Math 230 Calculus I / Precalc.

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Th 3:35 – 4:25

- math tutoring: 1S – 214
- students with disabilities

Text: Calculus, Early Transcendentals by Rogawski

HW: webworks

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§1.1 Numbers, functions, graphs

integers $\mathbb{Z}$ $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

rationals $\mathbb{Q}$ p/q $p, q \in \mathbb{Z}$ $(q \neq 0)$.

reals $\mathbb{R}$ · can be written as decimal numbers. e.g. 4 $4.0000\cdots$, $1/4$ $0.25000\cdots$ } finite decimal

rationals correspond to repeating decimals. $1/3$ $0.333\cdots$ $0.\overline{3}$

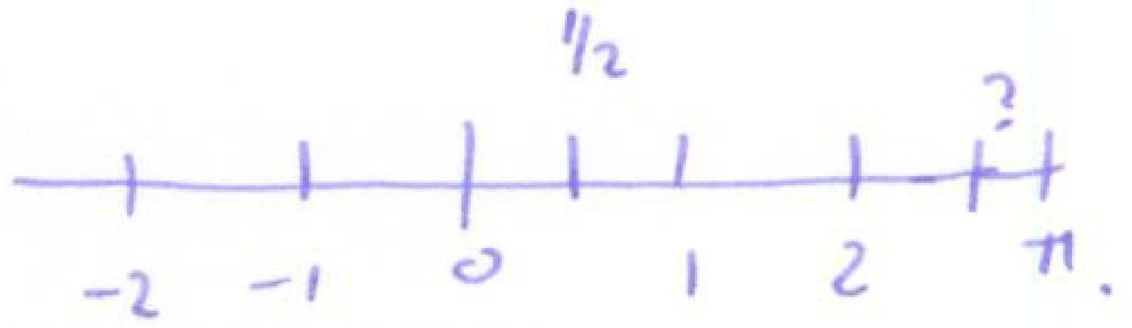
irrationals correspond to non-repeating decimals.

$\sqrt{2} = 1.41\cdots$

$\pi = 3.141\cdots$

$0.1010010001\cdots$

- reals numbers can be drawn as the number line



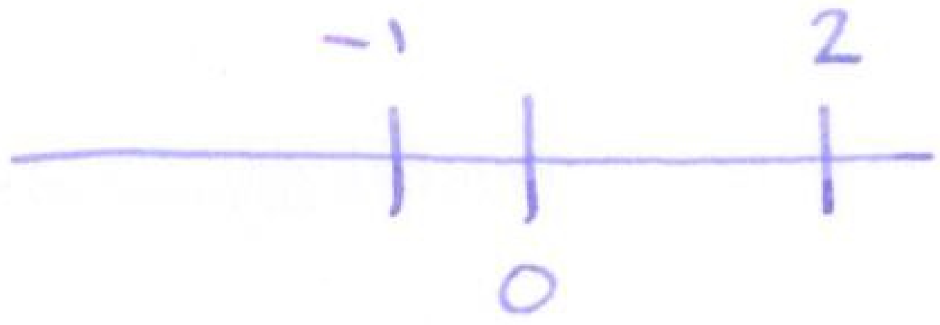
- decimal expansion of a number is unique, with the following exception:

every finite decimal is equal to an infinite decimal ending in 9's.

