sums and products.

<u>recall</u> $\sin(x+y) = \sin x \cos y + \sin y \cos x$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

so $\sin(x-y) = \sin x \cos y - \sin y \cos x$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

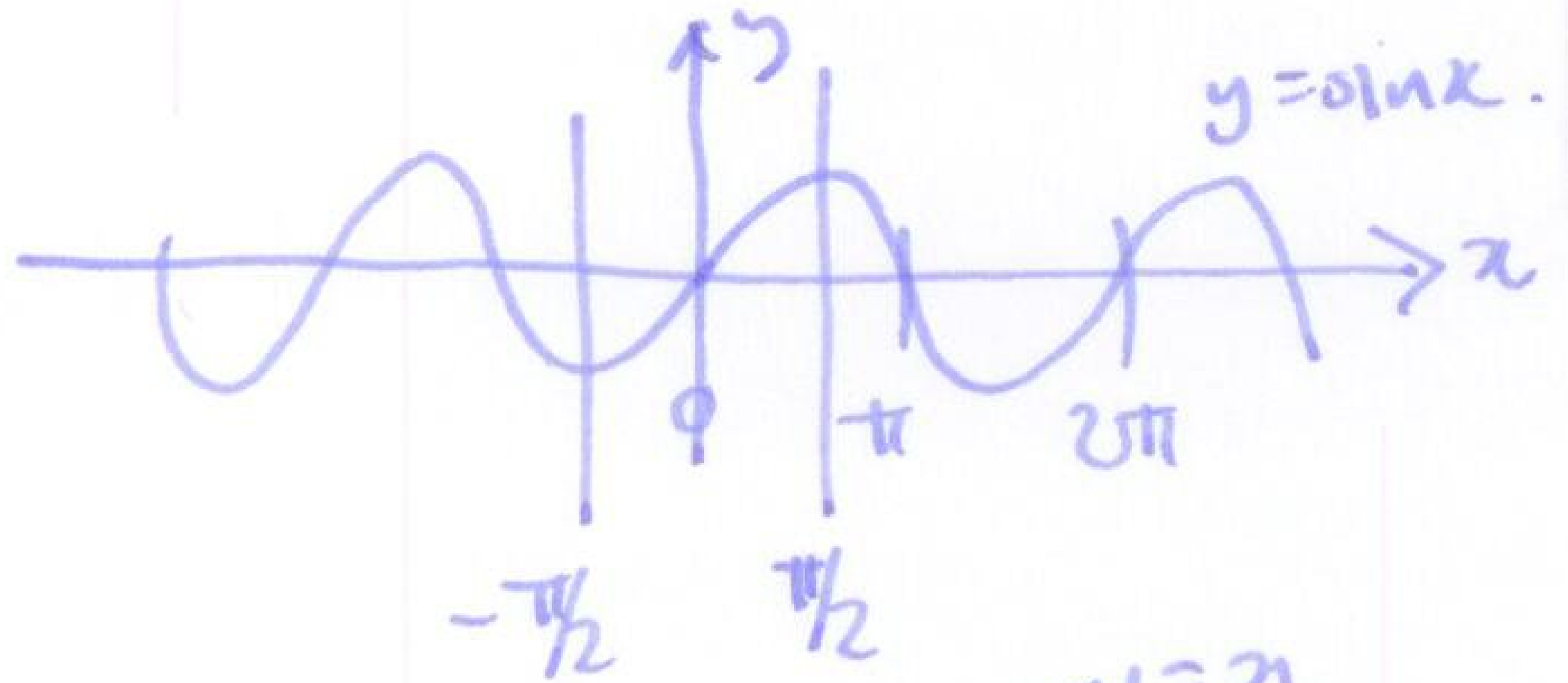
$\sin(x+y) + \sin(x-y) = 2 \sin x \cos y$

$\sin(x+y) - \sin(x-y) = 2 \sin y \cos x$

$\cos(x+y) + \cos(x-y) = 2 \cos x \cos y$

$\cos(x-y) - \cos(x+y) = 2 \sin x \sin y$.

