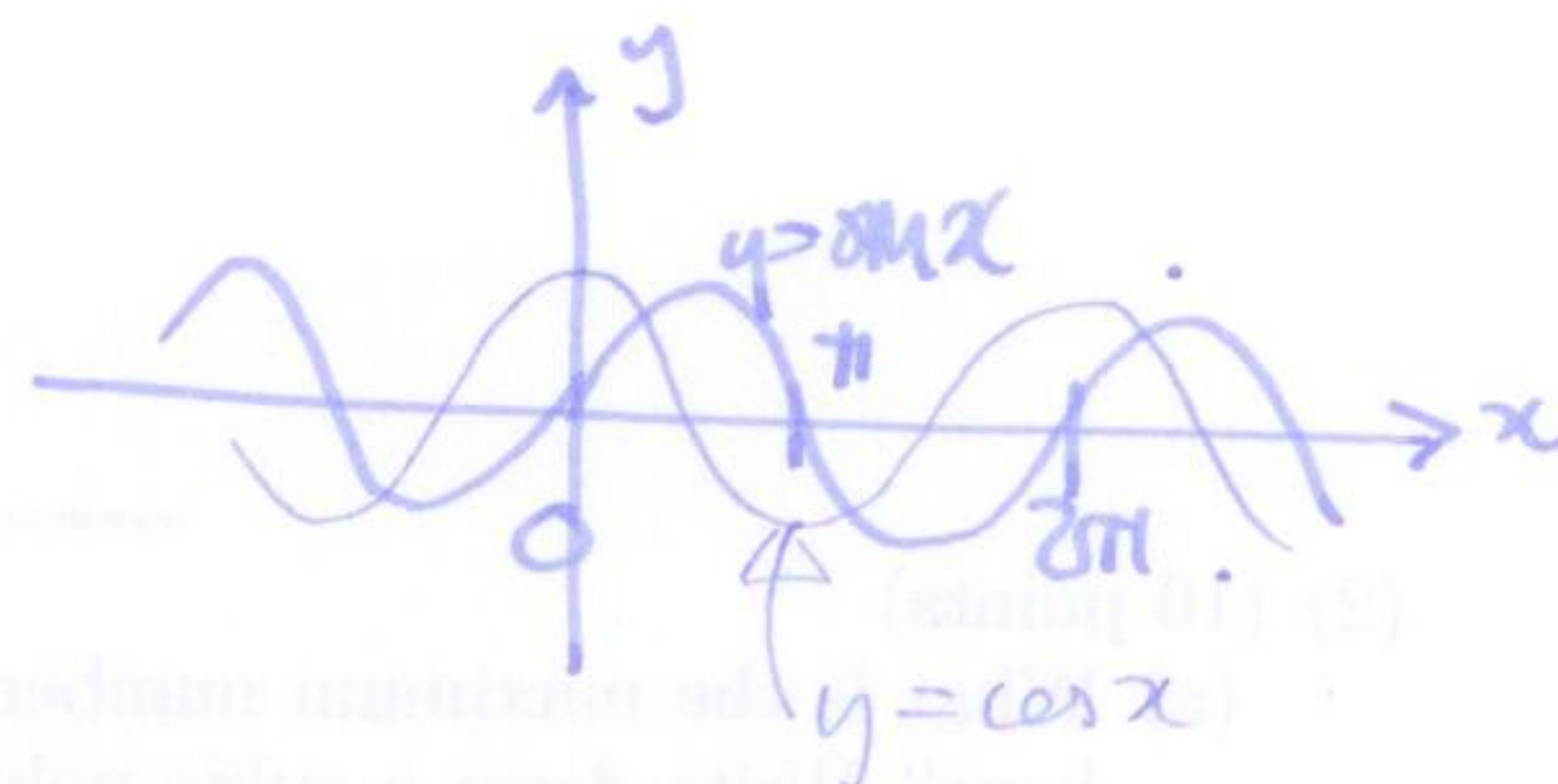


§7.2 Cofunction identities:

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$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Example show: $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

$$\begin{aligned} \sin\left(x + \frac{\pi}{2}\right) &= \sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2} && \text{(addition formula)} \\ &= \sin x \cdot 0 + \cos x \cdot 1 \\ &= \cos x \end{aligned}$$

so: $\sin\left(x + \frac{\pi}{2}\right) = \cos x$

$$\cos\left(x + \frac{\pi}{2}\right) = -\sin x$$

$$\sin\left(x - \frac{\pi}{2}\right) = -\cos x$$

$$\cos\left(x - \frac{\pi}{2}\right) = +\sin x$$

Example $\tan\left(x + \frac{\pi}{2}\right) = \frac{\sin\left(x + \frac{\pi}{2}\right)}{\cos\left(x + \frac{\pi}{2}\right)} = \frac{\cos x}{-\sin x} = -\cot x$

