NAME: _____

MATH 130 **FINAL EXAM** Spring 2007 Form M

Answer questions in the space provided below

Part I: Answer ALL ten questions worth 6 points each

1. If $f(x) = -x^2 - 3x + 1$ and $g(x) = 2x + 1$, compute and simplify $(f \circ g)(x) = ?$

2. If $f(x) = \dfrac{3x}{x+2}$, find and simplify $f^{-1}(x) = ?$

3. **Sketch** the **graph** of $f(x) = \sqrt{x + 20} + 30$. State **domain** and **range** of f.

4. Find an **equation** of the graph obtained by reflecting the graph of $f(x) = |x|$ upside down and then shifting it 3 units up and 4 units right. Do **not** graph.

5. **Solve** the inequality $\frac{x-2}{(x-3)(x+1)} \geq 0$. Write your answer in **interval notation**

6. **Prove** the identity: $\tan\theta + \cot\theta = \sec\theta \csc\theta$

7. **Sketch one period** of the graph $y = -200\cos(\frac{x}{2} + \frac{\pi}{8})$. Label the lowest points, the highest points and the x-intercepts of the graph with their coordinates.

8. A triangle has the following sides: a = 34.7 ft, b = 52.6 ft, c = 43.5 ft. Find the **measure** of its **biggest angle** only (round to two decimal places).

9. If $\sin u = -\frac{5}{13}$, (u in quadrant III) use a suitable identity to find the **exact value** of $\tan(2u)$. Write your answers as a simple fraction.

10. **Evaluate** (in terms of x) $\cos(\arctan\frac{x}{3}) = ?$

Part II: Answer ANY FIVE questions (worth 8 points each); cross out the two questions you choose not to answer.

11. If $f(x) = \frac{x(x-2)}{2x^2-2}$ find:

(if any item does not exist, write "NONE")

(a) the **coordinates** of the **x-intercept(s)**: _____

(b) the **coordinates** of the **y-intercept**: _____

(c) the **equation** of the **vertical asymptote**(s): _____

(d) the **equation** of the **horizontal asymptote**: _____

(e) **sketch** the **graph** of f together with all the points and lines found above:

12. Find all **solutions** x (in **radians**) in the interval $[0,2\pi)$:

$$2\cos^2 x - \cos x = 0$$

13. If $f(x) = 2x^3 - 5x^2 - 10x + 6$

(a) Give a complete list of **all** possible **rational zeros**:

(b) Use synthetic division to **check** that $x = \frac{1}{2}$ is a rational zero:

(c) Find **all** remaining zeros:

(d) Write f as a product of **linear factors**:

f(x) = _____

14. Use algebra to find **all solutions** of the system:

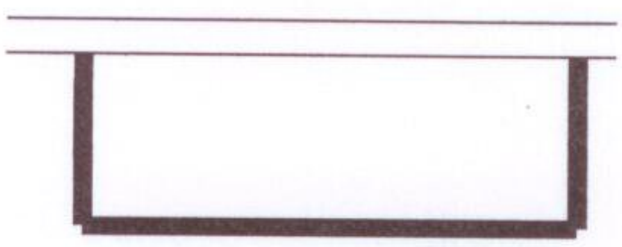
$$y^2 - x^2 = 16$$
$$2x - y = 1$$

15. **Sketch** the graph of: $4y^2 - 9x^2 - 8y - 36x = 68$. Label the **vertices** of the graph in your sketch with their coordinates.

16. A farmer has 1800 ft of fencing to enclose a rectangular field alongside a river. The side of the field along the river doesn't require any fence.

(a) Express the area of the field as a function of x only

(b) Use your calculator to find the dimensions of the field (length and width) for which the area is a maximum.



17. Given complex number $z = 2(\cos 30° + i \sin 30°)$, compute z^4 first in trigonometric form, then convert your answer to standard form.

Important Properties and Formulas

Basic Identities

$$\sin x = \frac{1}{\csc x}, \qquad \sin(-x) = -\sin x,$$

$$\cos(-x) = \cos x,$$

$$\cos x = \frac{1}{\sec x}, \qquad \tan(-x) = -\tan x$$

$$\tan x = \frac{1}{\cot x},$$

$$\tan x = \frac{\sin x}{\cos x},$$

$$\cot x = \frac{\cos x}{\sin x},$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1,$$

$$1 + \cot^2 x = \csc^2 x,$$

$$1 + \tan^2 x = \sec^2 x$$

Sum and Difference Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v,$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v,$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x,$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x,$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x,$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm\cos x,$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin x$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x,$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1,$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half-Angle Identities

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}},$$

$$\cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}},$$

$$\tan \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$$
$$= \frac{\sin x}{1 + \cos x}$$
$$= \frac{1 - \cos x}{\sin x}$$

Inverse Trigonometric Functions

Function	Domain	Range
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(continued)