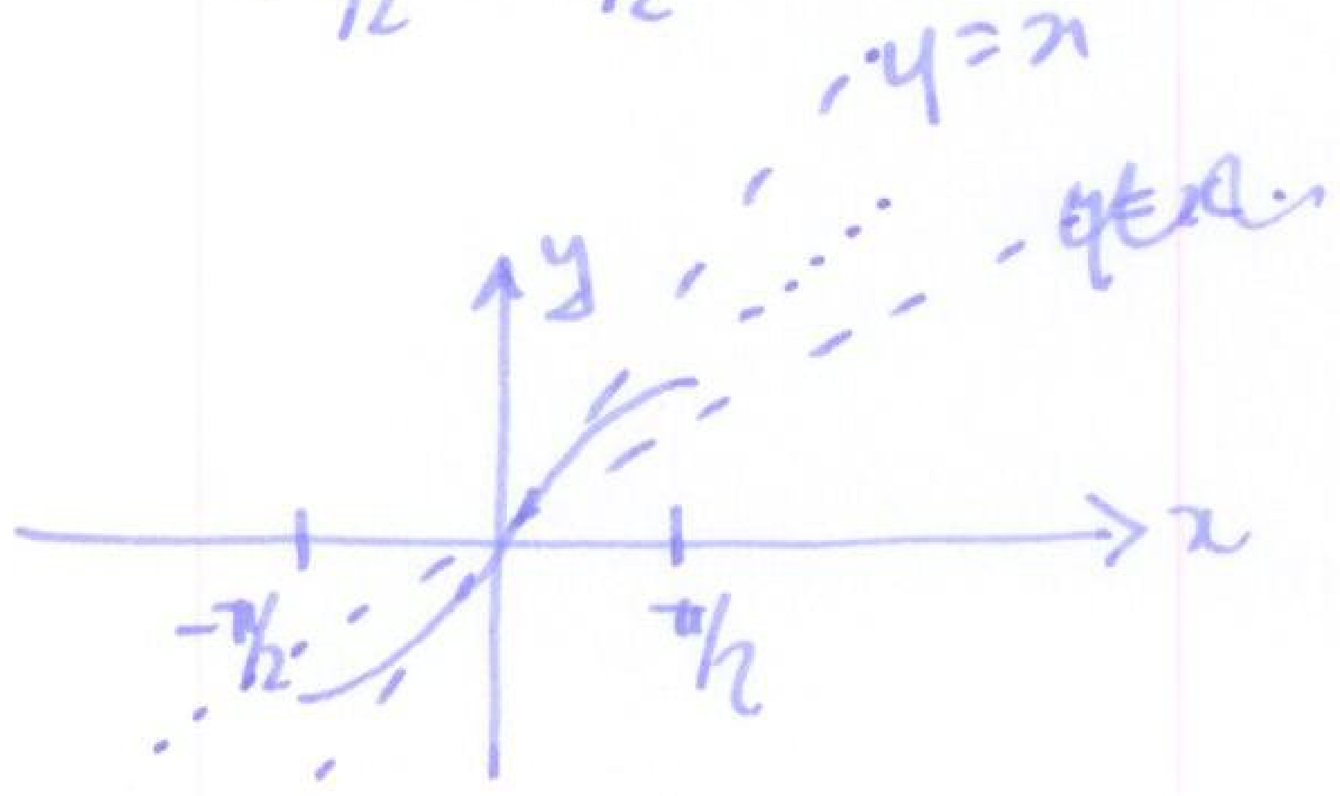
§7.4 Trig function inverses



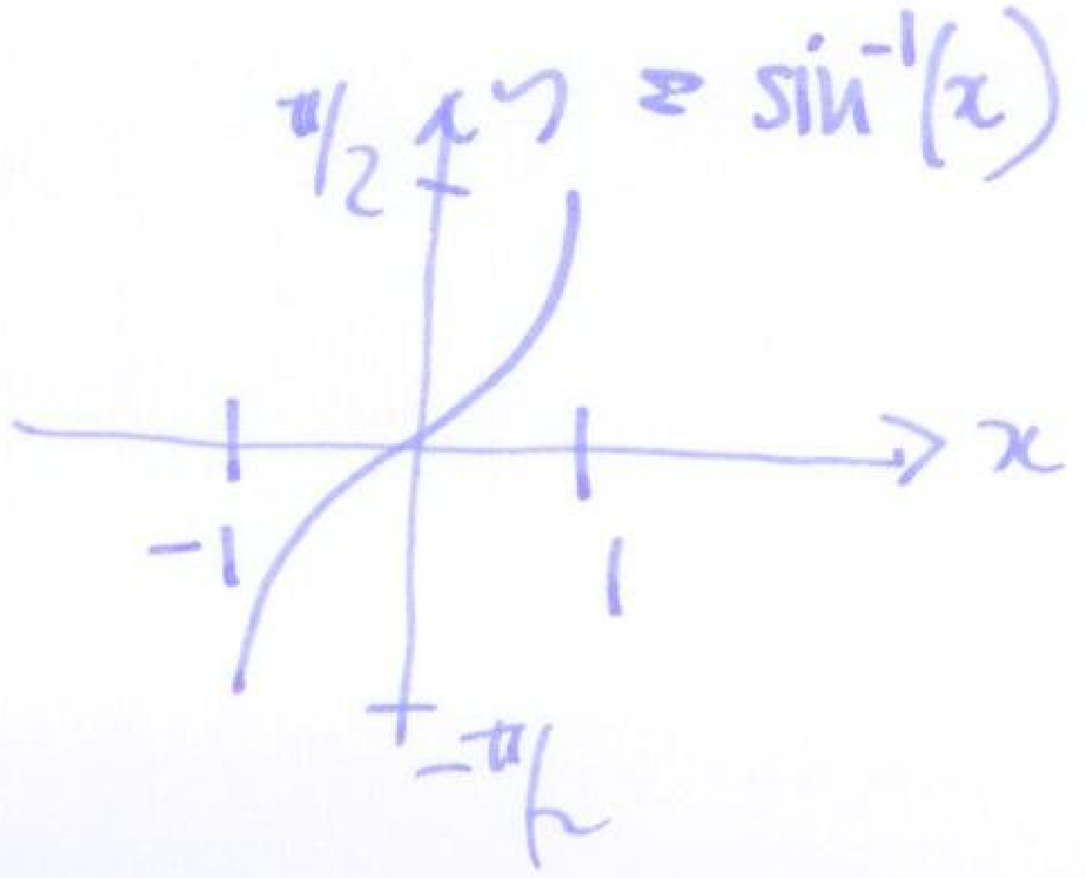
no inverse!

