

More examples

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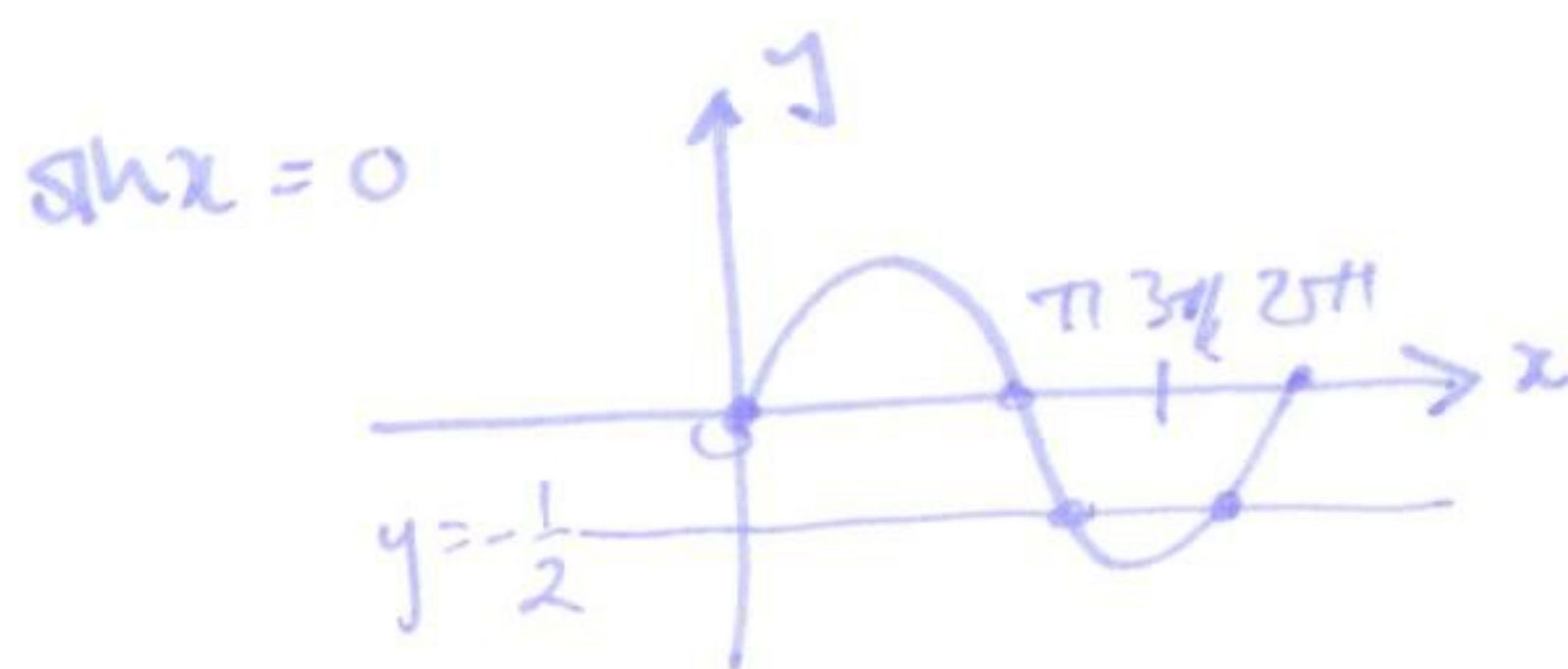
find all solutions to $\cos 2x - \sin x = 1$

