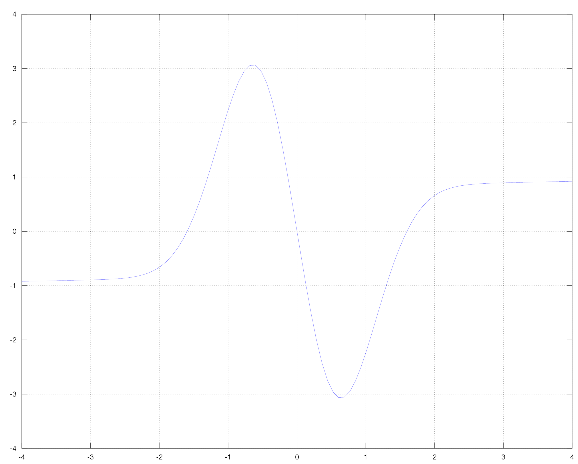
Math 230 Calculus 1 Fall 11 Sample Midterm 3

(1) The length of Big Ben's hour hand is 2.7m, and the length of its minute hand is 4.3m. How fast is the distance between the ends of the hands changing at 9 o'clock?

(2) The value of $\tan^{-1}(1)$ is $\pi/4$. Use a linear approximation to estimate $\tan^{-1}(0.9)$. Is this a good approximation?

(3) Consider the function $f(x)$ defined by the following graph.



(a) Label all regions where $f'(x) < 0$.
(b) Label all regions where $f'(x) > 0$.
(c) What is $\lim_{x\to\infty} f'(x)$?
(d) What is $\lim_{x\to-\infty} f''(x)$?
(e) Sketch a graph of $f'(x)$ on the figure.
(f) Label the approximate locations of all points of inflection.

(4) True or False. Indicate whether the following statements are True or False.
(a) Assume $f(x)$ is differentiable for all x. If $f'(c) = 0$ for some c, then $f(x)$ has a local maximum or minimum at $x = c$.
(b) If the relative maxima of some differentiable function $f(x)$ are $f(1) = 2$ and $f(5) = 10$ then $f'(c) = 2$ for some c in the interval $(1, 5)$.
(c) If $f'(x) > 0$ for all values of x, then $\lim_{x\to\infty} f(x)$ does not exist.
(d) If $f'(c) = 0$ and $f''(c) < 0$, the function $f(x)$ has a local minimum at $x = c$.

(5) Sketch a graph of a differentiable function f that satisfies the following conditions and has $x = 1$ as its only critical point.

$$\begin{aligned} f(1) &= 10 \\ f'(1) &= 0 \\ f'(x) &> 0 \text{ for } x < 1 \\ f'(x) &< 0 \text{ for } x > 1 \\ \lim_{x\to\infty} f(x) &= \lim_{x\to-\infty} f(x) = 5 \end{aligned}$$

(6) A function $f(x)$ has derivative

$$f'(x) = \frac{1}{x^4+1}.$$

Where on the interval $[1, 4]$ does it take its minimum value?

(7) Consider the following function:

$$g(x) = (x^2 - x)e^{-x}$$

(a) Find, if they exist, the coordinates of all relative maxima and minima.
(b) Determine the interval(s) where g is increasing and those where g is decreasing.
(c) Find, if they exist, the coordinates of all points of inflection.
(d) Determine the intervals where g is concave up and those where g is concave down.
(e) Sketch the curve as accurately as possible.

(8) Consider the function

$$f(x) = \frac{x}{x^3+1}$$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.
(d) Use the 2nd derivative test to attempt to identify all local maxima and minima.
(e) Sketch the function and label all relative maxima and minima.

(9) The equation for a circle of unit radius is $x^2 + y^2 = 1$. In the first quadrant, this implies that $y = \sqrt{1 - x^2}$. Find the dimensions of the rectangle of largest area, with sides parallel to the coordinate axes, that one can inscribe between the x-axis and the circle in the first quadrant. What is the maximum area?

(10) Compute the following limits. Show all work.

(a) $\lim_{x\to\infty} \frac{6x^5-12x^4-24}{2x^5+30}$

(b) $\lim_{x\to-\infty} \frac{x^2}{1-x^2}$

(c) $\lim_{x\to\infty} \frac{3x}{\sqrt{x^2+5}}$

(d) $\lim_{x\to 0} \frac{e^{2x^2}-1}{1-\cos(3x)}$