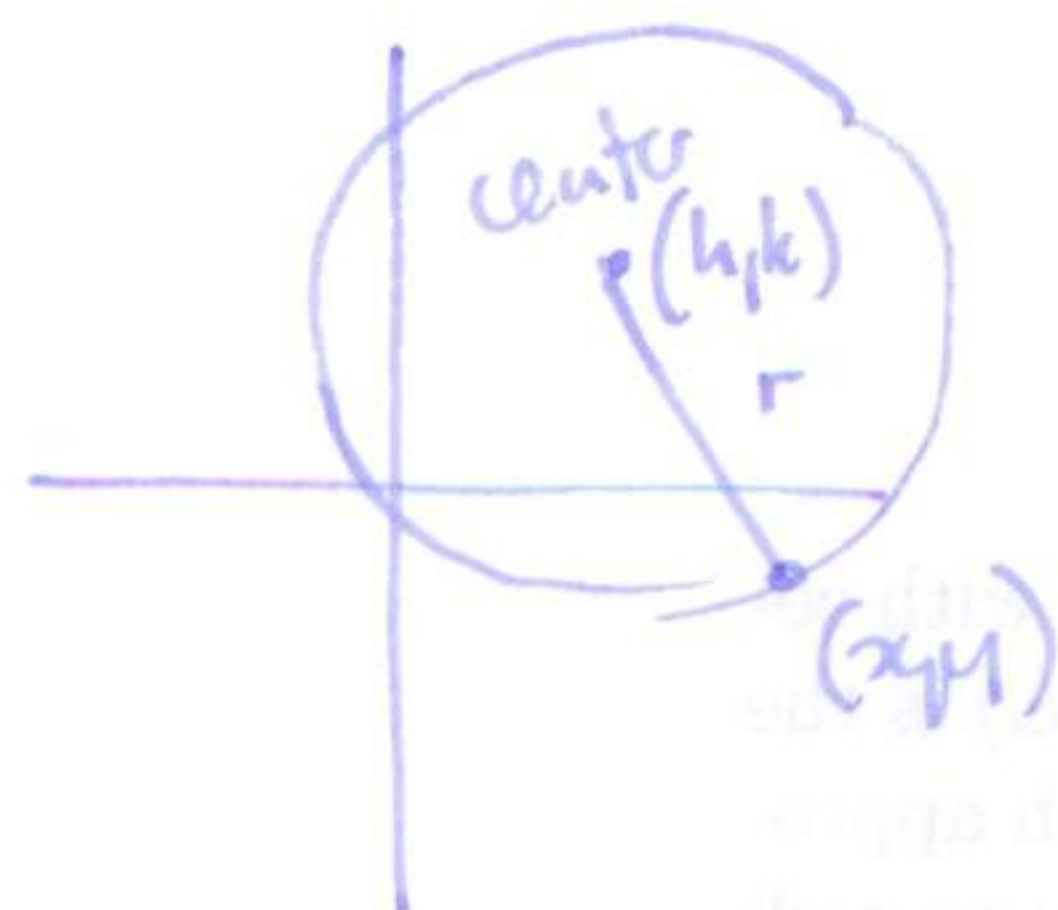


§10.2 Circles/ellipses

(56)

circle:

all points distance r from a center (h, k) .



$$r^2 = (x-h)^2 + (y-k)^2$$

Example

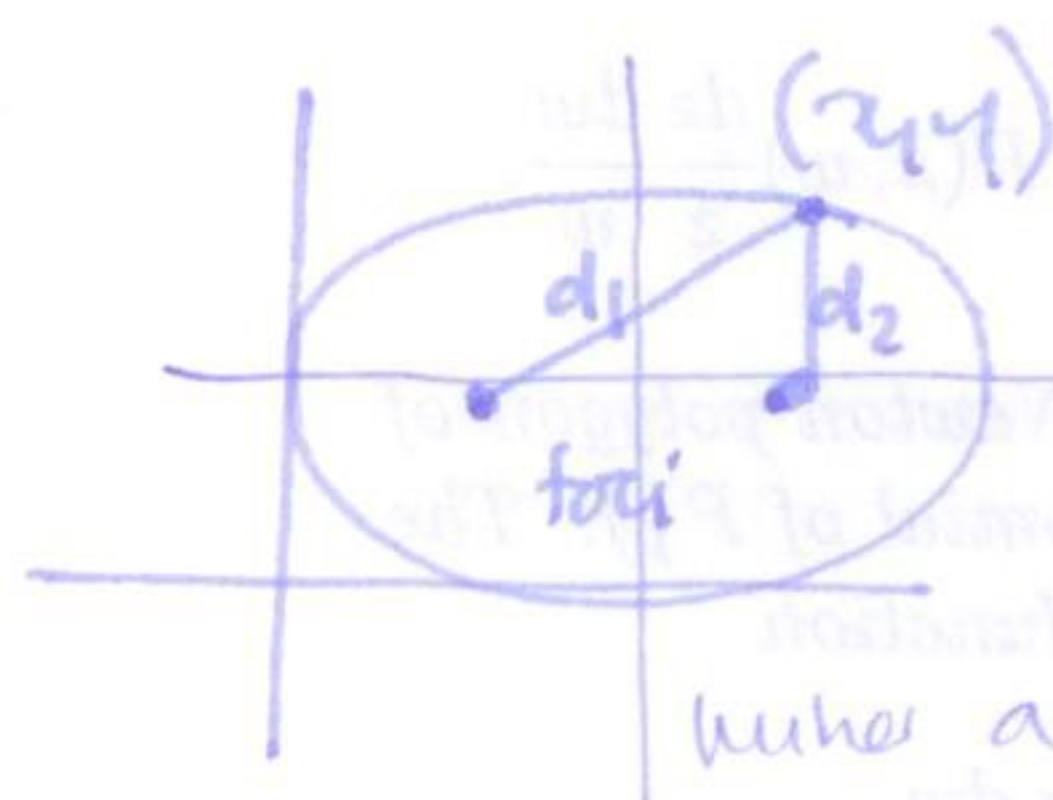
$$x^2 + y^2 - 16x + 14y + 32 = 0$$

Find center and radius: complete the square!