$$\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$$

$$1 - 2\sin^2 x - \sin x = 1$$

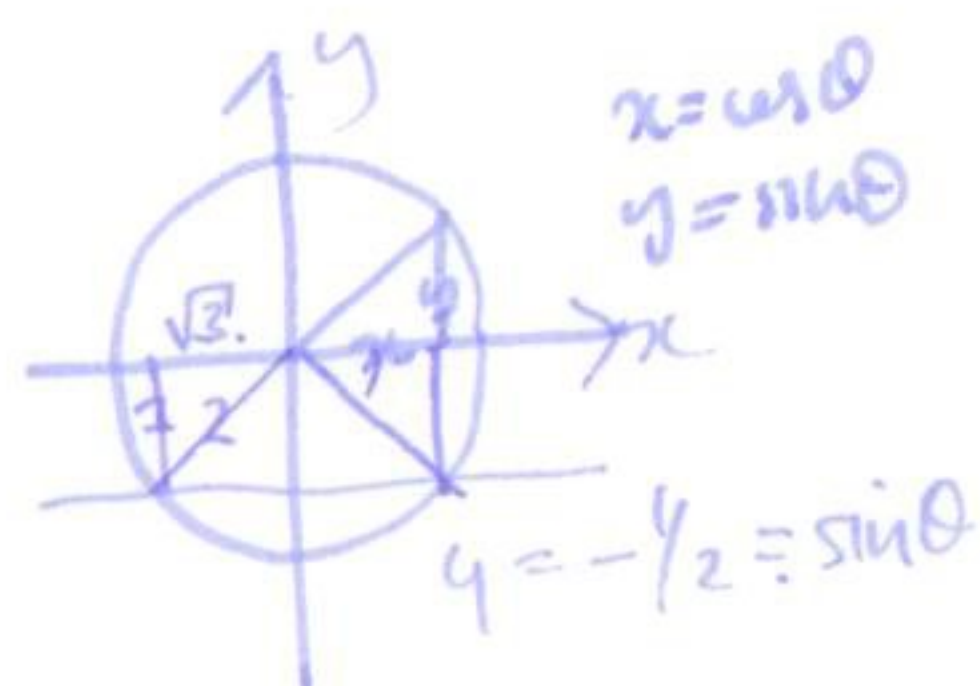
$$2\sin^2 x + \sin x = 0$$

$$\sin x (2\sin x + 1) = 0$$



$$\sin x = 0 : x = n\pi \quad n \in \mathbb{Z}$$

$$\sin x = -\frac{1}{2} \quad x = -\frac{\pi}{6} + 2\pi n, -\frac{5\pi}{6} + 2\pi n$$