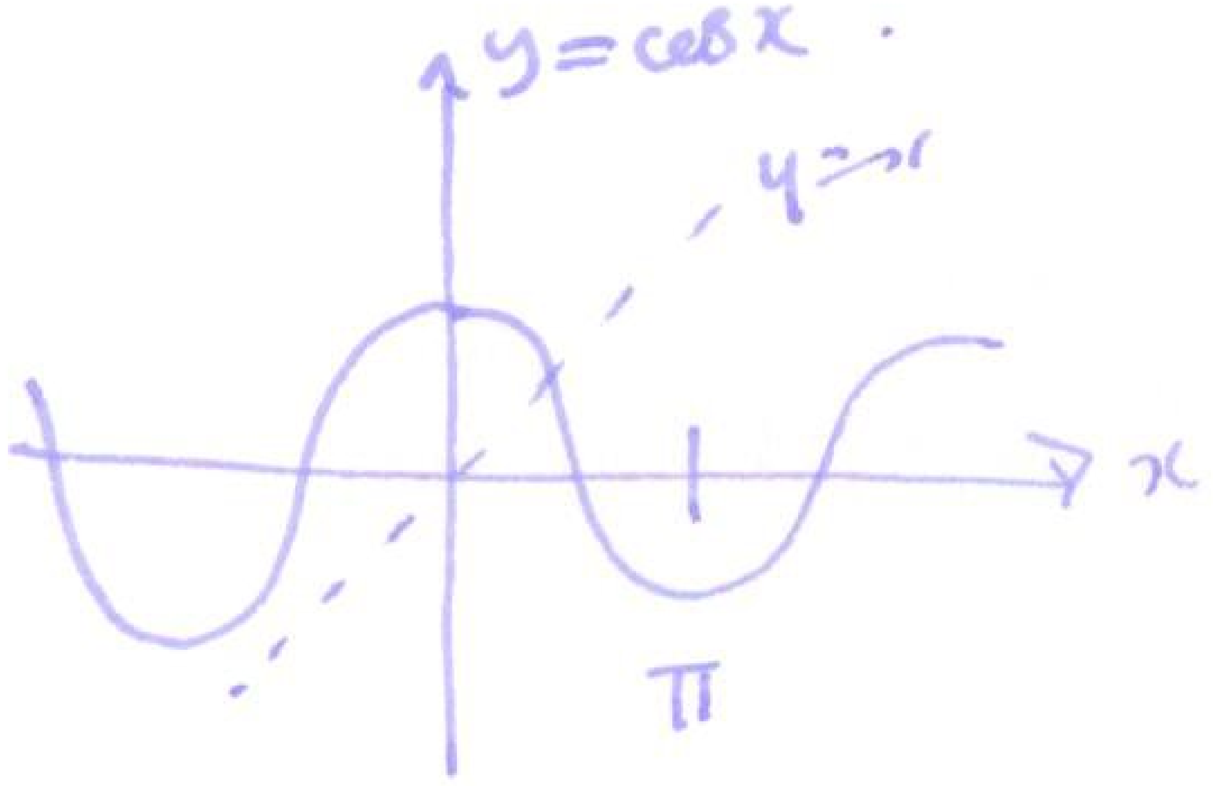
unless: restrict domain.

