completing the square: every quadratic polynomial can be written as $a(x+b)^2+c$

Example $2x^2+2x+1 = 2\left(x^2+x+\frac{1}{2}\right)$.

observation: $(x+b)^2 = x^2+2b+b^2$

$$= 2\left(\left(x+\frac{1}{2}\right)^2 + \left(\frac{1}{2}-\frac{1}{4}\right)\right).$$

$$x^2+x+\frac{1}{4}$$

$$= 2\left(\left(x+\frac{1}{2}\right)^2 + \frac{1}{4}\right)$$

why do this? Q: what value of x minimizes $2x^2+2x+1$? $(x=-\frac{1}{2})$

Q: what is the vertical line of symmetry $(x=-\frac{1}{2})$.

§1.3 Special functions

we're interested in functions $f: \mathbb{R} \to \mathbb{R}$

all such functions: very badly behaved.

eg. $f(x) = \begin{cases} 1 & x \text{ rational} \\ 0 & x \text{ irrational} \end{cases}$ ← yuck.

nice functions:

· polynomials: $\underbrace{a_n x^{n}}_{\text{leading term}} + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ (n: degree)

a_i coefficients

· rational functions: $\frac{p(x)}{q(x)}$ p, q polynomials

· algebraic functions: roots / fractional powers $\sqrt{x}$, $x^{4/3}$

$\sqrt{x^2+1}$ $x^{7/4}+1$

- exponential functions: 2^x e^x inverses, $\log(x)$ $\ln(x)$.
- trigonometric functions: $\sin(x)$, $\cos(x)$, combinations of these, inverses etc.
- <u>elementary functions</u>: functions made from combining these functions above by
  - addition/subtraction $f(x)+g(x)$
  - multiplication/quotients $f(x)/g(x)$
  - composition $f(g(x))$
  - inverses $f^{-1}(x)$

<u>e.g.</u> $\dfrac{1+e^{\sqrt{x}}}{\sin^{-1}(\tan(x))}$

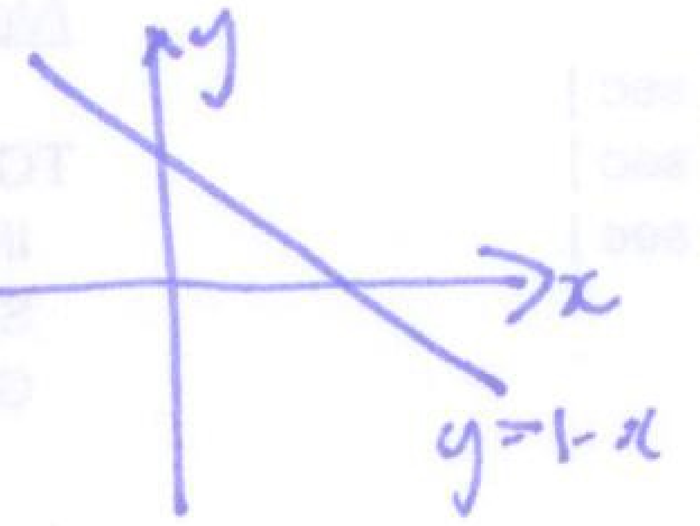
<u>Example</u>: composition of functions

$f(x) = 1-x$ "do f then g"

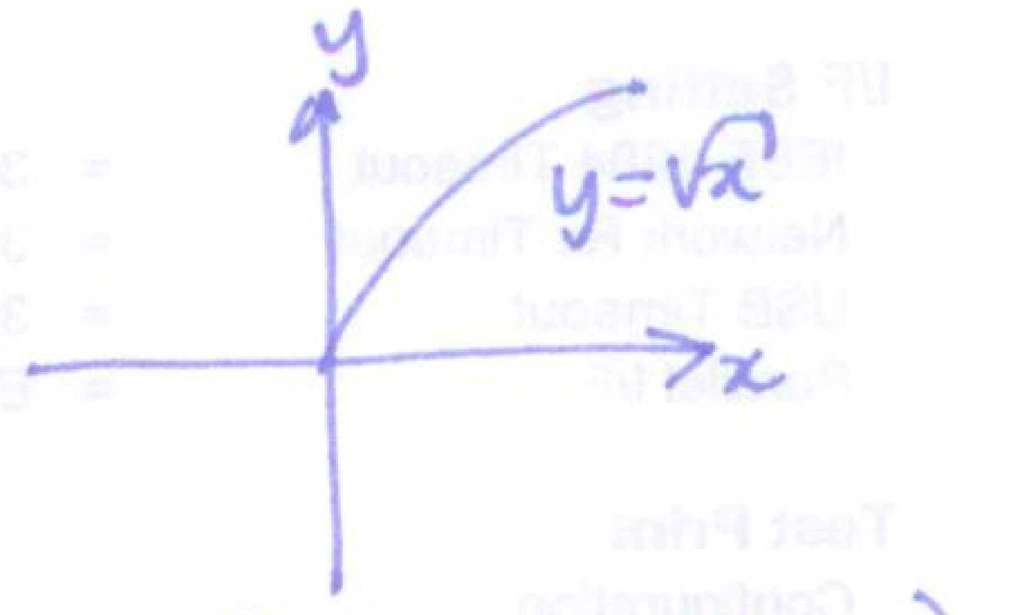
$g(x) = \sqrt{x}$ $x \mapsto g(f(x)) = g \circ f(x)$