$$(x-8)^2 - 64 + (y+7)^2 - 49 + 32 = 0$$

$$(x-8)^2 + (y+7)^2 = 81 \quad \text{center } (8, -7) \text{ radius } 9.$$

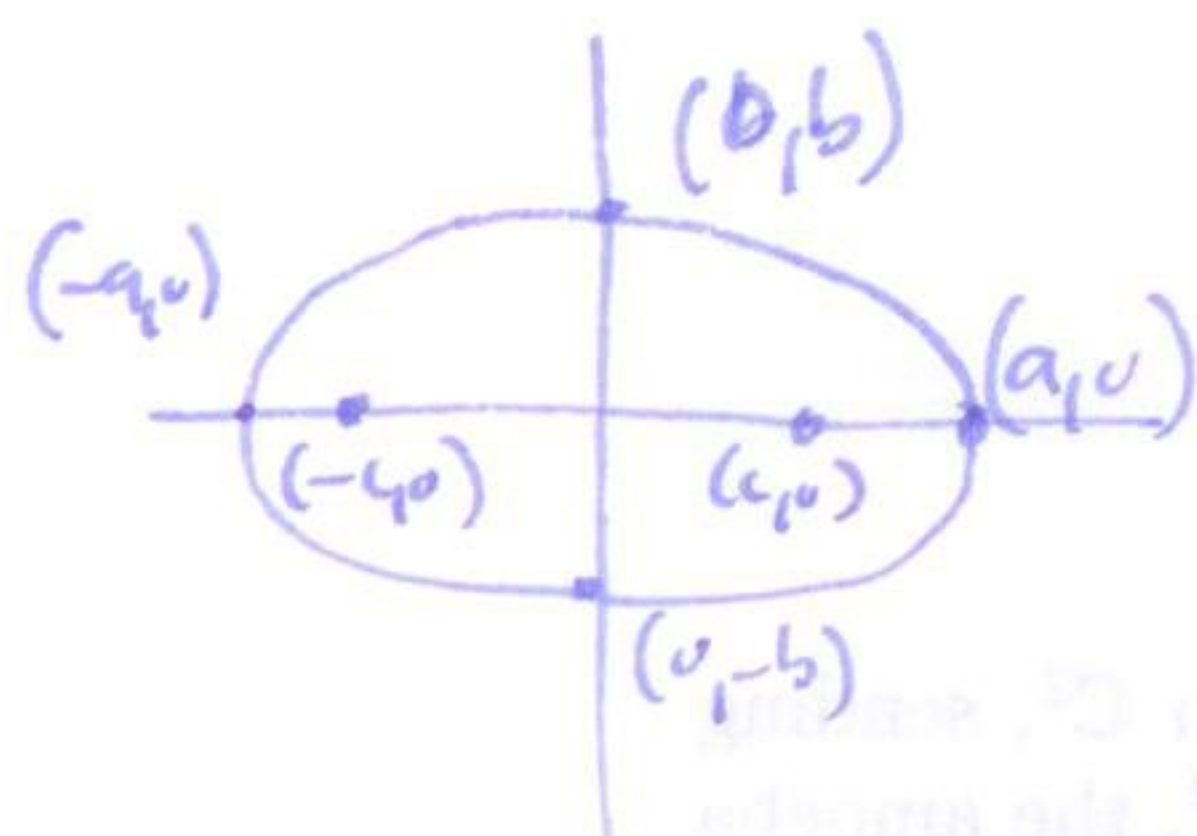
Ellipses



$$d_1 + d_2 = \text{constant}$$

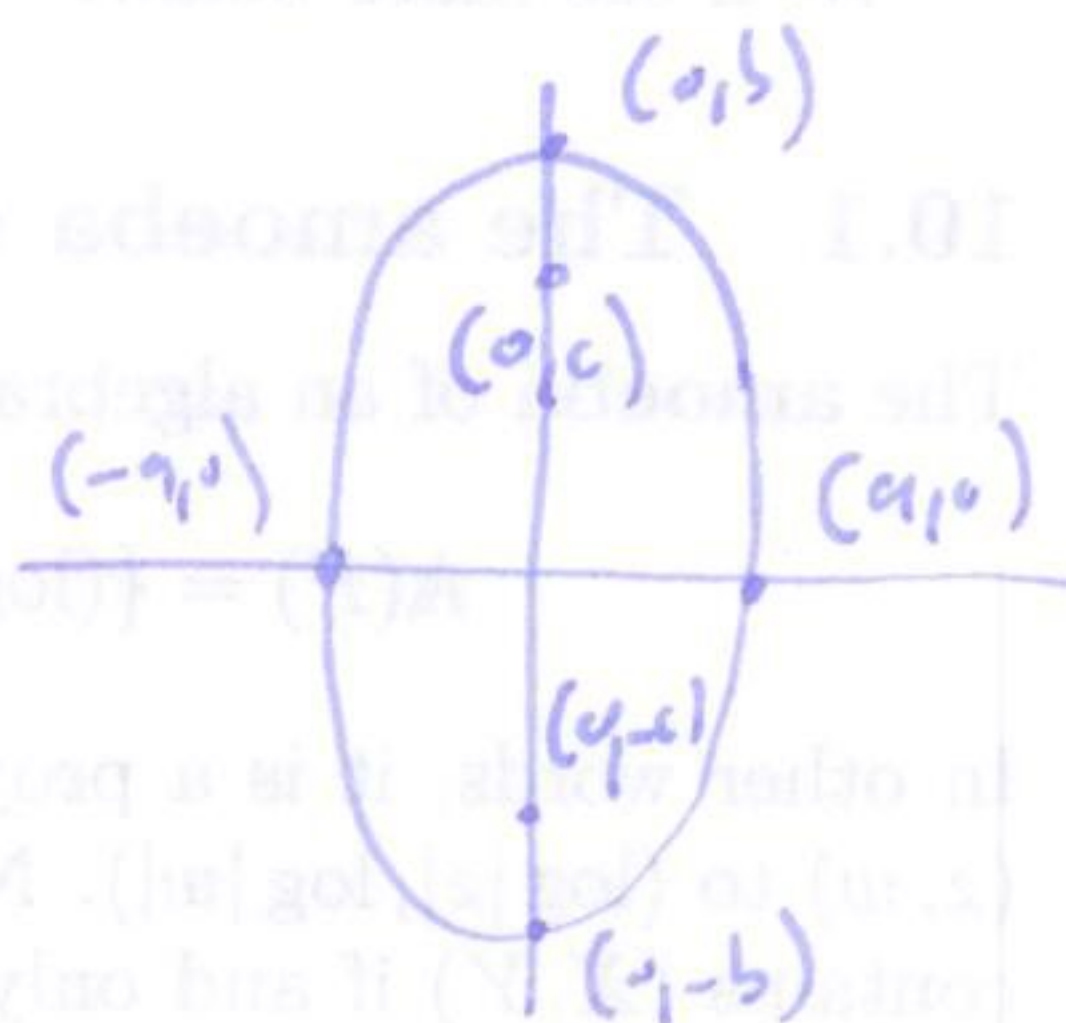
