example f is increasing on $(-\infty, -1]$

decreasing on $(-1/10, 1/10)$

neither increasing nor decreasing on $[-10, +10]$

strictly increasing means $x_1 < x_2$ then $f(x_1) < f(x_2)$

strictly decreasing means $x_1 < x_2$ then $f(x_1) > f(x_2)$

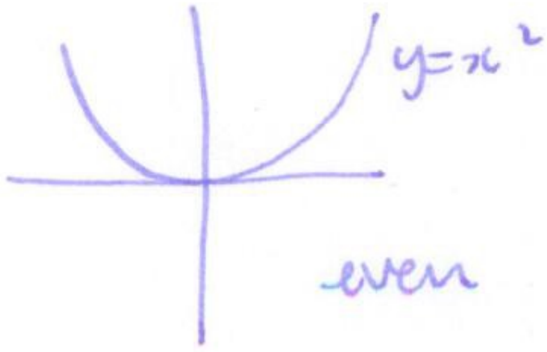
increasing but not strictly increasing

decreasing but not strictly decreasing.

Parity $f(x)$ is even if $f(-x) = f(x)$ reflectional symmetry

$f(x)$ is odd if $f(-x) = -f(x)$ half turn symmetry

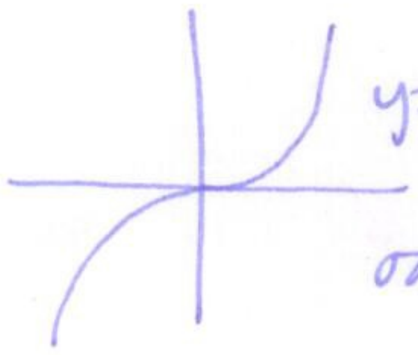
Examples



even

check $f(x) = x^2$

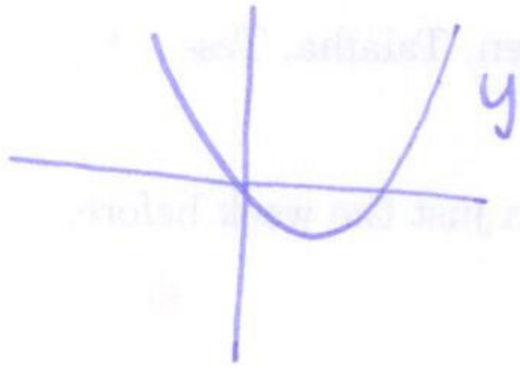
$f(-x) = (-x)^2 = x^2$ ✓.



odd

check $f(x) = x^3$

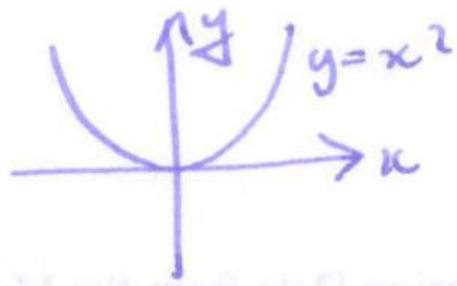
$f(-x) = (-x)^3 = -x^3$.



neither odd nor even.