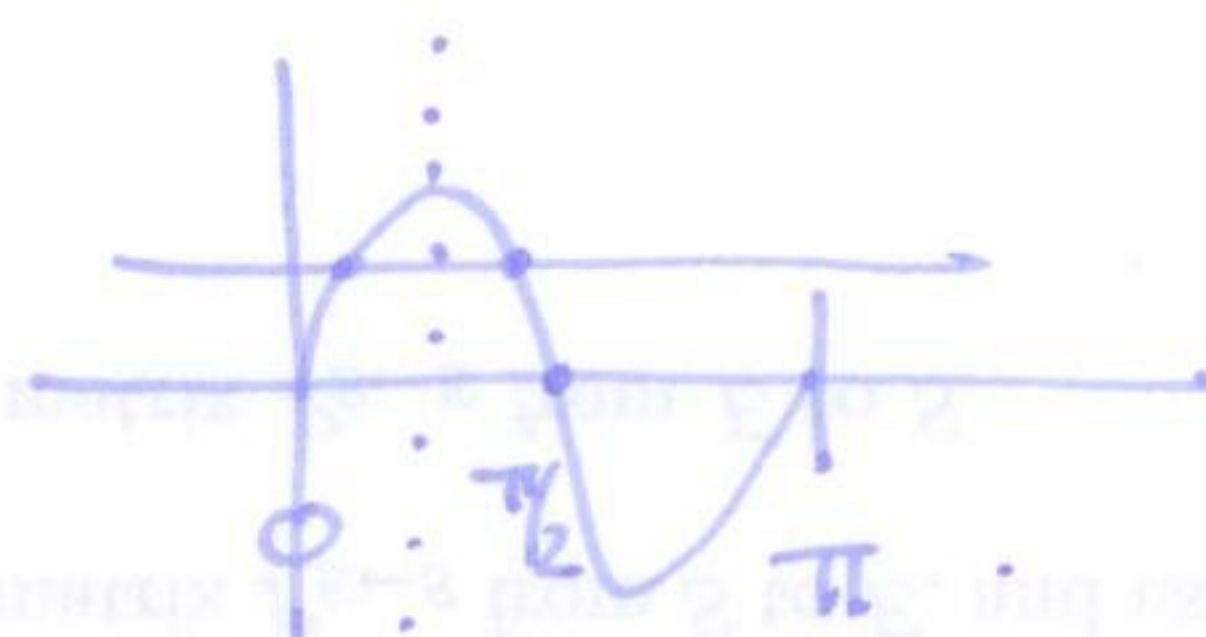


find all solutions to $\sin x = \tan \frac{x}{2}$

solve $\sin 2x = 1/2$

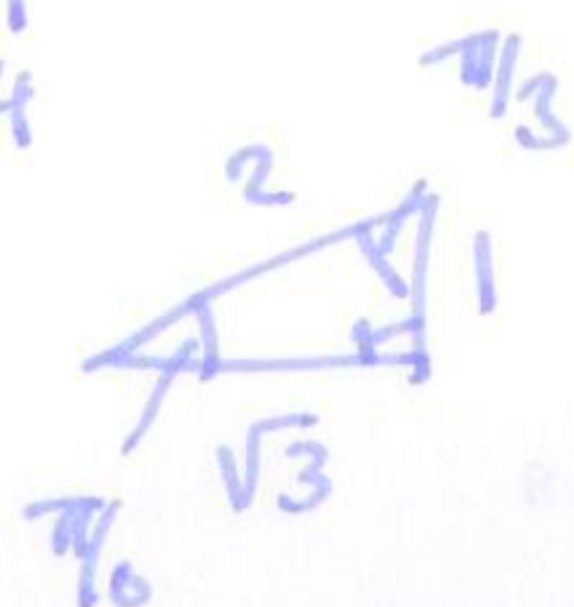
$$2x = \sin^{-1}(1/2)$$

$$x = \frac{1}{2} \sin^{-1}(1/2)$$



$$x = \frac{\pi}{12}$$

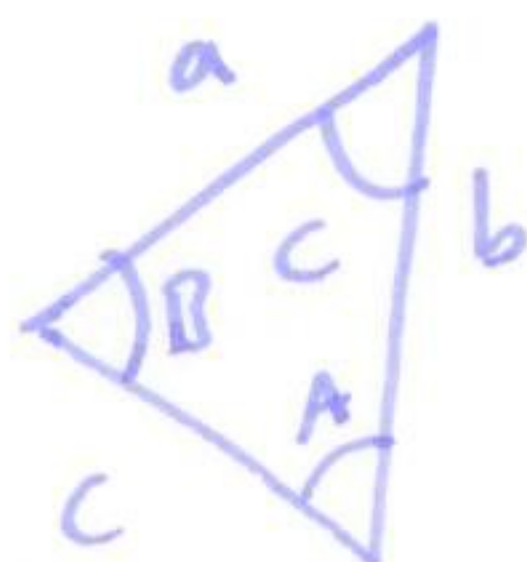
$$x = \frac{\pi}{12}$$



$$2x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x = \frac{\pi}{12}, \frac{5\pi}{12}$$

§8.1 Law of sines

solving triangles



$$A+B+C = 180^\circ = \pi \text{ radians}$$

given enough information (angles, side lengths) you can find the other angles/side lengths.

(52)

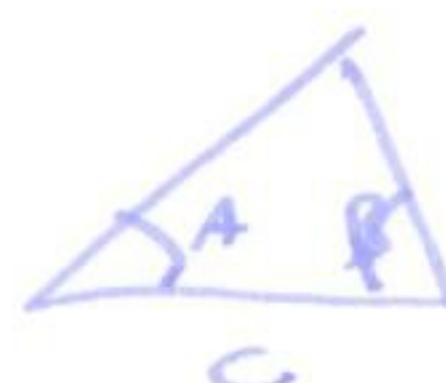
Cases:

AAS:



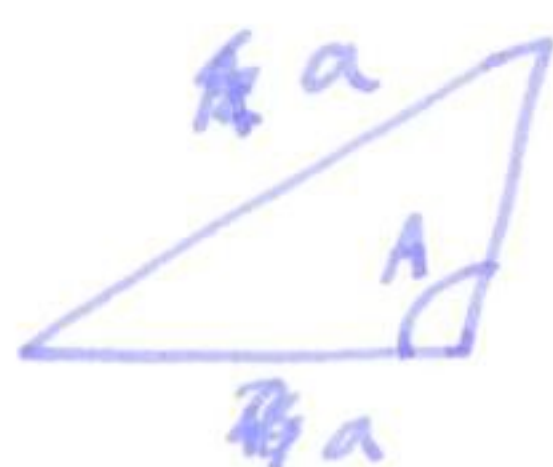
unique solution

ASA:



unique solution

SSA:



either 0, 1 or 2 solutions

SAS:



unique solution.

