Part I: Answer all ten questions worth 6 points each.

1. If \( f(x) = 3x^2 - 2x + 1 \) and \( g(x) = 3x + 1 \), compute and simplify \( (f \circ g) \).

2. If \( f(x) = \frac{x - 1}{x + 2} \), find and simplify \( f^{-1}(x) \).

3. Sketch the graph of \( f(x) = 1 - \sqrt{2-x} \). State domain and range of \( f \).
4. Find an **equation** of the graph obtained by reflecting the graph of \( f(x) = x^2 \) upside down and then shifting it 15 units up and 9 units to the right. Do not graph.

5. **Solve** the inequality \( \frac{x - 4}{x+3} - \frac{x + 2}{x-1} \leq 0 \). Write your answer in **interval notation**.

6. **Prove** the identity: \( \tan \Theta + \cot \Theta = \sec \Theta \cdot \csc \Theta \)
7. Sketch the graph of \( y = 3 \sin \left( 2x + \frac{\pi}{2} \right) + 1 \). Find the amplitude, the period, and the phase shift.

8. A triangle has the following sides: \( a = 25.4 \), \( b = 73.8 \) and \( c = 51.2 \). Find the measure of the smallest angle.

9. If \( \tan \Theta = -15 \), \( \Theta \) in quadrant II. Use a suitable identity to find the exact value of \( \cos 2\Theta \). Write your answer as a simple fraction.
10. Evaluate: $\sin \left( \frac{\cot^{-1} x}{2} \right)$

Part II. Answer any five questions (worth 8 points each). Cross out the three questions you choose not to answer.

11. If $f(x) = \frac{2x + 3}{3x^2 + 7x - 6}$ find:

   (if any item does not exist, write "NONE")

   (a) the coordinates of the $x$-intercept(s):

   (b) the coordinates of the $y$-intercept(s):

   (c) the equation of the vertical asymptote(s):

   (d) the equation of the horizontal asymptote(s):

   (e) sketch the graph of $f$ together with all the points and lines found above
12. Find all **solutions** $x$ (in **radians**) in the interval $[0, 2\pi]$:

$$\cos 2x - \sin x = 1$$
13. If \( f(x) = x^3 - 7x^2 + 11x + 3 \)
   (a) Give a complete list of all possible rational zeros:

   (b) Use synthetic division to check that \( x = 3 \) is a rational zero:

   (c) Find all remaining zeros:

   (d) Write \( f \) as a product of linear factors:

   \[ f(x) = \]
14. Use algebra to find all solutions of the system:

\[ x^2 + 4y^2 = 25 \]

\[ x + 2y = 7 \]

15. Find the center, the vertices, and the foci of:

\[ 9x^2 + 4y^2 + 18x - 16y = 11 \]. Then draw the graph. Label all points found.
16. A farmer has 160 feet of fencing to enclose two adjacent rectangular pig pens.
(a) Express the enclosed area as a function of $x$ only:

```
|-----x-----|-----x-----|
```

\[
y \quad y \\
\hline
x \quad x
\]

(b) What dimensions should be used so that the enclosed area will be a maximum?

17. Given complex number: $z = 6(\cos 60^0 + i \sin 60^0)$, compute $z^4$ in trigonometric form, then convert your answer to standard form.
18. Prove the following identity:

\[ 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n + 1)(2n + 1)}{6} \]
18. Prove the following identity:

\[
\frac{(1 + n)(1 + n + 1)}{2} = \frac{1^2 + 2^2 + 3^2 + \ldots + n^2}{n}
\]