major axis horizontal

major axis vertical



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a > b > 0.$$

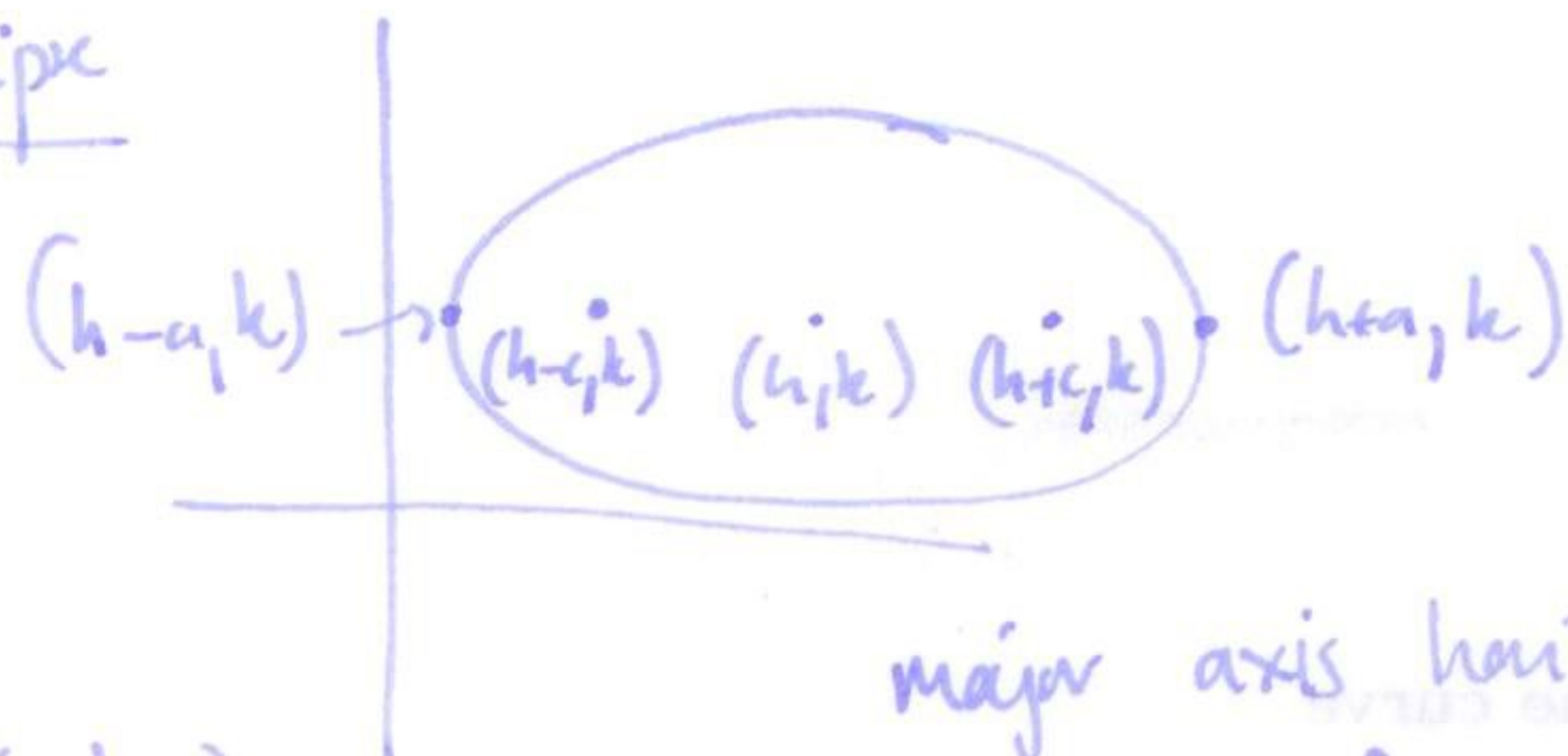
$$c^2 = a^2 - b^2$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad b > a > 0.$$