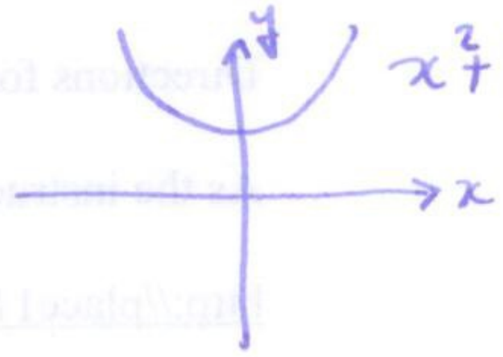
New graphs from old

Example



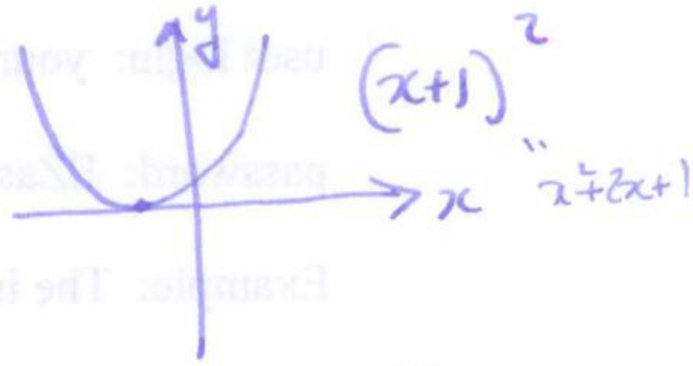
vertical translation $f(x)+c$

$c>0$ moves up
$c<0$ moves down



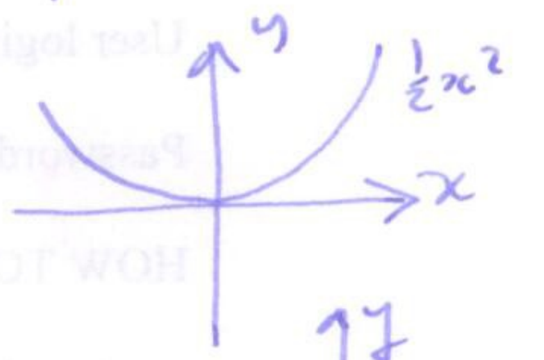
horizontal translation $f(x+c)$

$c>0$ left
$c<0$ right



vertical scaling $kf(x)$

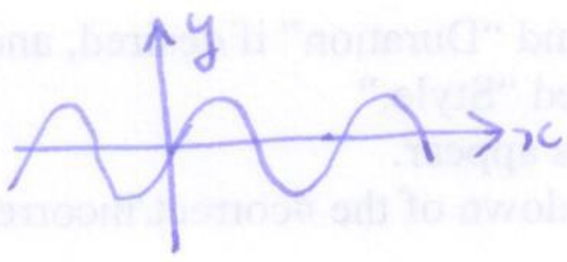
$k>1$ expansion
$0<k<1$ contraction
$k=-1$ reflection in x-axis



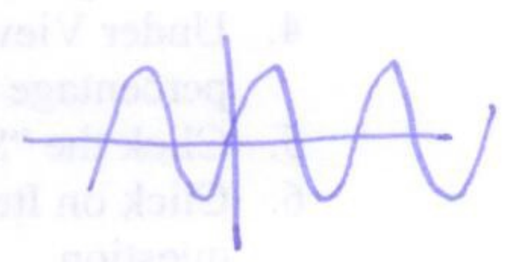
horizontal scaling $f(kx)$

$k>1$ contraction
$0<k<1$ expansion
$k=-1$ reflection in y-axis.