SSS:



all three sides.

unique solution

AAA:

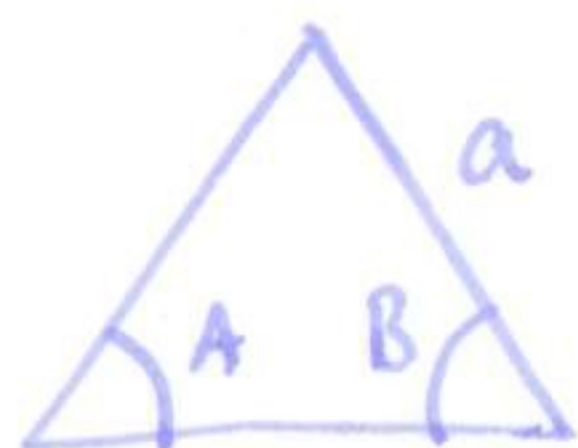


infinitely many solutions!

Law of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

AAS:



$$C = \pi - A - B$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

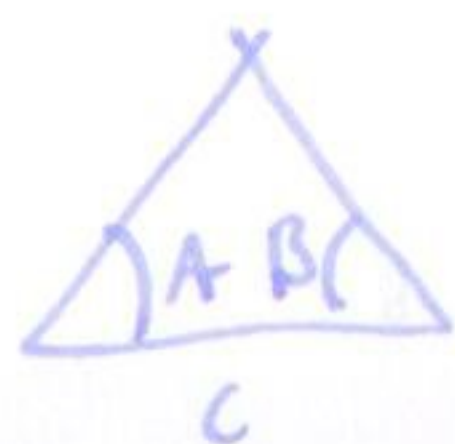
known known known

can find b, c.



find...

ASA:

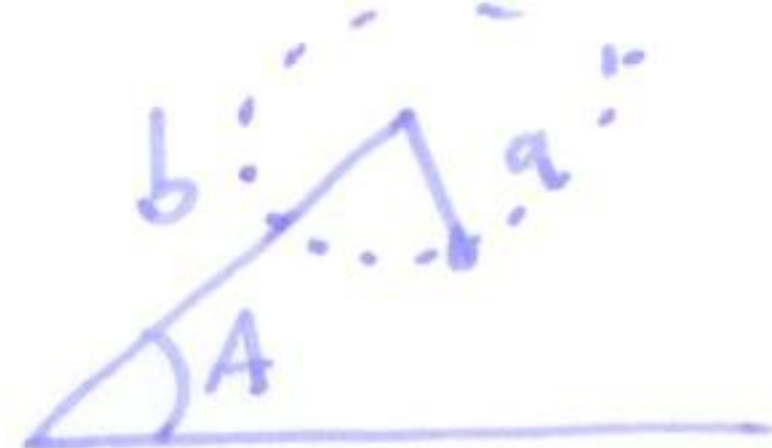
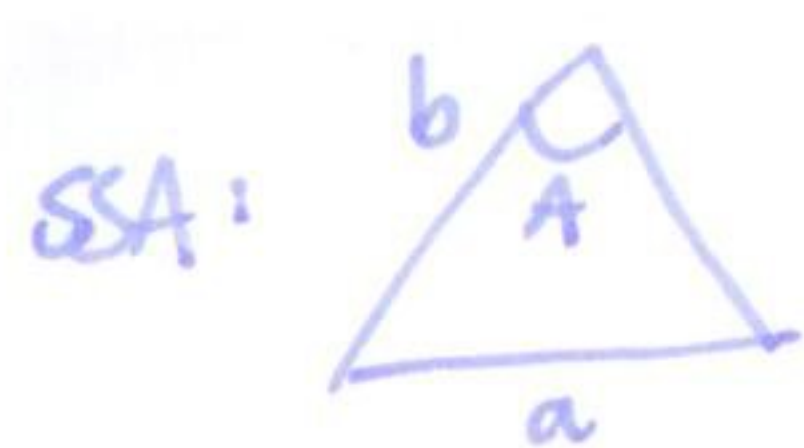