$$c^2 = b^2 - a^2$$

general ellipse

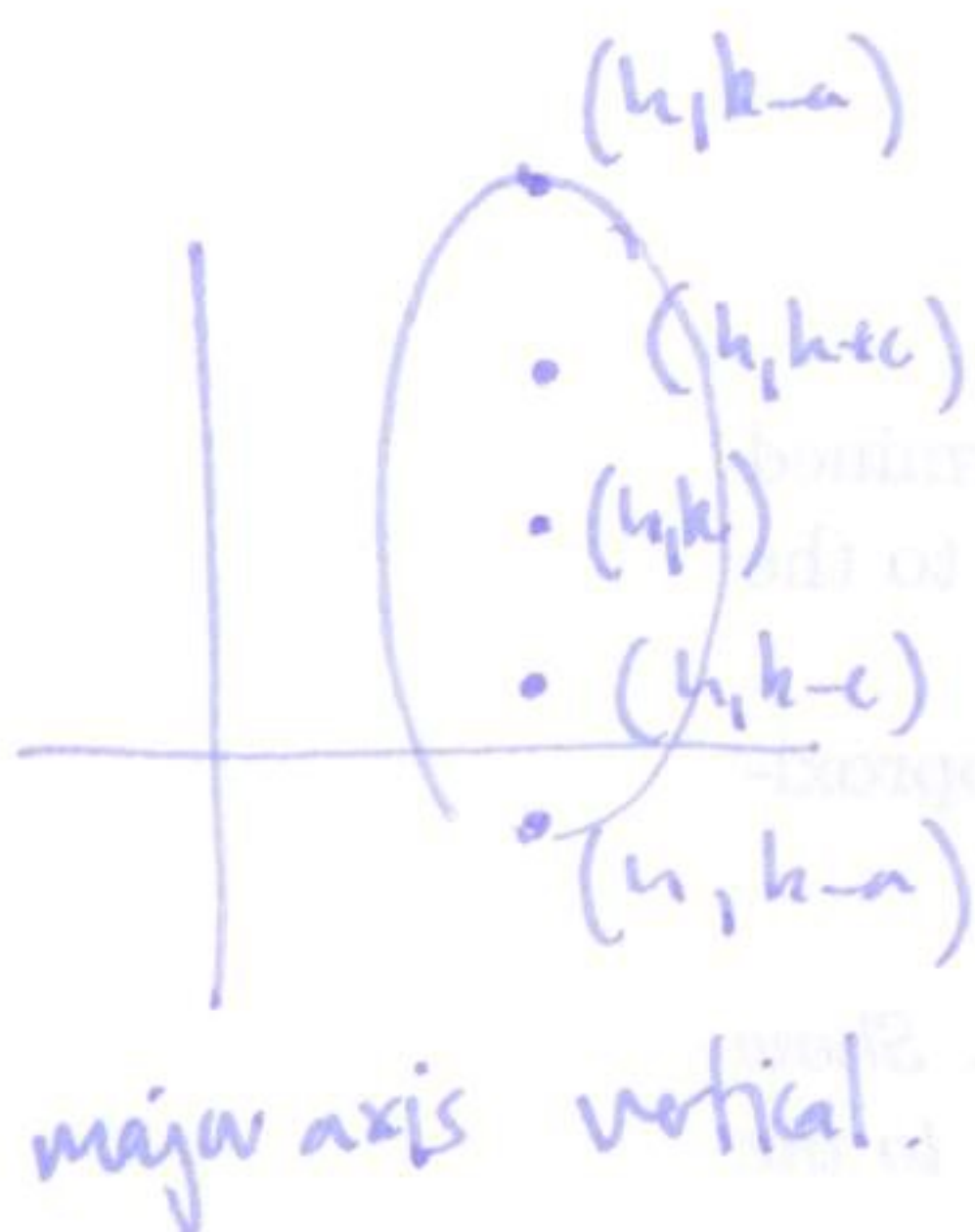


$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$a > b > 0$$

major axis horizontal.

$$\text{foci } (h-c, k) \text{ and } (h+c, k) \quad c^2 = a^2 - b^2$$



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad b > a > 0$$

major axis vertical.