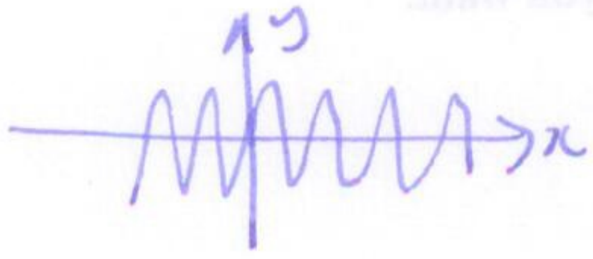
Example $y=\sin(x)$



$y=2\sin(x)$

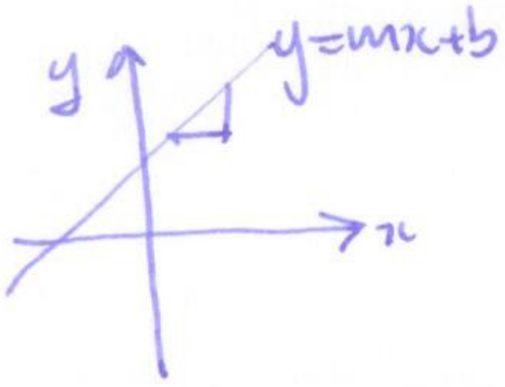


$y=\sin(2x)$



§1.2 Linear and quadratic functions

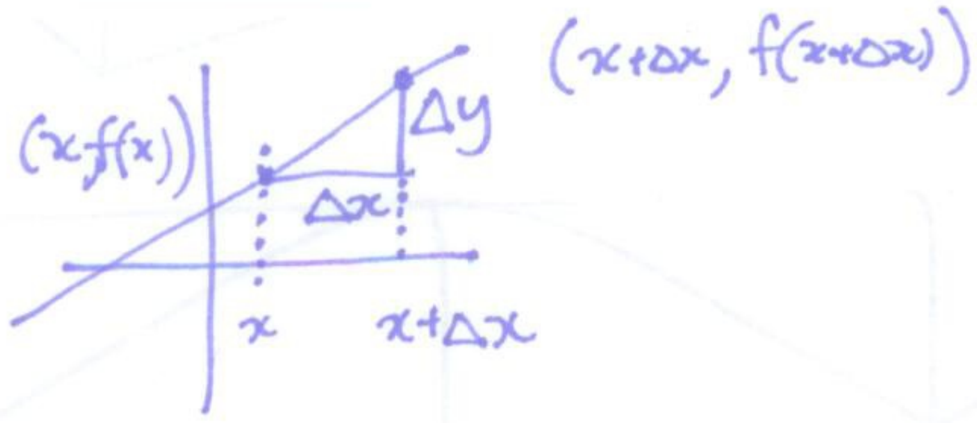
A linear function $f:\mathbb{R}\to\mathbb{R}$ is of the form $f(x)=mx+b$ (m,b numbers)

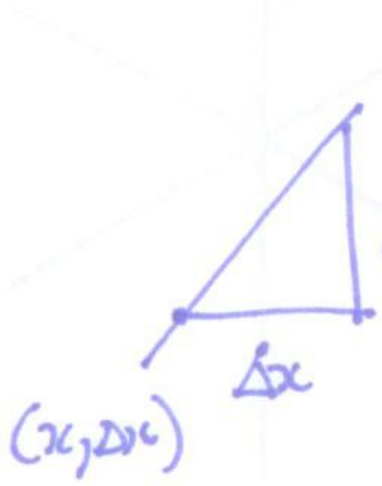


slope $=m$

y-intercept $(0,b)$

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta y}{\Delta x}$$



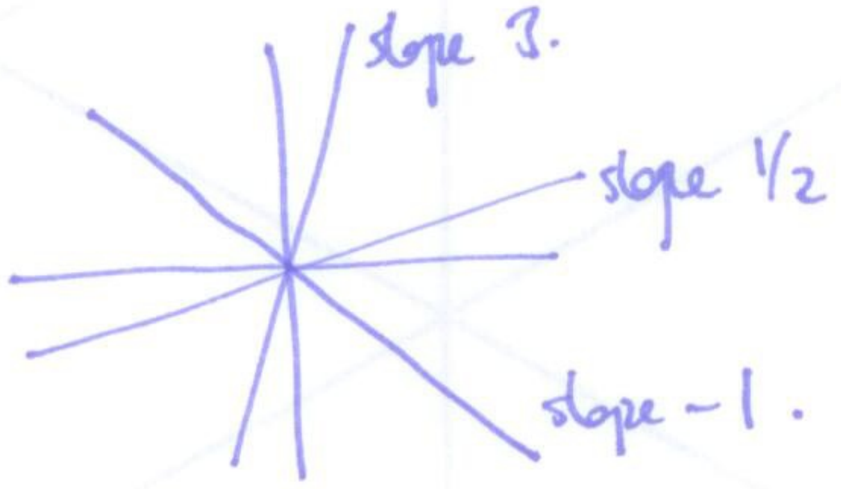


$\Delta y = f(x+\Delta x) - f(x)$ so $\Delta y = m(x+\Delta x)+b-(m(x)+b)$

$= m\Delta x.$

so slope $= \frac{m\Delta x}{\Delta x} = m$

examples

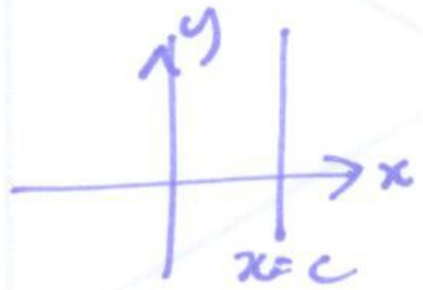


$m=0$ horizontal line

$m>0$ slopes up from left to right

$m<0$ " down " "

problem: vertical line has equation $x=c$



can't write this as $y=mx+b$

(vertical line is not graph of a function of x!)