i.e. $1 = 0.9999\cdots$ $\frac{1}{4} = 0.25 = 0.249999\cdots$

- absolute value : if $a \in \mathbb{R}$ then $|a|$ = distance to 0

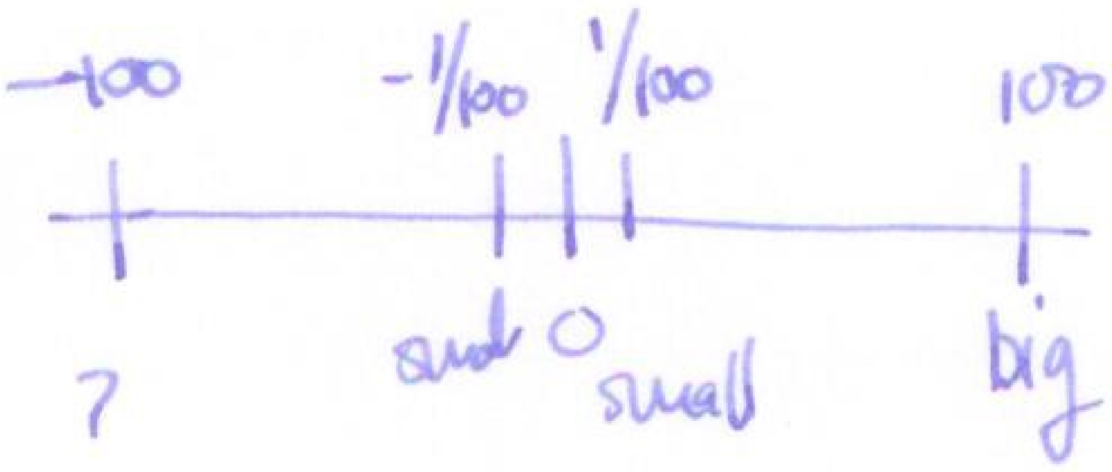
$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$



$|2| = 2.$

$|-1| = 1.$

Language problem :



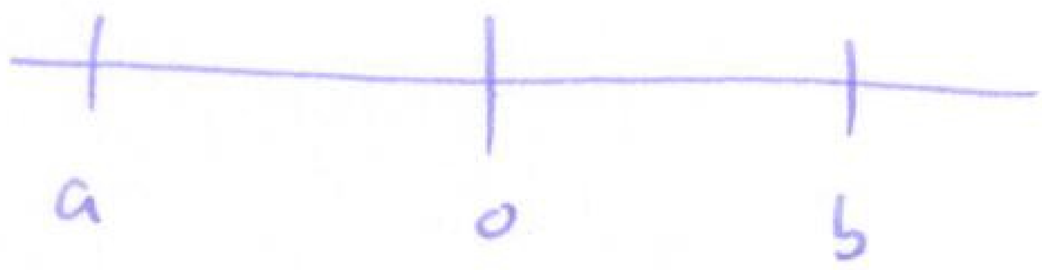
big magnitude
but negative

useful facts :

$|-a| = |a|$

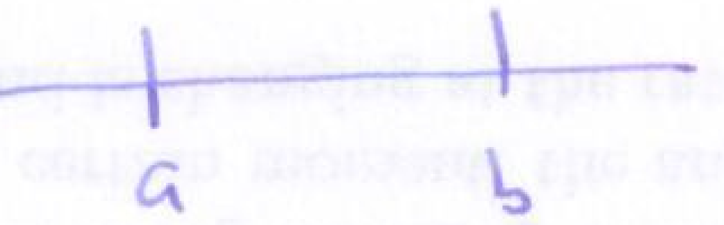
$|ab| = |a||b|$

$|a+b| \leq |a| + |b|$



distance from a to b is $|b-a| = |a-b|$

interval notation :

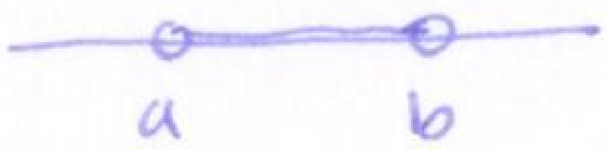


closed interval $[a,b]$

$\{x \mid a \leq x \leq b\}$

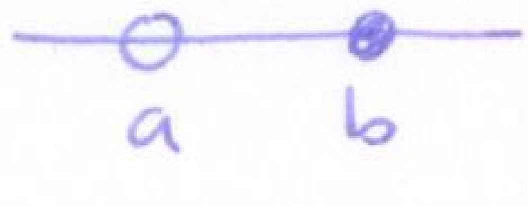
includes endpoints

open interval (a,b)