$$\text{foci } (h, k-c) \text{ and } (h, k+c) \quad c^2 = b^2 - a^2$$

Example

$$4x^2 + y^2 + 24x - 2y + 21 = 0 \quad \text{complete the square!}$$

$$4(x^2 + 6x) + y^2 - 2y + 21 = 0$$

$$4(x+3)^2 - 9 + (y-1)^2 - 1 + 21 = 0$$

$$4(x+3)^2 + (y-1)^2 = 1 - 21 + 36 = 16$$

$$\frac{(x+3)^2}{2^2} + \frac{(y-1)^2}{4^2} = 1$$

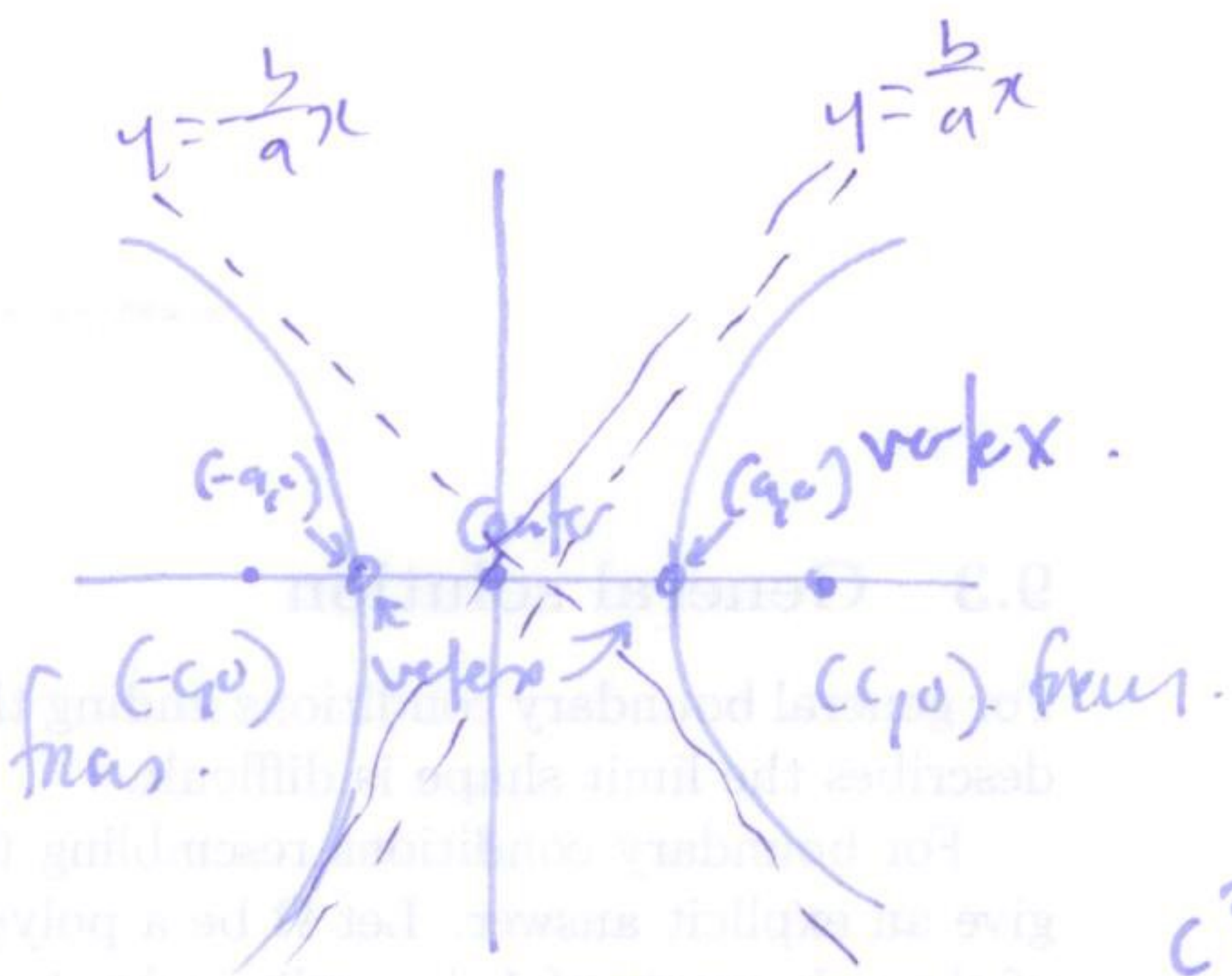
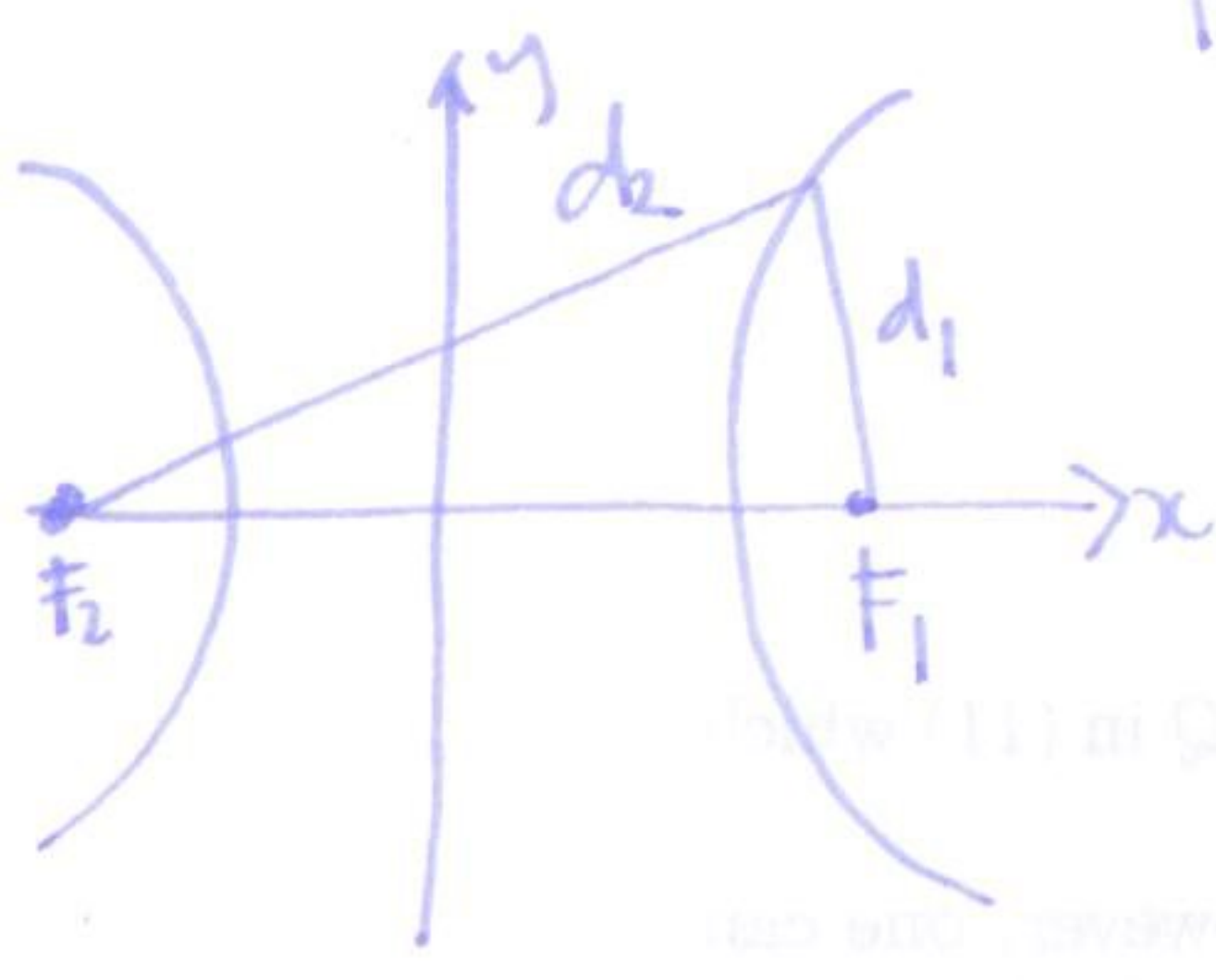
$$Z(\lambda) = \frac{1}{2}(\lambda m + \lambda m + \lambda m + \lambda m)$$

where

$$\lambda_{\alpha, \beta} = \prod_{\alpha \in \Delta^+} \prod_{\beta \in \Delta^+} p(\alpha, \beta)$$