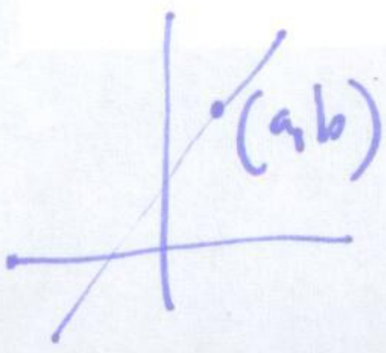
sometimes say vertical line has "infinite" slope

general linear equation: $ax+by=c$ (a,b not both zero)

(also known as symmetric equation of a line)

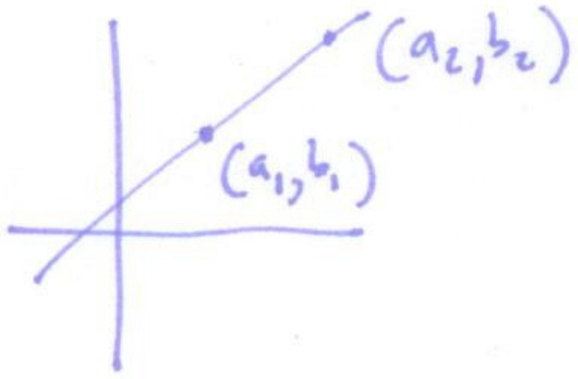
can write any line in this form

useful ways of writing down equations of lines:



line through (a,b) with slope m:

$$y-b = m(x-a)$$

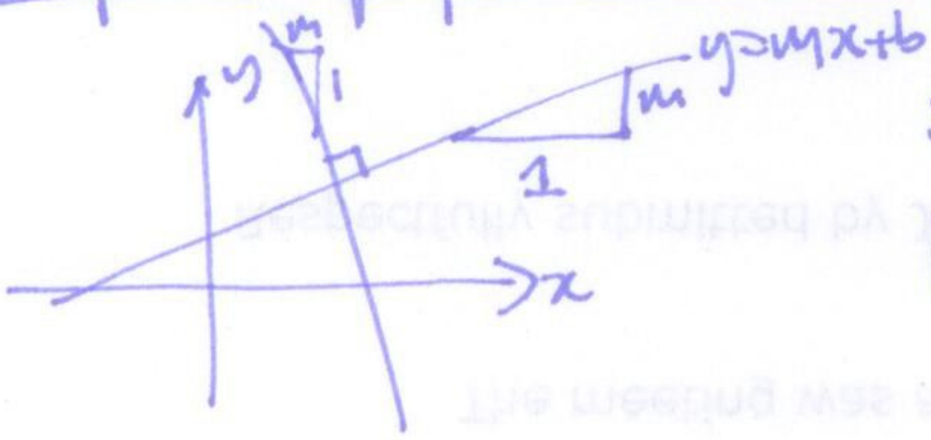


Line through two points:

find slope: $m = \dfrac{b_2-b_1}{a_2-a_1}$

then $y-b_1 = m(x-a_1)$

Slope of perpendicular lines

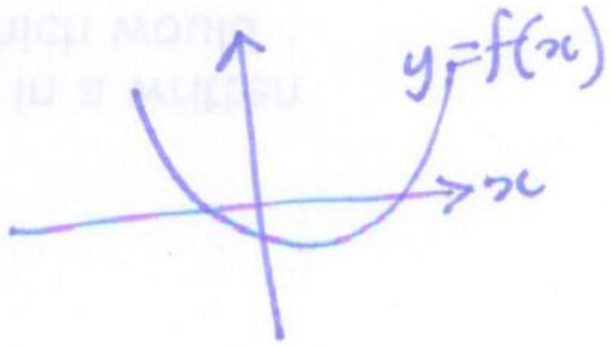


slope of perpendicular line to $y=mx+b$ has slope $-\frac{1}{m}$.

Quadratic functions

A <u>quadratic function</u> is a graph defined by a quadratic function equation: $f(x) = ax^2+bx+c$ (a,b,c numbers, $a\neq 0$)

the graph of a quadratic function is a parabola



there are at most two distinct real soln's to $f(x)=0$, given by $x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

no real solutions if $b^2-4ac<0$ one real soln if $b^2-4ac=0$ (always two complex soln's (may be the same thing)).

<u>factorization</u>: $ax^2+bx+c = a(x-r_1)(x-r_2)$ r_1,r_2 roots.