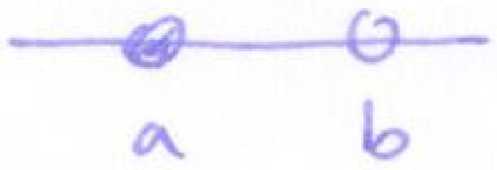


$\{x \mid a < x < b\}$ doesn't include endpoints

half-open intervals



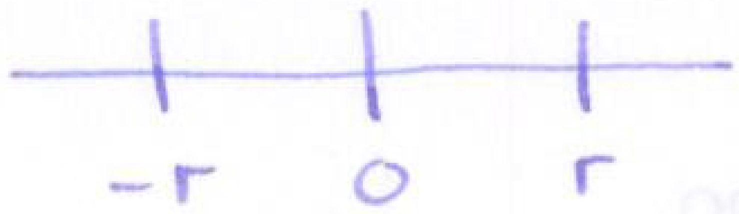
$(a,b]$ $\{x \mid a < x \leq b\}$.



$[a,b)$ $\{x \mid a \leq x < b\}$

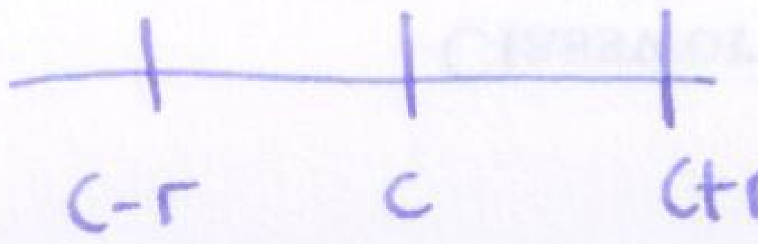
describing intervals with inequalities :

$\{x \mid |x| \leq r\}$

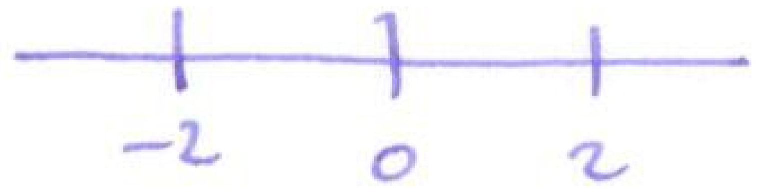


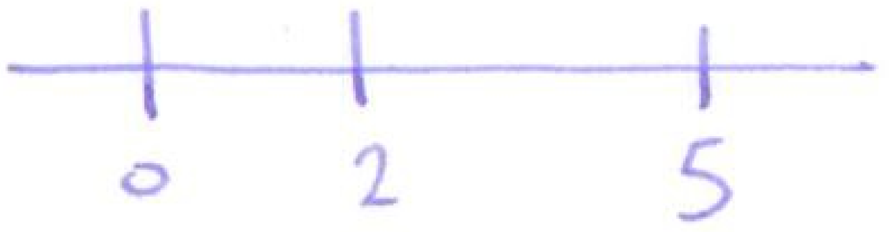
$|x|$ = distance from zero

$\{x \mid |x-c| \leq r\}$



$|x-c|$ = distance from c.

Examples  $[-2,2]$ $|x| \le 2$ & $-2 \le x \le 2$

 $[2,5]$ $2 \le x \le 5$ $|x - \frac{7}{2}| \le \frac{3}{2}$.

$|x - 3\frac{1}{2}| \le 1\frac{1}{2}$.

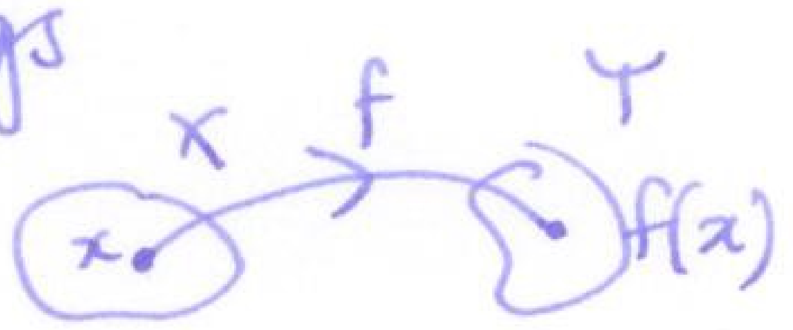
Exercise convert $[a,b]$ to distance notation.