§10.3 Hyperbolas

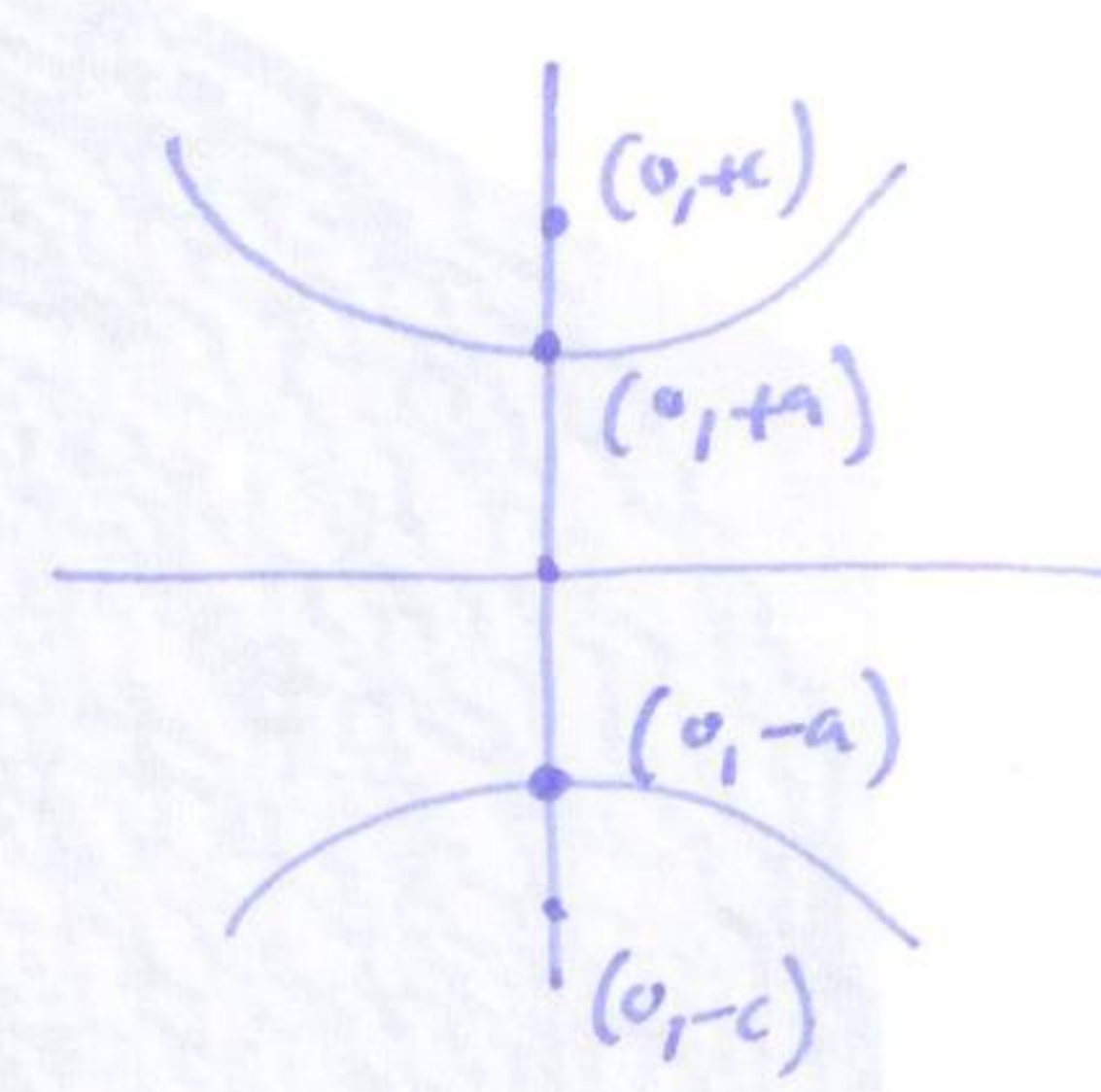
$$|d_2 - d_1| = \text{const}$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

