Part I: Answer all questions in this part in the space provided. No credit will be allowed if work is not shown. Each question is worth 6 points.

NAME:

1. If \( f(x) = \frac{4}{x + 7} \)
   
   a. Sketch a graph of the function and determine that the function is one-to-one.
   
   b. Find \( f^{-1}(x) \) and simplify your answer.

2. A triangle has sides which measure \( a = 4.3, \quad b = 2.5, \quad c = 3.8 \) Find the measure of the largest angle only to the nearest tenth of a degree.

   1. 
   
   b. 

   2.
3. Prove the identity: \[ \frac{\tan(\theta) + \cot(\theta)}{\csc(\theta)} = \sec(\theta) \].

4. Given the complex number \( z = 4(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right)) \), compute \( z^4 \) first in trigonometric form and then convert your answer to standard form.

4. Trig form \[ \text{__________} \]
Standard form \[ \text{__________} \]

5. Find \( \tan(\cos^{-1}\left(\frac{3}{x}\right)) \)
6. If \( \tan(\theta) = -\frac{3}{4} \) and \( \theta \) is in quadrant IV, use suitable identities to find the values of the following:

(No credit will be allowed unless answers are written as fractions.)

\[
\begin{align*}
\text{a) } \sin 2\theta & \quad \text{b) } \cos 2\theta \\
\end{align*}
\]

\[
\begin{align*}
\sin 2\theta &= \\
\cos 2\theta &= \\
\end{align*}
\]

7. Use synthetic division to find the quotient and the remainder.

\[
(x^3 + 2x^2 - 8) \div (x + 3)
\]

7. Quotient

Remainder
8. Solve the inequality \( \frac{x + 1}{x - 2} \geq 3 \). Write your answer in interval notation.

9. Write an equation of a function that has the shape of \( y = \sqrt{x} \), but stretched vertically by a factor of 4, then shifted left 2 units and then down 1 unit.

10. Determine whether the functions are even, odd or neither. Explain your answer.

   a. \( f(x) = x^3 - 5x \)

   b. \( g(x) = x^4 + 3x \)
Math 130 Final Exam (continued)

Part II: Answer only five questions in this part (8 points each). Cross out those questions that you do not wish to answer. If you answer more than five questions, only the first five will be graded.

11. Solve: \( \sin(2x) + \cos(x) = 0 \) for all solutions in \([0, 2\pi]\).

12. In the accompanying diagram of \( \triangle ABC \), \( A = 60^\circ, C = 75^\circ \) and the side opposite vertex C is 10m. Find the length of the side opposite vertex B and find the area of \( \triangle ABC \).

\[ \text{12. Side} \]  
\[ \text{Area} \]
13. For the ellipse find the center, vertices and foci. Then draw the graph.

\[ 9x^2 + y^2 - 18x + 4y - 23 = 0 \]

13. center ____________

Vertices ____________

Foci ____________

14. Solve the following system of equations for x and y:

\[ x^2 + 4y^2 = 25 \]

\[ x + 2y = 7 \]

14. ______________
15. Simplify:
   a) \( \frac{2 + 5i}{4 - i} \)
   b) \((3 + i)(6 - 4i)\)

16. Consider the \( p(x) = x^3 - 2x + 4 \).
   a) Give a complete list of all possible rational zeros.
   b) Find all the zeros
Math 130 Final Exam (continued)

17. Consider the function: 
\[ f(x) = \frac{2x - 5}{x^2 + 3x - 10} \]

a) The coordinates of the x-intercept(s) 

b) The coordinates of the y-intercept(s) 

c) The equation(s) of the vertical asymptote(s) 

d) The equation(s) of the horizontal asymptote(s) 

e) Sketch the graph of \( f(x) \) together with all the points and lines found above.
18. Given that $y = 4 \cos \left( 2x - \frac{\pi}{4} \right)$,

find the following:

a) The amplitude

b) The period

c) The phase shift

d) Sketch the graph

19 a) 

b) 

c) 

d) 