$-\frac{\pi}{2} \le x \le \pi/2$



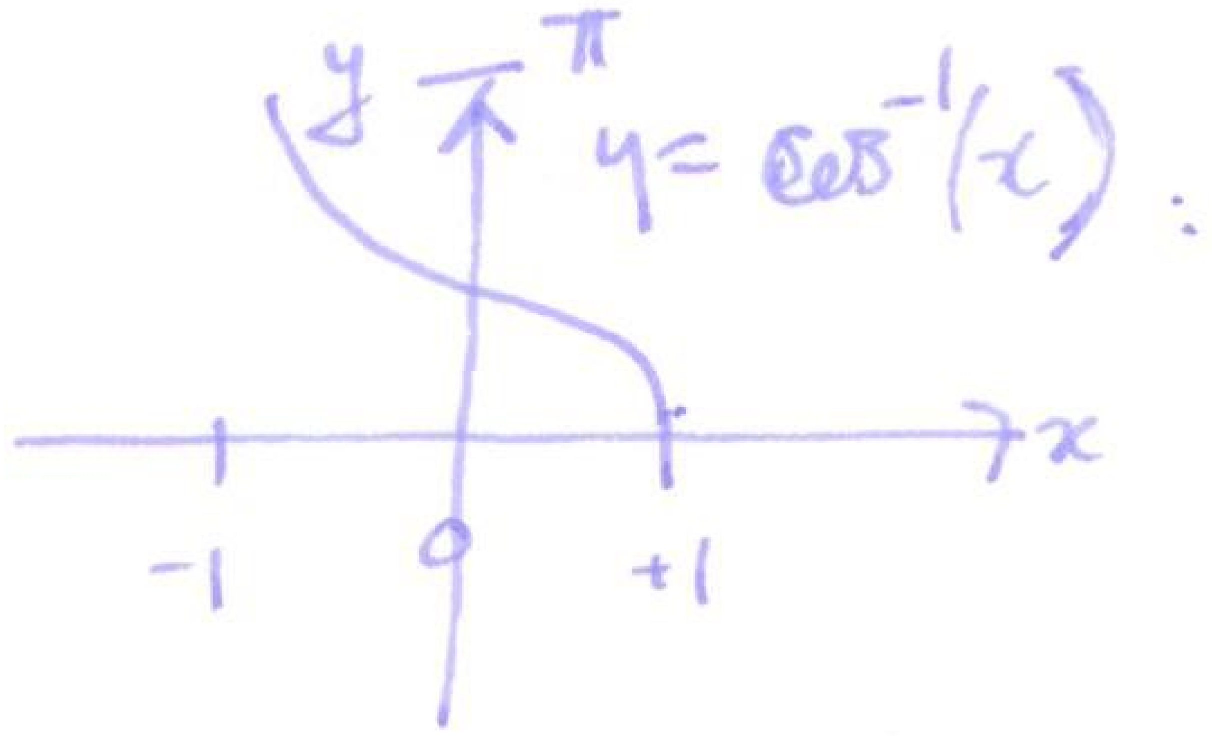
inverse

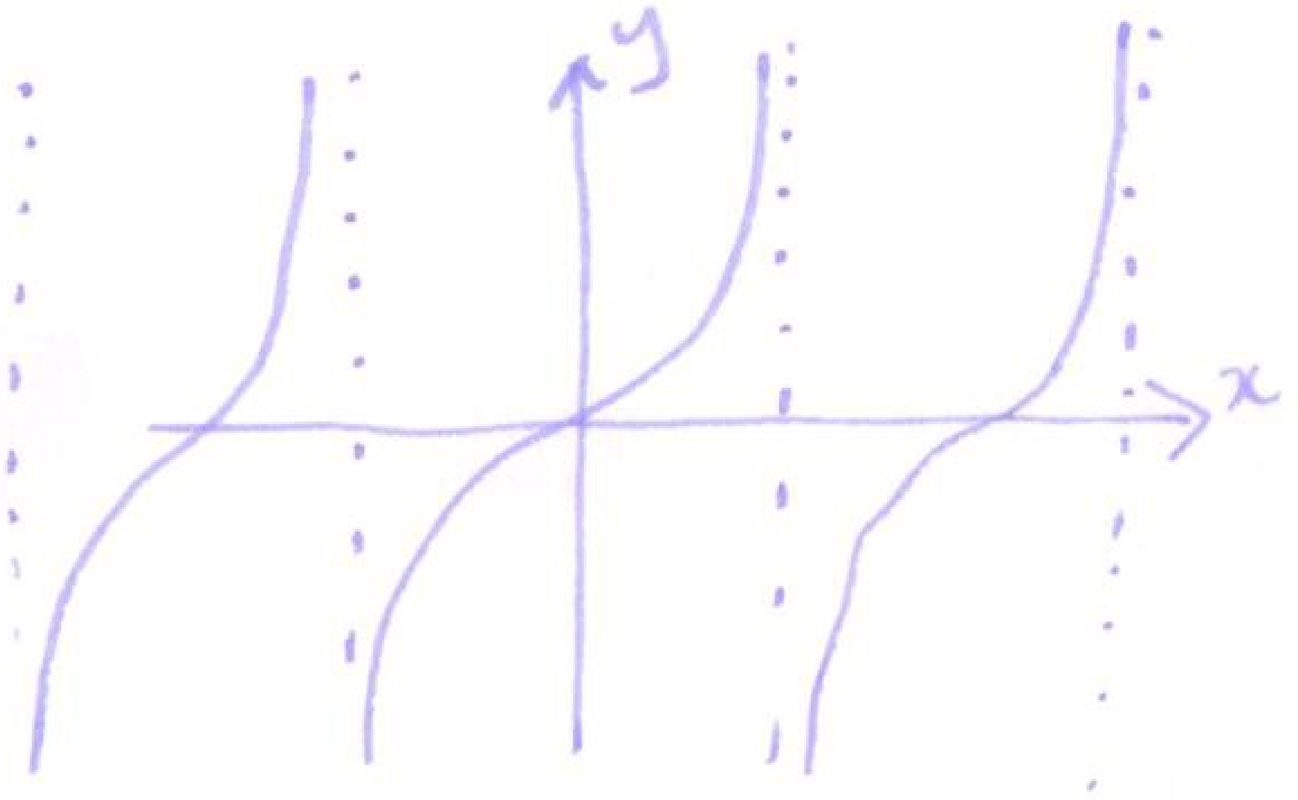




no inverse!

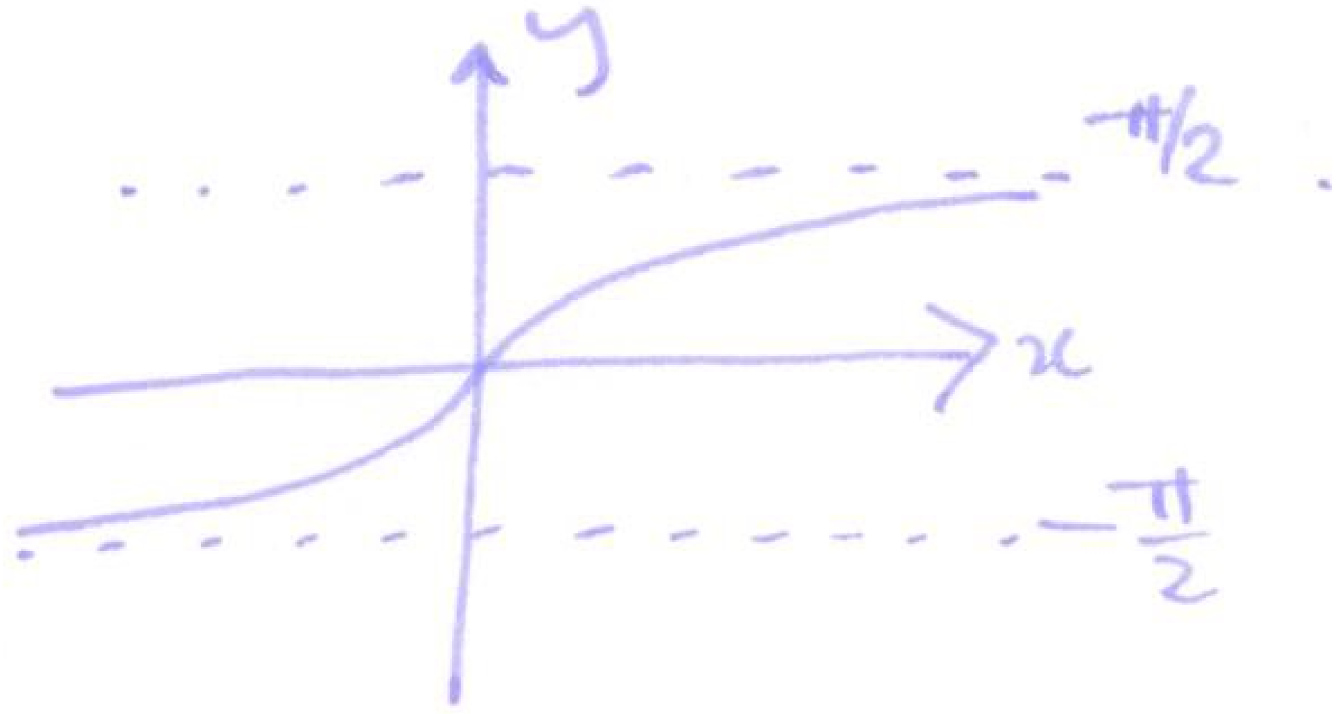
unless: restrict domain $0 \leq x \leq \pi$





no inverse!

unless: restrict domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$



<u>warning</u> $\sin^{-1}(x) \neq \frac{1}{\sin(x)}$

$\cos^{-1}(x) \neq \frac{1}{\cos(x)}$

$\tan^{-1}(x) \neq \frac{1}{\tan(x)}$

$\sin^{-1}(x) = \arcsin(x)$ inverse $\sin(x)$ function

$\cos^{-1}(x) = \arccos(x)$

$\tan^{-1}(x) = \arctan(x)$.