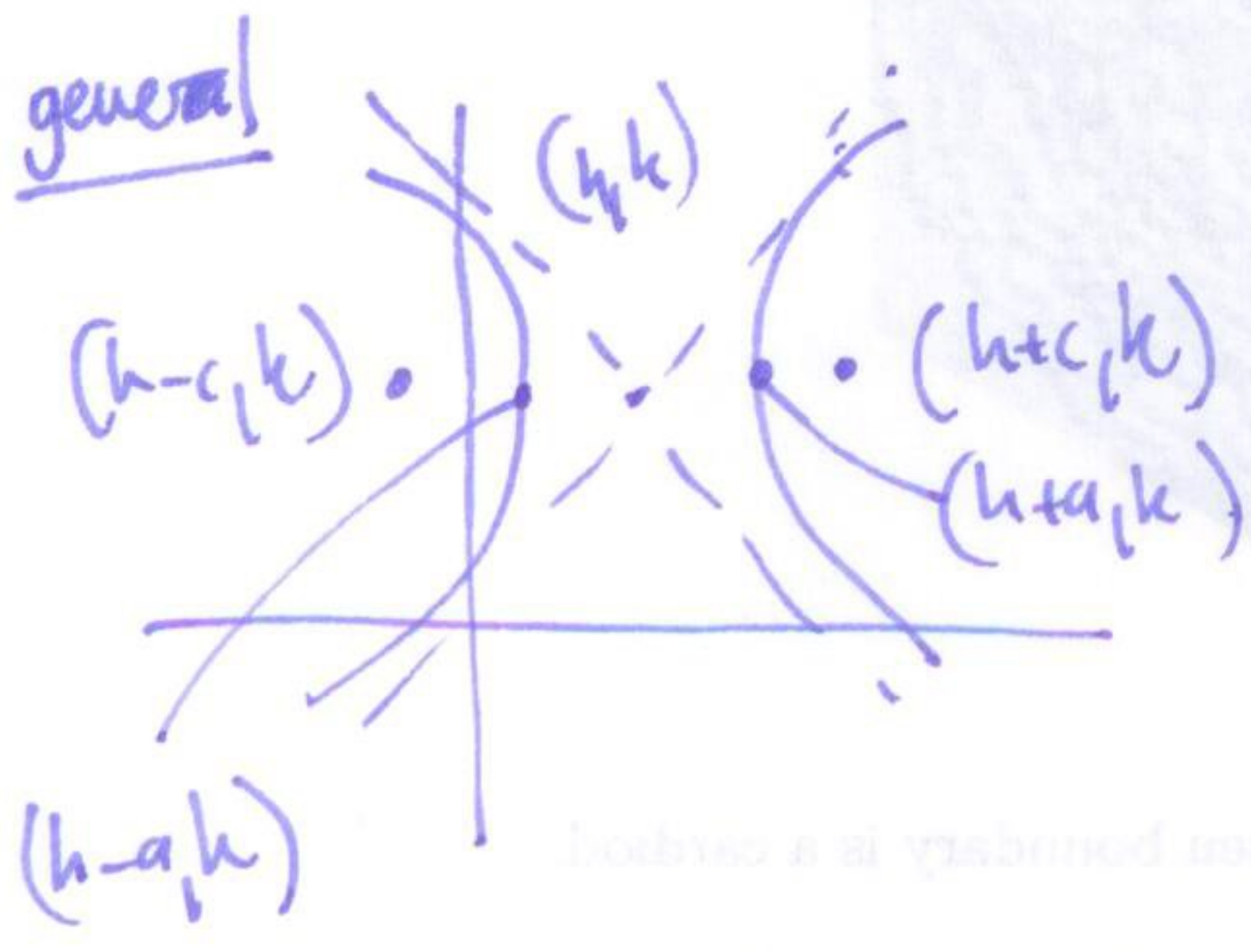
$$c^2 = a^2 + b^2$$

Example $9x^2 - 16y^2 = 144$
 general case: complete the square.



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$



$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

vertices: $(h-a, k)$ $(h+a, k)$

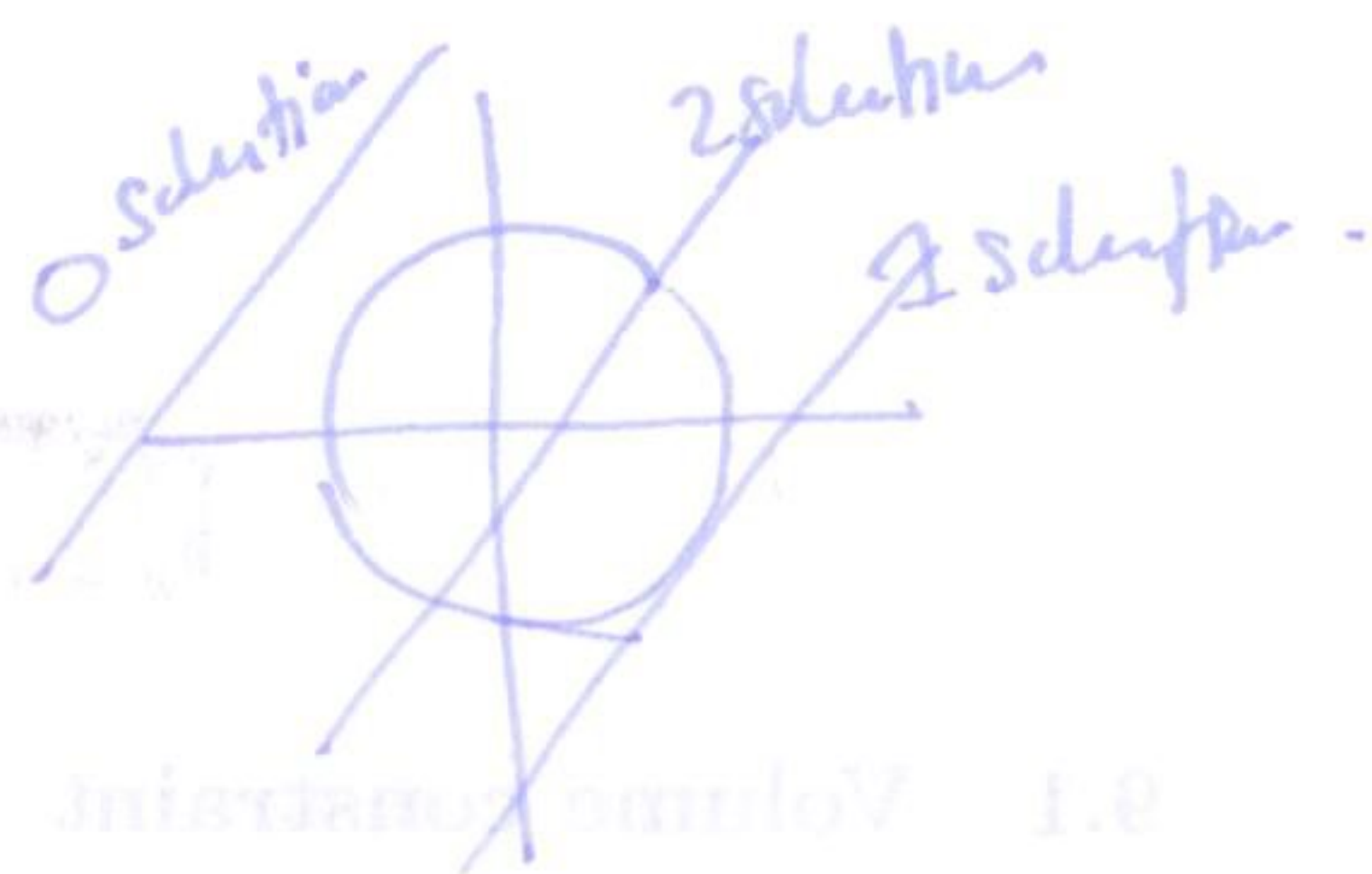
asymptotes: $y-k = \pm \frac{b}{a}(x-h)$

foci: $(h-c, k)$ $(h+c, k)$ $c^2 = a^2 + b^2$

§10.4 Nonlinear equations

(9)