Formulas

Area of Triangle

\[ K = \frac{1}{2} ab \sin C \]

Functions of the Sum of Two Angles

\[ \sin(A + B) = \sin A \cos B + \cos A \sin B \]
\[ \cos(A + B) = \cos A \cos B - \sin A \sin B \]

Functions of the Difference of Two Angles

\[ \sin(A - B) = \sin A \cos B - \cos A \sin B \]
\[ \cos(A - B) = \cos A \cos B + \sin A \sin B \]

Law of Sines

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

Law of Cosines

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

Functions of the Double Angle

\[ \sin 2A = 2 \sin A \cos A \]
\[ \cos 2A = \cos^2 A - \sin^2 A \]
\[ \cos 2A = 1 - 2 \sin^2 A \]

Functions of the Half-Angle

\[ \sin \frac{1}{2} A = \pm \sqrt{\frac{1 - \cos A}{2}} \]
\[ \cos \frac{1}{2} A = \pm \sqrt{\frac{1 + \cos A}{2}} \]

Conic Sections

Circle:

\[ (x - h)^2 + (y - k)^2 = r^2 \]

Ellipse:

\[ \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1, \]
\[ \frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1 \]

Parabola:

\[ (x - h)^2 = 4p(y - k), \]
\[ (y - k)^2 = 4p(x - h) \]

Hyperbola:

\[ \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1, \]
\[ \frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1 \]
Important Properties and Formulas

Basic Identities
\[
\begin{align*}
\sin x &= \frac{1}{\csc x}, & \sin (-x) &= -\sin x, \\
\cos x &= \frac{1}{\sec x}, & \cos (-x) &= \cos x, \\
\tan x &= \frac{1}{\cot x}, & \tan (-x) &= -\tan x \\
\cot x &= \frac{\cos x}{\sin x}, & \\
\csc x &= \frac{1}{\sin x}.
\end{align*}
\]

Pythagorean Identities
\[
\begin{align*}
\sin^2 x + \cos^2 x &= 1, \\
1 + \cot^2 x &= \csc^2 x, \\
1 + \tan^2 x &= \sec^2 x.
\end{align*}
\]

Sum and Difference Identities
\[
\begin{align*}
\sin (u \pm v) &= \sin u \cos v \pm \cos u \sin v, \\
\cos (u \pm v) &= \cos u \cos v \mp \sin u \sin v, \\
\tan (u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}.
\end{align*}
\]

Cofunction Identities
\[
\begin{align*}
\sin \left( \frac{\pi}{2} - x \right) &= \cos x, \\
\tan \left( \frac{\pi}{2} - x \right) &= \cot x, \\
\sec \left( \frac{\pi}{2} - x \right) &= \csc x.
\end{align*}
\]

Double-Angle Identities
\[
\begin{align*}
\sin 2x &= 2 \sin x \cos x, \\
\cos 2x &= \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1, \\
\tan 2x &= \frac{2 \tan x}{1 - \tan^2 x}.
\end{align*}
\]

Half-Angle Identities
\[
\begin{align*}
\sin \frac{x}{2} &= \pm \sqrt{\frac{1 - \cos x}{2}}, \\
\cos \frac{x}{2} &= \pm \sqrt{\frac{1 + \cos x}{2}}, \\
\tan \frac{x}{2} &= \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}.
\end{align*}
\]

Inverse Trigonometric Functions
\[
\begin{align*}
\text{FUNCTION} & \quad \text{DOMAIN} & \quad \text{RANGE} \\
y = \sin^{-1} x & \quad [-1, 1] & \quad \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \\
y = \cos^{-1} x & \quad [-1, 1] & \quad [0, \pi] \\
y = \tan^{-1} x & \quad (-\infty, \infty) & \quad \left( -\frac{\pi}{2}, \frac{\pi}{2} \right)
\end{align*}
\]

(continued)