$\sin(\sin^{-1}(x)) = x \qquad -1 \leq x \leq 1$

$\cos(\cos^{-1}(x)) = x \qquad -1 \leq x \leq 1$

$\tan(\tan^{-1}(x)) = x \qquad$ all x!

<u>Q</u> what is $\sin^{-1}(\sin(x + 100\pi))$? $-\pi/2 \leq x \leq \pi/2$

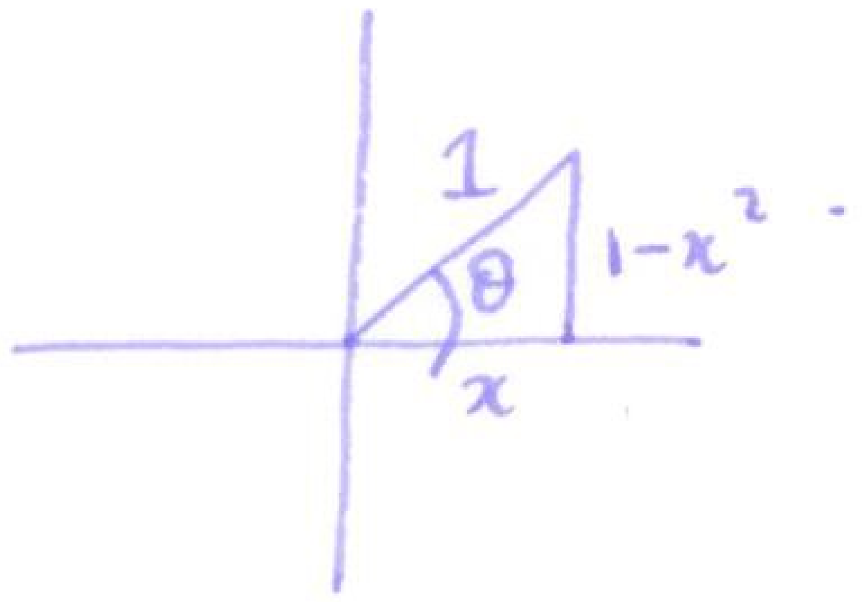
$\sin^{-1}(\sin(x)) = x \qquad -\pi/2 \leq x \leq \pi/2$

$\cos^{-1}(\cos(x)) = x \qquad 0 \leq x \leq \pi$

$\tan^{-1}(\tan(x)) = x \qquad -\frac{\pi}{2} \leq x \leq \pi/2$.

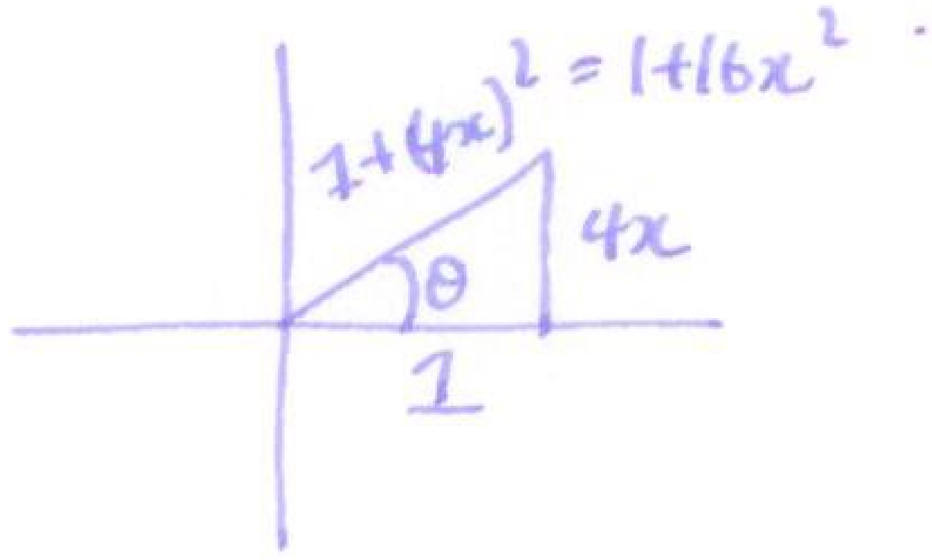
Find ~~$\sin^{-1}(\cos x)$~~ as a function of x.