Example ① $x^2 + y^2 = 25 \leftarrow \text{circle}$
 ② $3x - 4y = 0 \leftarrow \text{line}$



by substitution: write $4y = 3x$
 $y = \frac{3}{4}x$

now sub into ①: $x^2 + \left(\frac{3}{4}x\right)^2 = 25$

$$x^2 + \frac{9x^2}{16} = 25$$

$$\frac{25}{16}x^2 = 25 \quad x^2 = 16 \quad x = \pm 4$$

$$(+4, 3) \quad (-4, -3)$$

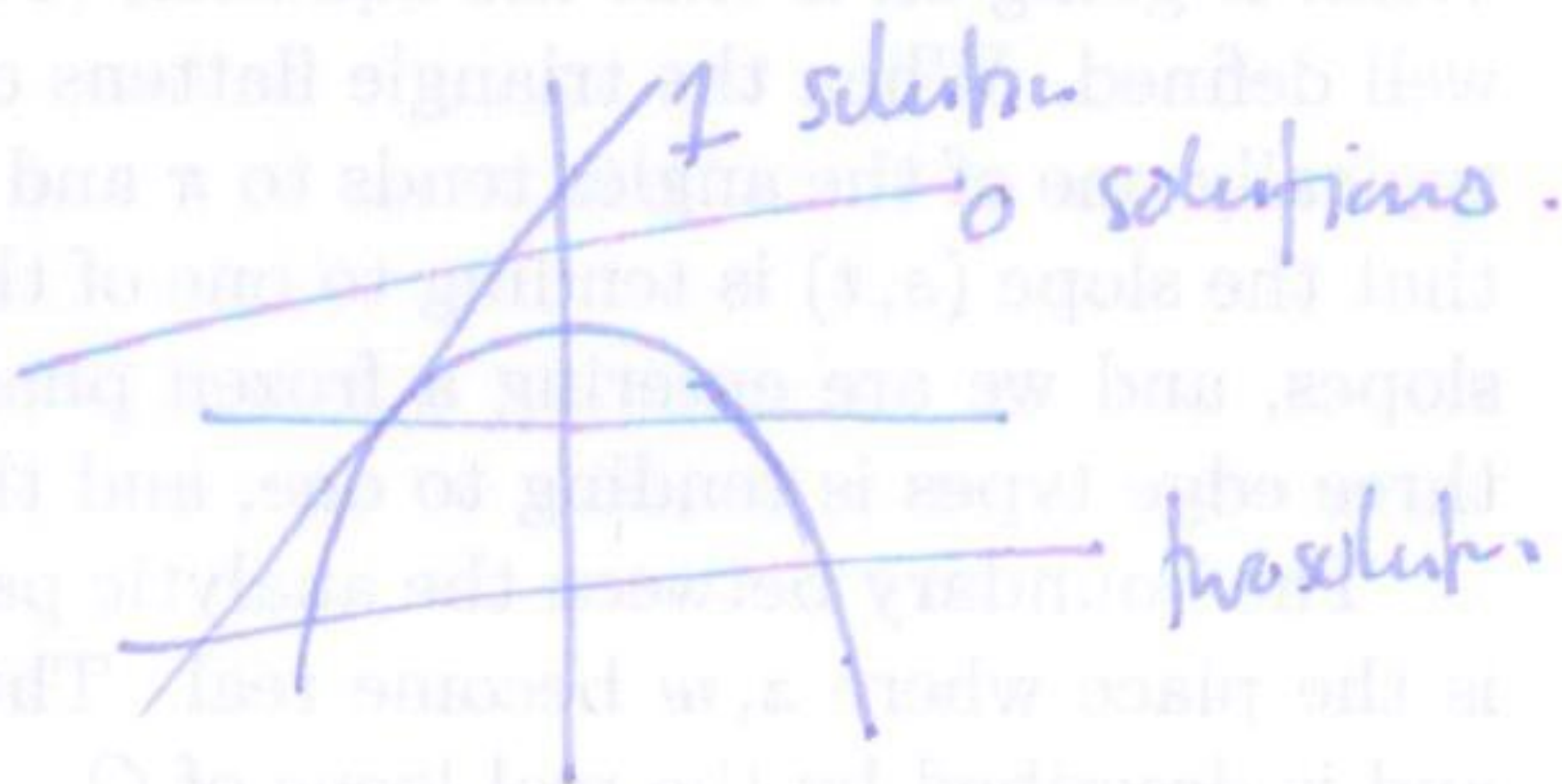
check: $4^2 + 3^2 = 16 + 9 = 25$

$$(-4)^2 + (-3)^2 = 16 + 9 = 25$$

$$3 \cdot 4 - 4 \cdot 3 = 0$$

$$3 \cdot (-4) - (-4) \cdot (-3) = 0$$

Example ① $x + y = 5$ line
 ② $y = 3 - x^2$ parabola



$$\textcircled{1}: y = 5 - x$$

sub into ②: $(5 - x) = 3 - x^2$

$$x^2 - x + 2 = 0 \quad x = \frac{1 \pm \sqrt{1 - 8}}{2} \quad \text{no (real) solutions}$$