case (shift): $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

$$\cos\left(x + \frac{\pi}{2}\right) = -\sin x .$$

§1.5 Inverse functions

$f: X \to Y$

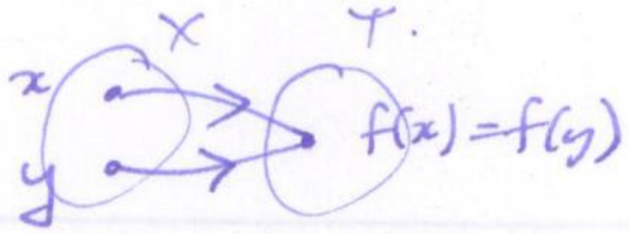
$x \mapsto f(x)$

$X \leftarrow Y : f^{-1}$

$x \longleftarrow f(x)$

want: the inverse of the function should do the reverse, i.e send $f(x) \mapsto x$

problem:



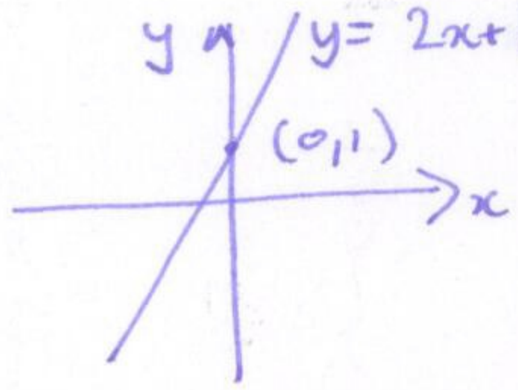
doesn't work unless f is **one-to-one**.

solution: restrict domain of f until it is one-to-one.

Example (one-to-one function)

$f: \mathbb{R} \to \mathbb{R}$

$x \mapsto 2x+1$



one-to-one $\longleftrightarrow$ ~~vertical~~ horizontal line test passes

i.e. every horizontal line hits the graph exactly once.

equivalently: for every value of $f(x)$ there is a unique x such that $x \mapsto f(x)$

so this function has an inverse.

Q how do we find a formula for the inverse?

1. write down $y = f(x)$
2. try and solve for x in terms of y.
3. swap x and y
4. check!

Example $f(x) = 2x+1$

$y = 2x+1$

$y-1 = 2x$

$\frac{y-1}{2} = x.$

so $f^{-1}(x) = \frac{x-1}{2}$

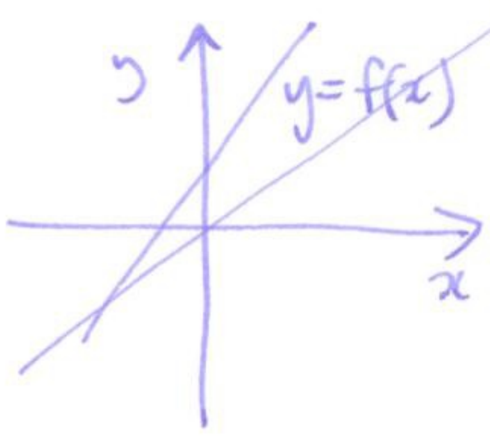
check: $f(f^{-1}(x)) = x$

$f^{-1}(f(x)) = x$

$x \mapsto 2x+1 \mapsto \frac{2x+1-1}{2} = x$ ✓

Q: how do we draw a graph of the inverse?

① find $f^{-1}(x)$ draw graph.

② 

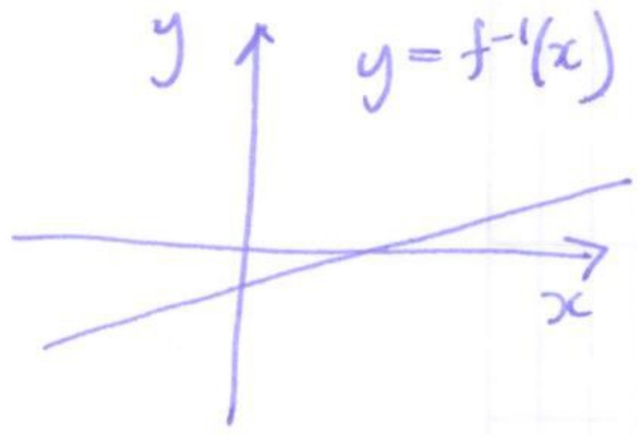
graph $(x, f(x))$

(input, output)

$(x, f(x)) \longleftrightarrow (f(x), x)$

graph of f graph of f^{-1}

$(x,y) \longleftrightarrow (y,x)$ is reflection in $y=x$



graph $(x, f^{-1}(x))$

(output, input)

