completing the square every quadratic polynomial can be written as $a(x+b)^2+c$

Example $2x^2+2x+1 = 2\left(x^2+x+\frac{1}{2}\right)$.

observation: $(x+b)^2 = x^2+2bx+b^2$

$$= 2\left((x+\tfrac{1}{2})^2 + (\tfrac{1}{2}-\tfrac{1}{4})\right).$$

$$x^2+x+\tfrac{1}{4}$$

$$= 2\left((x+\tfrac{1}{2})^2+\tfrac{1}{4}\right)$$

why do this? Q. what value of x minimizes $2x^2+2x+1$? $(x=-\frac{1}{2})$

Q: what is the vertical line of symmetry $(x=-\frac{1}{2})$.

§1.3 Special functions

we're interested in functions $f:\mathbb{R}\to\mathbb{R}$

all such functions: very badly behaved.

eg. $f(x) = \begin{cases} 1 & x \text{ rational} \\ 0 & x \text{ irrational} \end{cases}$ ← yuck.

nice functions:

- polynomials: $\underbrace{a_n x^{(n)}}_{\text{leading term}} + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ (n: degree; a_i: coefficients)
- rational functions: $\frac{p(x)}{q(x)}$ p,q polynomials
- algebraic functions: roots/fractional powers $\sqrt{x}$, $x^{4/3}$, $\sqrt{x^2+1}$, $x^{7/4}+1$