$\sin(\cos^{-1}(x))$



if $\cos\theta = x$ then $\sin\theta = 1 - x^2$

find $\cos^{-1}\left(\tan^{-1}(4x)\right)$



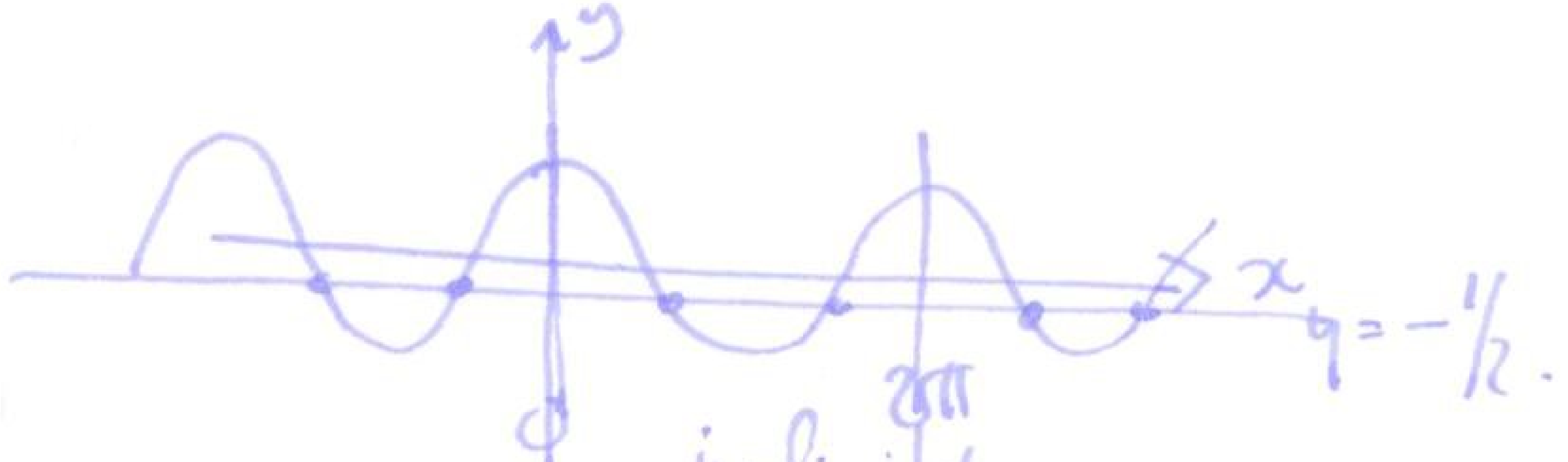
$\tan\theta = 4x$

th $\cos\theta = \dfrac{1}{1+16x^2}$.

§7.5 Solving trig equations

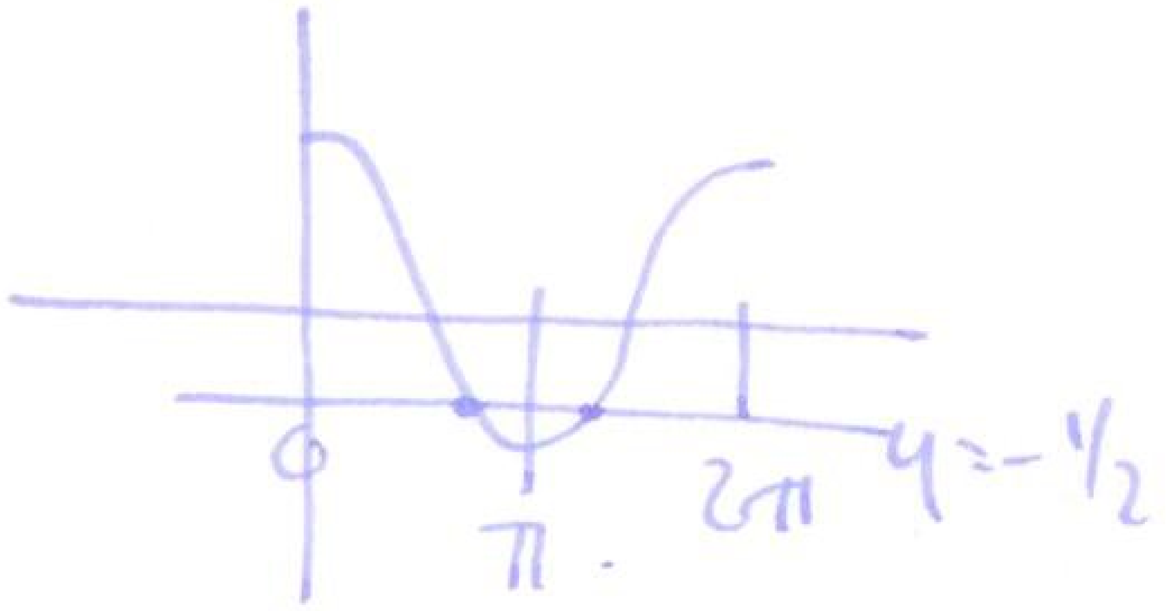
Example $2\cos x = -1$

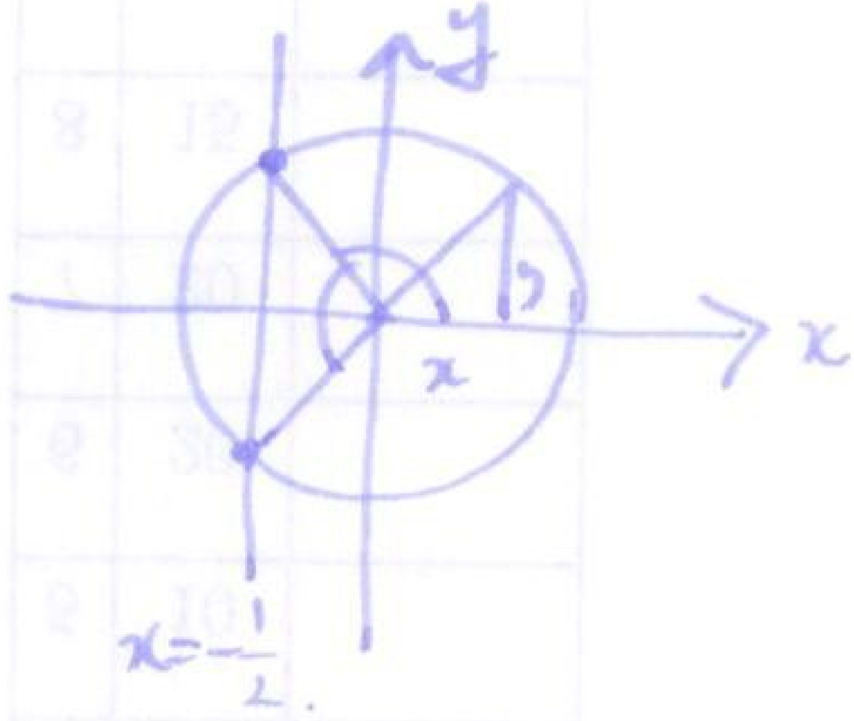
$\cos x = -\frac{1}{2}$



infinitely many solutions!

however $\cos x$ is 2π-periodic, so find all solutions $0 \le x \le 2\pi$ and add on all multiples of 2π.