$x \xmapsto{f} 1-x \xmapsto{g} \sqrt{1-x}$

<u>Q</u>: domain and range?



$f: \mathbb{R} \to \mathbb{R}$
all values



$f: [0,\infty) \to [0,\infty)$

domain: values of x such that $1-x$ is ≥ 0.

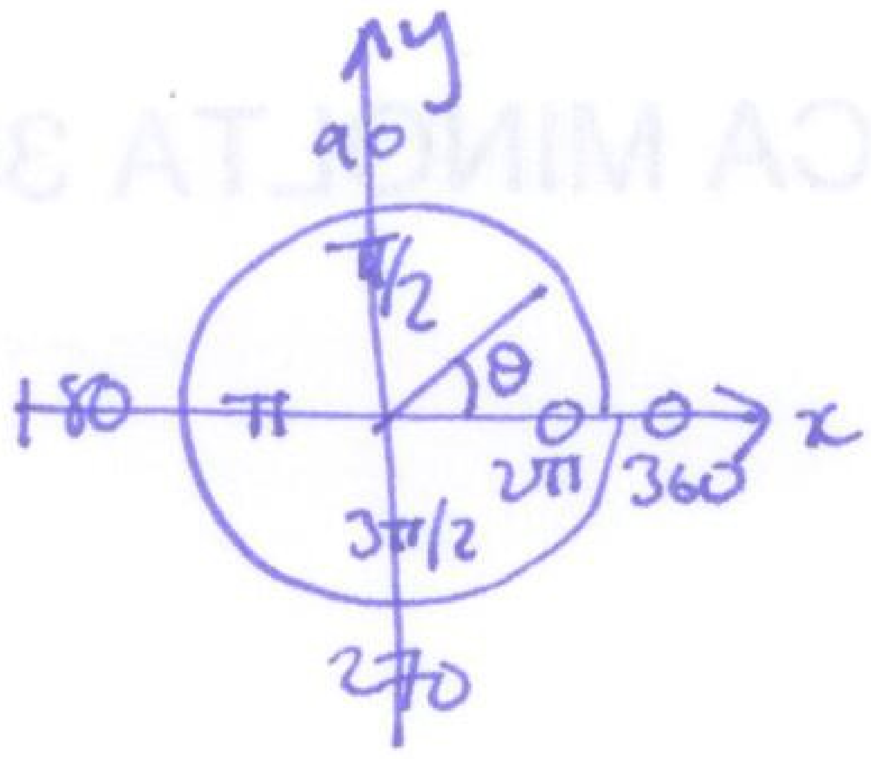
$$1-x \geq 0$$
$$-x \geq -1$$
$$x \leq 1$$

range: $[0,\infty)$

so $g(f(x)) : (-\infty,1] \longrightarrow [0,\infty)$

§1.4 Trigonometric functions

- angles : radians / degrees

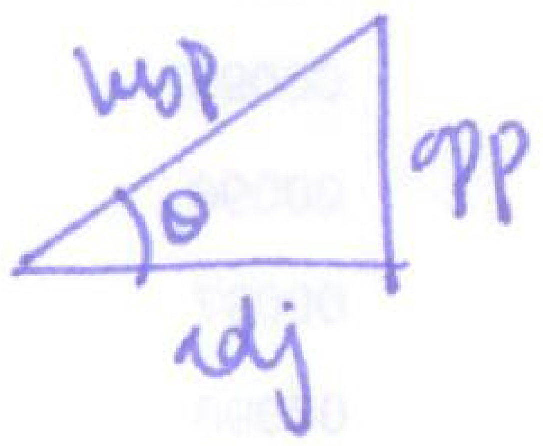


radians = distance travelled round unit circle.