Also allow infinite intervals:  $[-2, \infty)$ $-2 \le x$.

Functions to define a function need 3 things

- domain X
- target set Y
- for each element $x \in X$ a unique $f(x) \in Y$. called the *image* of x.

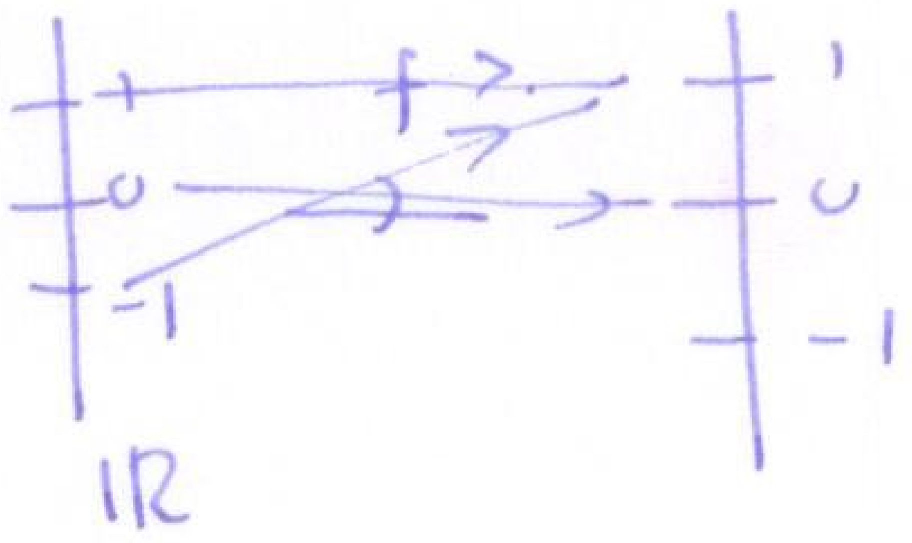
$X \xrightarrow{f} Y$.



Examples $f: \mathbb{R} \to \mathbb{R}$

$x \mapsto x^2$

$f(x) = x^2$



other examples

$\cos(x)$

$1/x$ $\sqrt{x}$.

$f(x) = \begin{cases} 0 & x \notin \mathbb{Q} \\ 1 & x \in \mathbb{Q} \end{cases}$

- the *range* of a function is all possible values of $f(x)$

(may be smaller than the target set!).

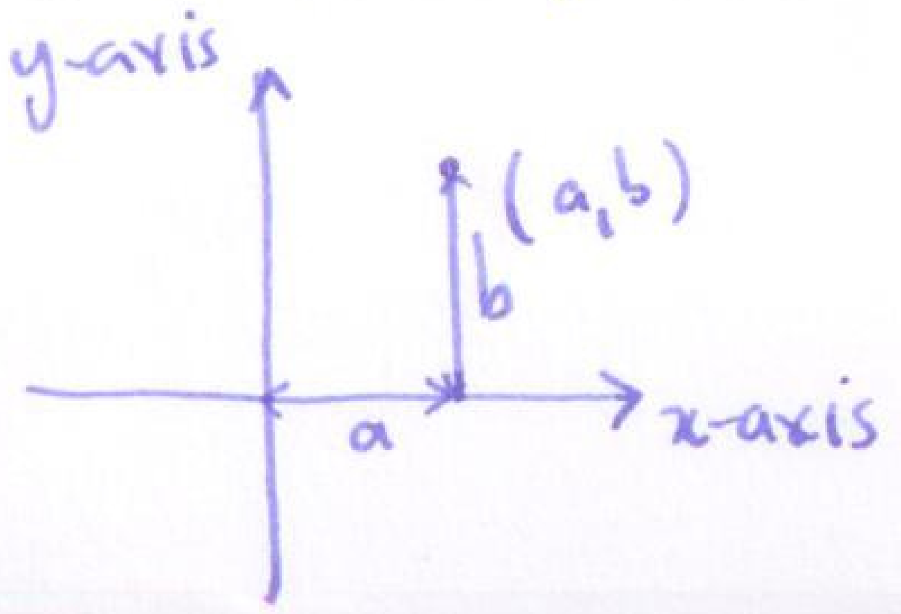
Example $f(x) = x^2$ [*warning*: often expected to guess domain and target set!]