$x = \cos\theta$
$y = \sin\theta$.

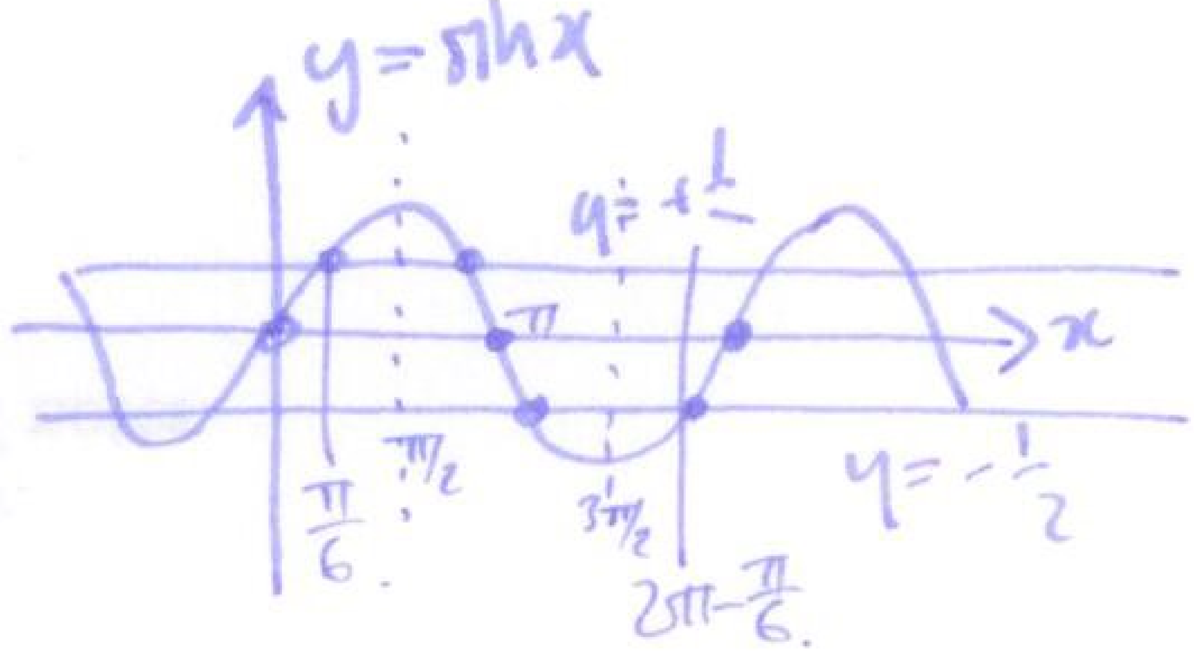
solutions $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$.

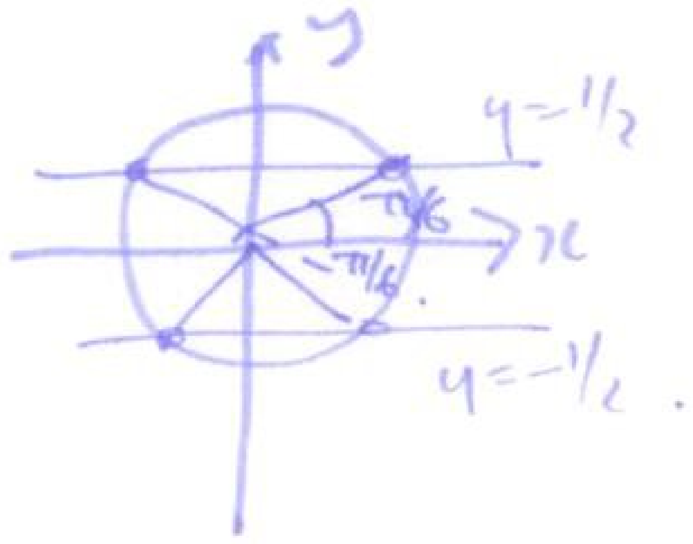
all solutions: $\frac{2\pi}{3} + 2\pi n, \quad \frac{4\pi}{3} + 2\pi n$.

Example $4\sin^2 x = 1$

$\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$





$\sin x$: 2π-periodic, so want to find all solutions in $[0, 2\pi]$.

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \quad , \quad \pi - \frac{\pi}{6}$

$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6} \longleftrightarrow 2\pi - \frac{\pi}{6} = \quad , \left(-\pi + \frac{\pi}{6}\right) + 2\pi = \pi + \frac{\pi}{6}.$

so solutions are $\frac{\pi}{6}+2\pi n$, $\frac{5\pi}{6}+2\pi n$, $\pi+\frac{\pi}{6}+2\pi n$, $2\pi-\frac{\pi}{6}+2\pi n$.