$(z, f^{-1}(z))$

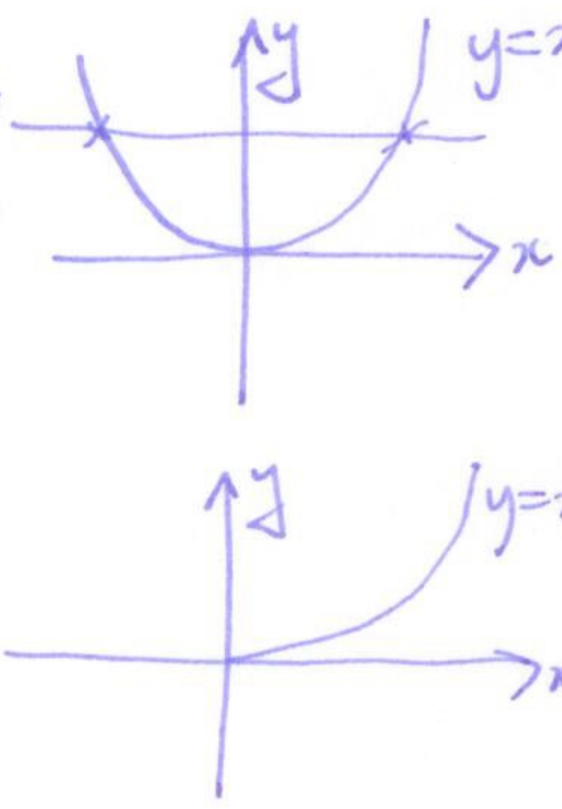
$z = f(x)$ for some x

$(f(x), f^{-1}(f(x)))$

$(f(x), x)$

② graph of f^{-1} is graph of f reflected in line $y=x$.

Example



problem: no inverse!

soln: restrict domain

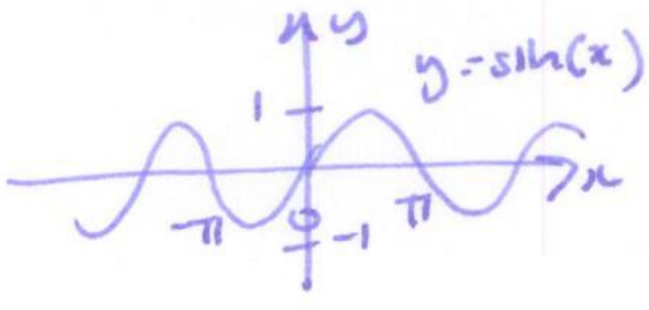
by $x \geq 0$.

$y = x^2$

$\sqrt{y} = x$.

inverse is $f^{-1}(x) = \sqrt{x}$.

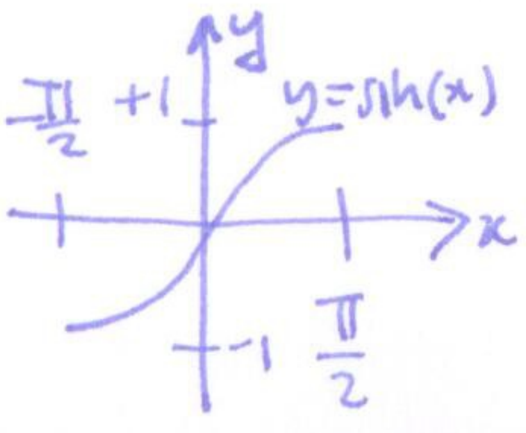
Example $y = \sin(x)$

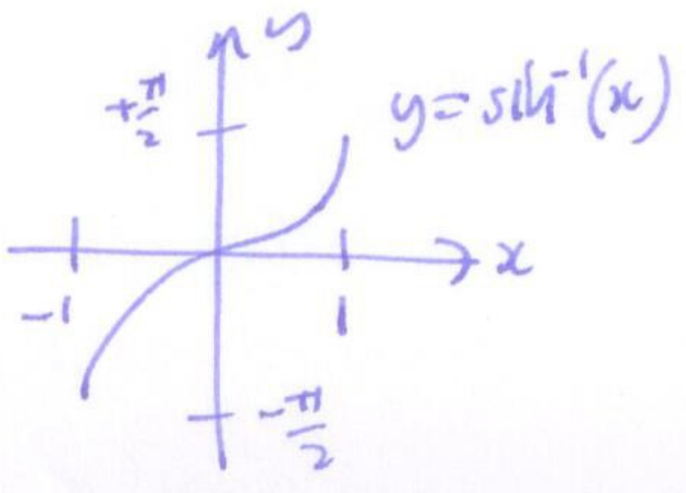


problem: no inverse!

soln: restrict domain.

usually use $[-\frac{\pi}{2}, \frac{\pi}{2}]$





Note

$\sin^{-1}(\sin(\frac{\pi}{3})) = \frac{\pi}{3}$.

$\sin^{-1}(\sin(\frac{7\pi}{3})) = \frac{\pi}{3}$!