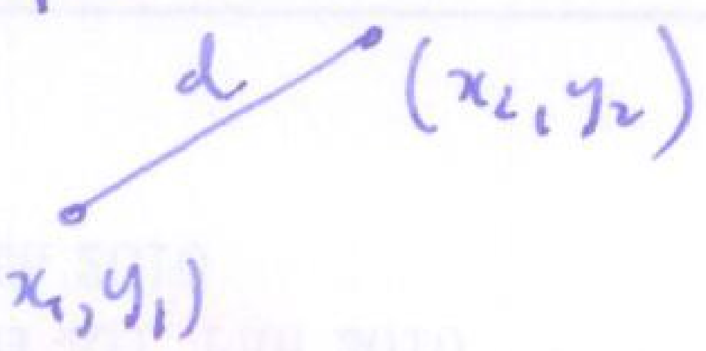
domain: $\mathbb{R}$ range: all $y \in \mathbb{R}$ s.t. $y = x^2$ for some $x \in \mathbb{R}$.

so range is $[0, \infty)$

Graphing / pictures of functions

Points in the plane: cartesian coordinates.



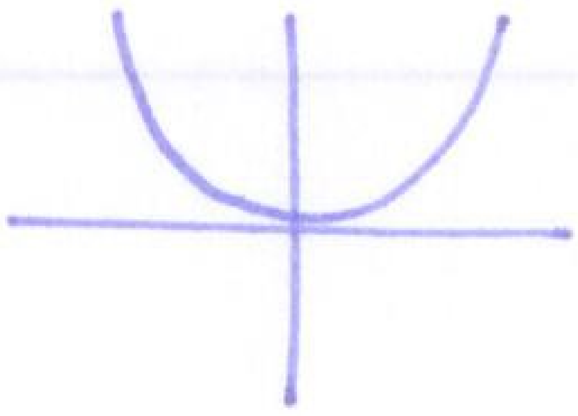
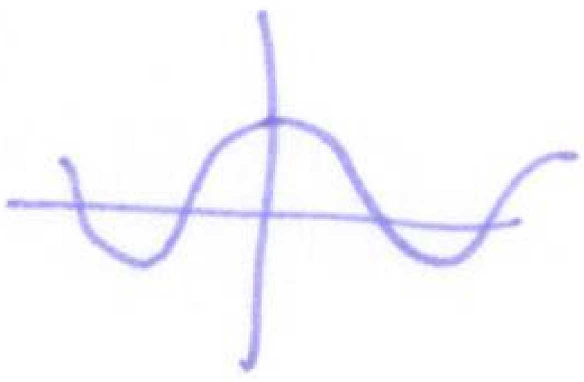
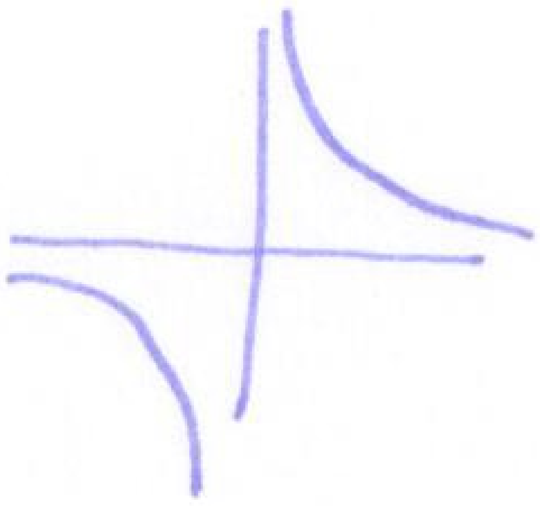
distance: 

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

given a function $f: \mathbb{R} \to \mathbb{R}$ we can draw the graph of the function.

