Name:.............................

1. (15 points) Let $p(x) = 2x^3 + 3x^2 - 14x - 21$.

   (a) Give a complete list of all possible rational zeros.

   (b) Check, using synthetic division, whether $x = 3$ and $x = -\frac{3}{2}$ are actual rational zeros and find all remaining zeros.

   (c) Sketch the graph of p.

2. (15 points) For the following function, find its period and amplitude. Then graph one period of the function and indicate its x-intercepts, its maximum and minimum.

$$f(x) = \frac{1}{2} \cos(4x - \pi)$$

3. (10 points) If $\cos(\phi) = 12/13$ in quadrant 4, find $\sin(\phi)$ and $\sin(2\phi)$ and $\cos(2\phi)$. In which quadrant does the angle 2ϕ lie?

4. (10 points) Simplify the following term:

$$\sin(\arctan(x/5))$$

5. (15 points) Consider the function

$$f(x) = \frac{3x}{x-2}$$

and find its inverse function $f^{-1}(x)$. Sketch the graphs of both functions.

6. (10 points) Prove the following identity:

$$\frac{1 - 2\sin^2 \phi}{\sin \phi \cos \phi} = \frac{1}{\tan \phi} - \tan \phi$$

7. (10 points) Find the foci and the center of the following ellipse given by

$$x^2 + 4y^2 - 4x + 8y + 4 = 0$$

8. (15 points) Consider the function

$$f(x) = \frac{2x^2 - 4}{x^2 - 3x - 10}$$

(a) Find the domain of f, all asymptotes of f, and the zeros of f and $f(0)$.

(b) Sketch the graph of f.