$$C = \pi - A - B$$

(1) (10 degree) from the horizontal line

$$\frac{c}{\sin C} = \frac{b}{\sin B} = \frac{a}{\sin A}$$

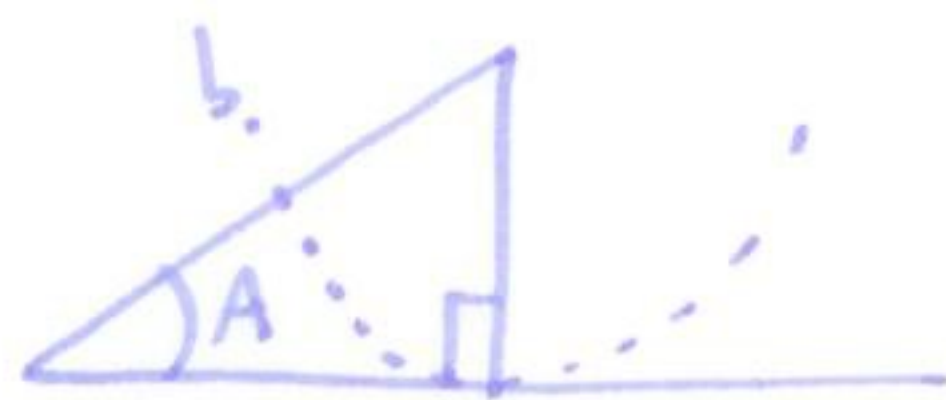
known



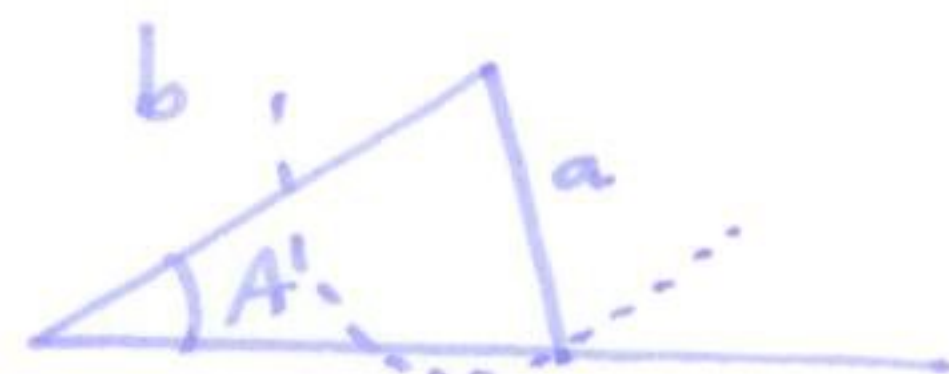
no solutions

53

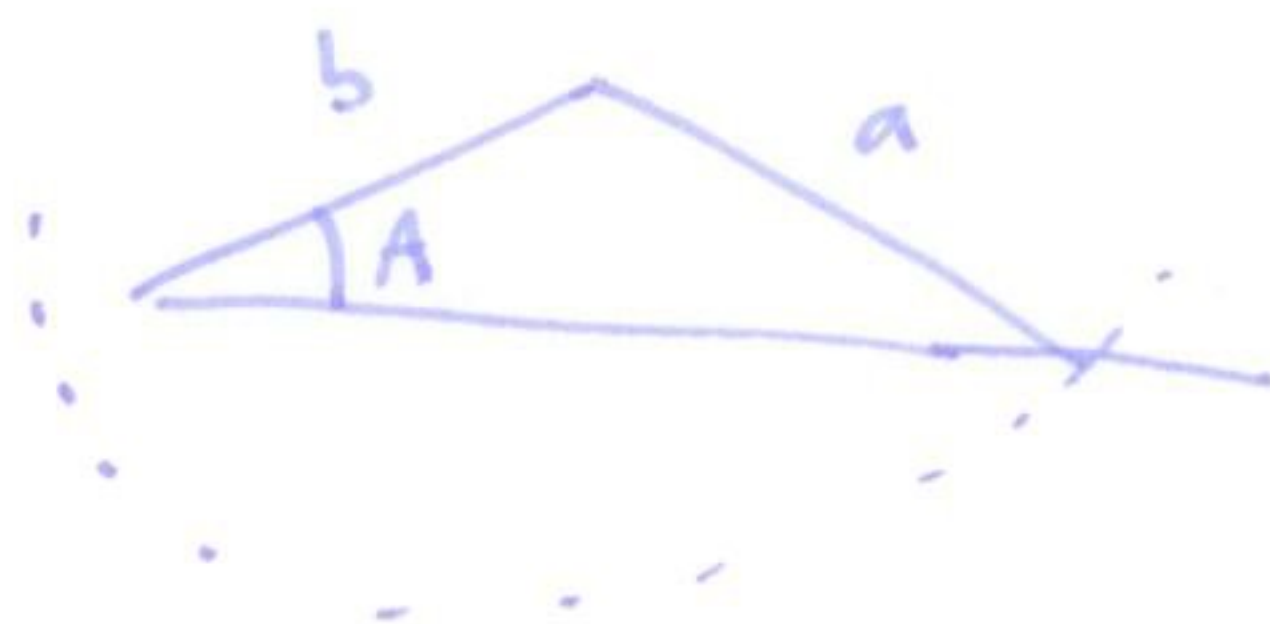
A acute:



1 solution.

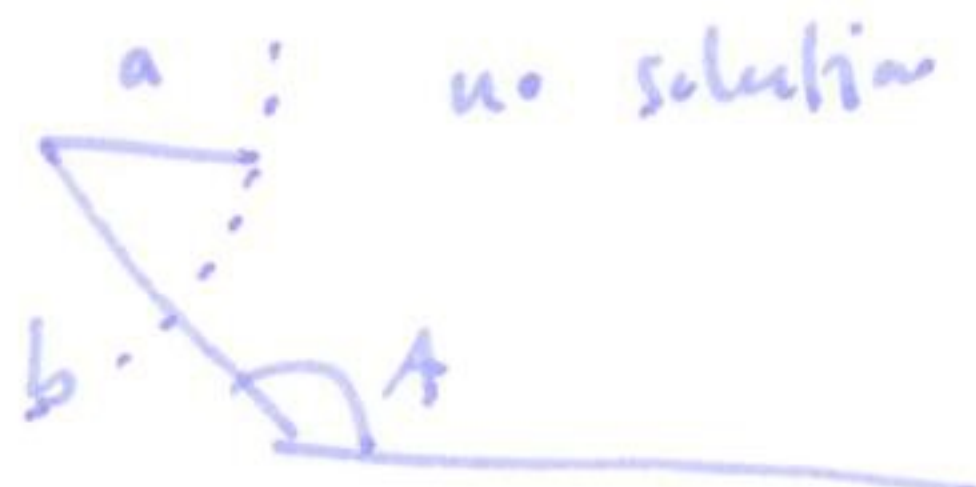


2 solutions.



1 solution.

A obtuse:

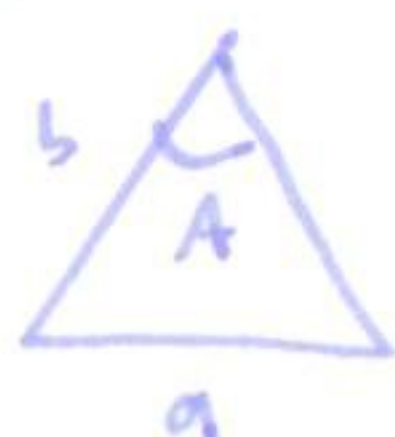


no solution



1 solution.

Examples

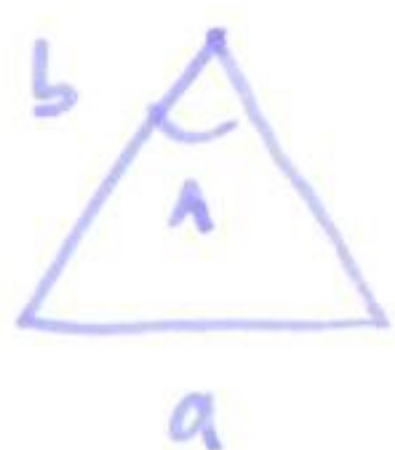


$A = 0.4$
 $b = 100$
 $a = 1$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin B = \frac{b}{a} \sin 0.4 > 1$$

no solutions!



$A = 0.4$
 $b = 2$
 $a = 1$

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin B = \frac{b}{a} \sin A$$