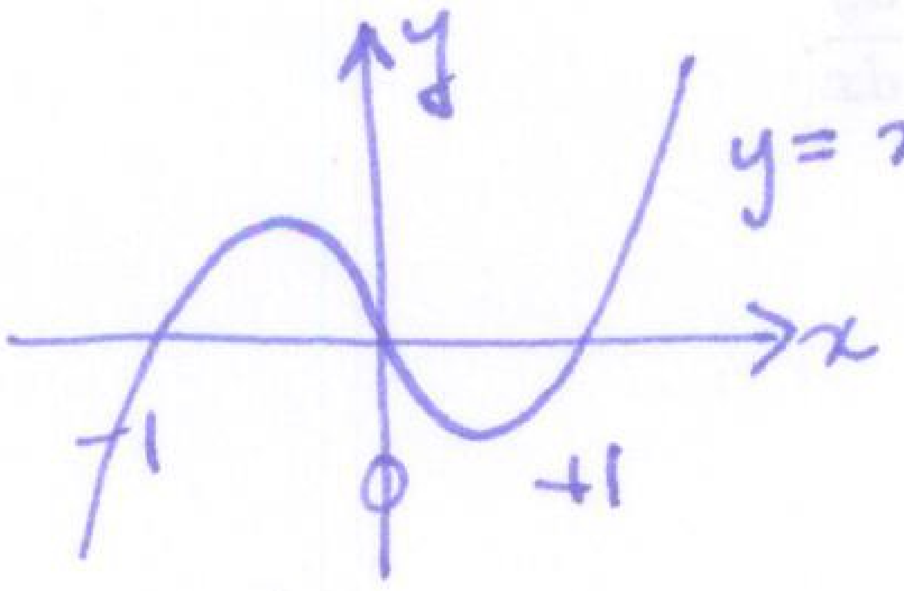
graph of f: $\{ (x, f(x)) \in \mathbb{R}^2 \mid x \in \mathbb{R} \}$

graph vs equation

Examples		domain	range.
$x \mapsto x^2$		$\mathbb{R}$	$[0,\infty)$
$x \mapsto \cos(x)$		$\mathbb{R}$	$[-1,1]$
$x \mapsto 1/x$		$\mathbb{R} \setminus \{0\}$. or $(-\infty,0) \cup (0,\infty)$	$\mathbb{R} \setminus \{0\}$ $(-\infty,0)\cup(0,\infty)$.

Properties of functions

Example $f(x) = x^3 - x$



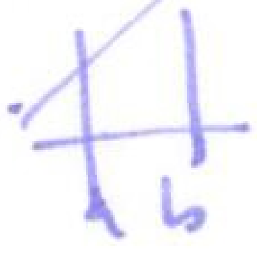
$y = x^3 - x$

Roots/zeros values of x for which $f(x) = 0$.

solve: $f(x) = 0$ example:

$$x^3 - x = 0$$
$$x(x^2 - 1) = 0$$
$$x(x-1)(x+1) = 0 \qquad x = 0, -1, +1.$$

A function f is increasing on an interval (a,b) if for any $x_1, x_2 \in (a,b)$ with $x_1 \le x_2$ then $f(x_1) \le f(x_2)$



f is decreasing on an interval (a,b) if for any $x_1, x_2 \in (a,b)$ with $x_1 \le x_2$ then $f(x_1) \ge f(x_2)$