Example solve $3\tan 2x = -3$ in $[0, 2\pi)$

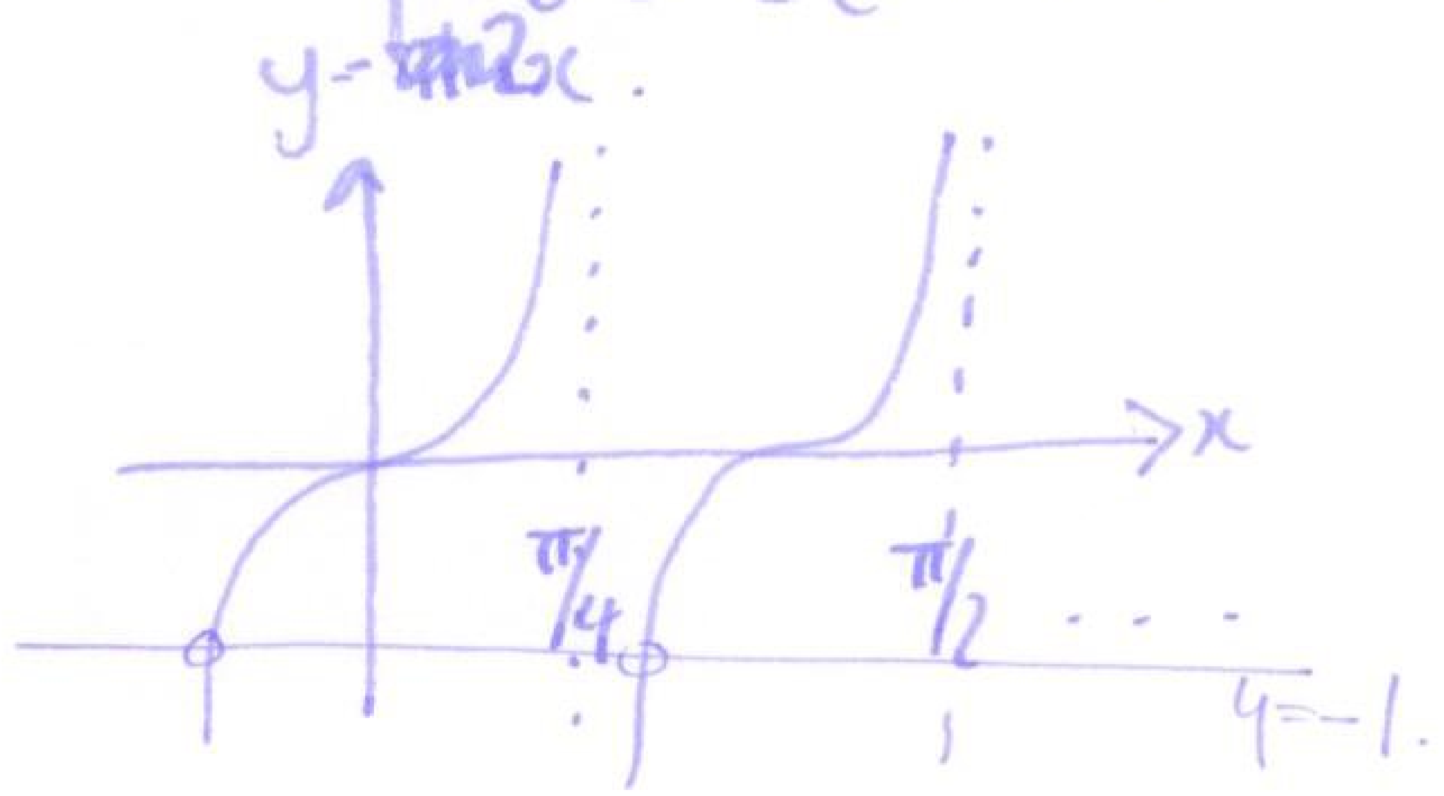
$$\tan 2x = -1$$

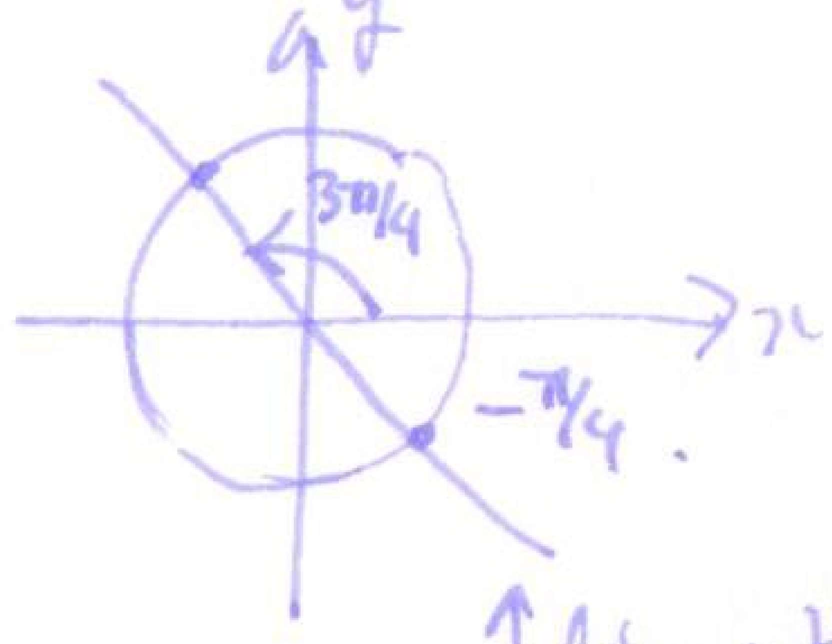
want $0 \le x < 2\pi$

$0 \le 2x < 4\pi$.

$\tan^{-1}(-1) = \cancel{\frac{3\pi}{4}}\ -\frac{\pi}{4}$ also $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

i.e. $2x = \frac{3\pi}{4}$ works $x = \frac{3\pi}{8}$.





↑ line with slope -1!

all solutions to $\tan^{-1}(-1)$ are $-\frac{\pi}{4} + n\pi$.

all solutions to $\tan^{-1}(-1)$ in $[0, 4\pi)$ are $2x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$.

so $x = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$.

Example solve $2\cos^2 u = 1 - \cos u$ in $[0, 2\pi)$.

$$2\cos^2 u + \cos u - 1 = 0$$

$$(2\cos u - 1)(\cos u + 1) = 0$$

$\underline{\cos u = \frac{1}{2}}$ or $\underline{\cos u = -1}$. solve these two!