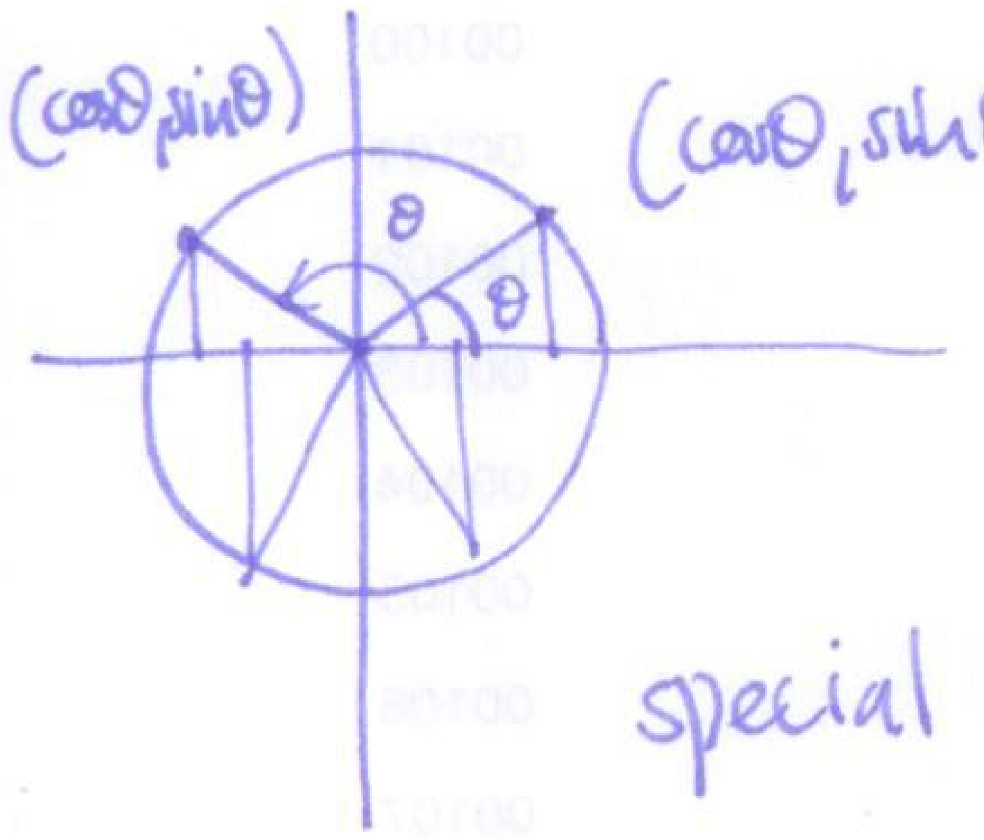
$$\text{angle in degrees} = \frac{360}{2\pi}\,\text{angle in radians}$$

- on the unit circle angle $\theta = \theta + 2\pi = \theta + 2\pi n,\ n \in \mathbb{Z}$.

recall: for right angled triangles



$$\left.\begin{aligned} \sin\theta &= \frac{opp}{hyp} \\ \cos\theta &= \frac{adj}{hyp} \end{aligned}\right\}$$ can extend these definitions to work for all angles $\theta \in \mathbb{R}$.



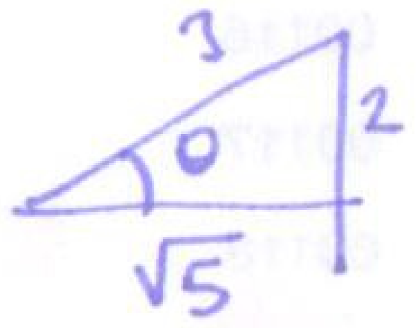
note: $\sin(-\theta) = -\sin(\theta)$ (odd)
$\cos(-\theta) = \cos(\theta)$ (even)

special values:

θ	0	$\pi/2$	$\pi/4$	$\pi/3$	$\pi/6$
$\sin\theta$	0	1	$\sqrt{2}/2$	$\sqrt{3}/2$	$1/2$
$\cos\theta$	1	0	$\sqrt{2}/2$	$1/2$	$\sqrt{3}/2$

$$\left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

Examples suppose $\sin\theta = 2/3$ find $\cos\theta$, $\tan\theta$



$$\cos\theta = \frac{\sqrt{5}}{3} \qquad \tan\theta = \frac{2}{\sqrt{5}}$$

Q: what about $\sin 2\theta$? use $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{2}{3}\cdot\frac{\sqrt{5}}{3} = \frac{4\sqrt{5}}{9}$