Double angle

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Half angle identities

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start with

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - \sin^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x.\end{aligned}$$

so

$$\sin^2 x = \frac{1 - \cos 2x}{2}.$$

replace x by $\frac{x}{2}$

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}.$$

take square root:

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

so:

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

sums and products.

recall $\sin(x+y) = \sin x \cos y + \sin y \cos x$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

so $\sin(x-y) = \sin x \cos y - \sin y \cos x$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

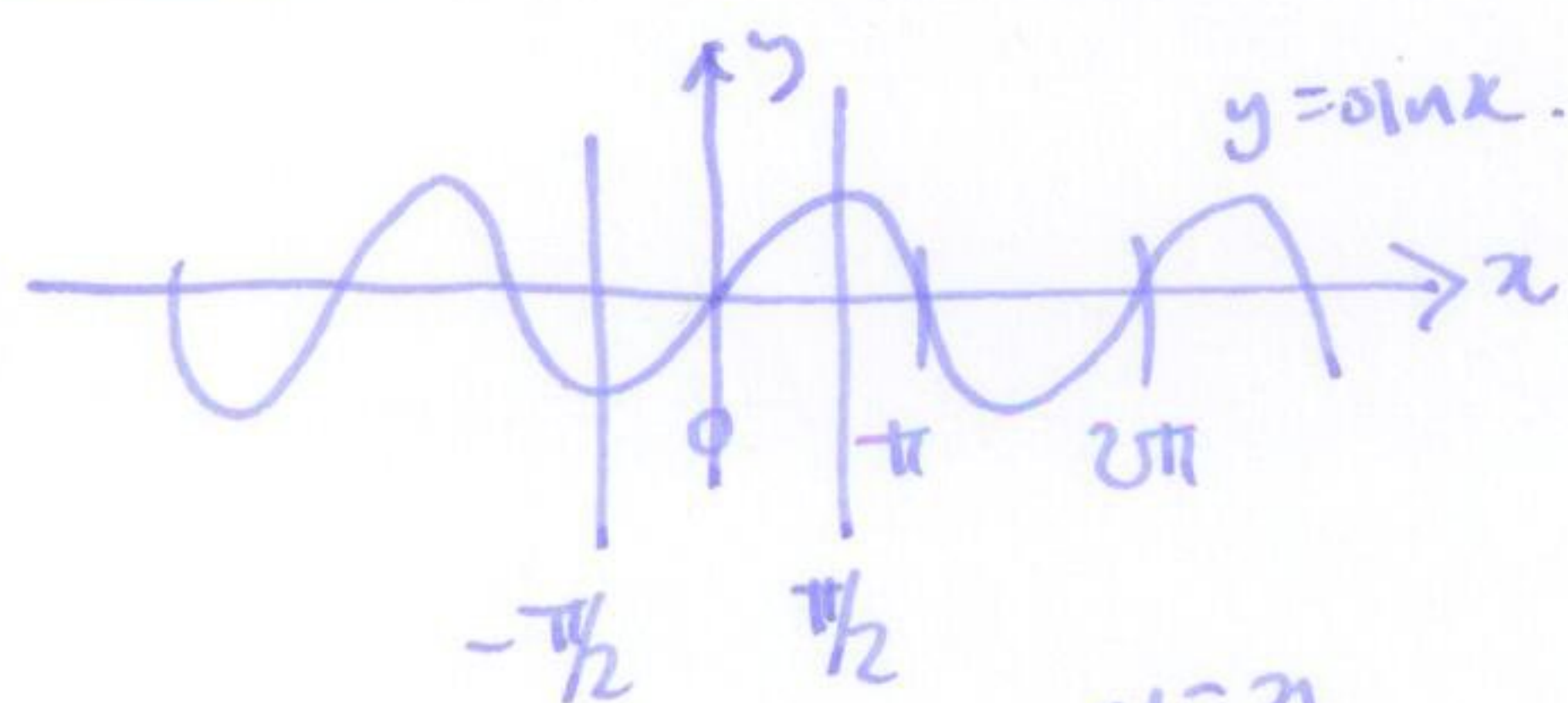
$\sin(x+y) + \sin(x-y) = 2 \sin x \cos y$

$\sin(x+y) - \sin(x-y) = 2 \sin y \cos x$

$\cos(x+y) + \cos(x-y) = 2 \cos x \cos y$

$\cos(x-y) - \cos(x+y) = 2 \sin x \sin y$

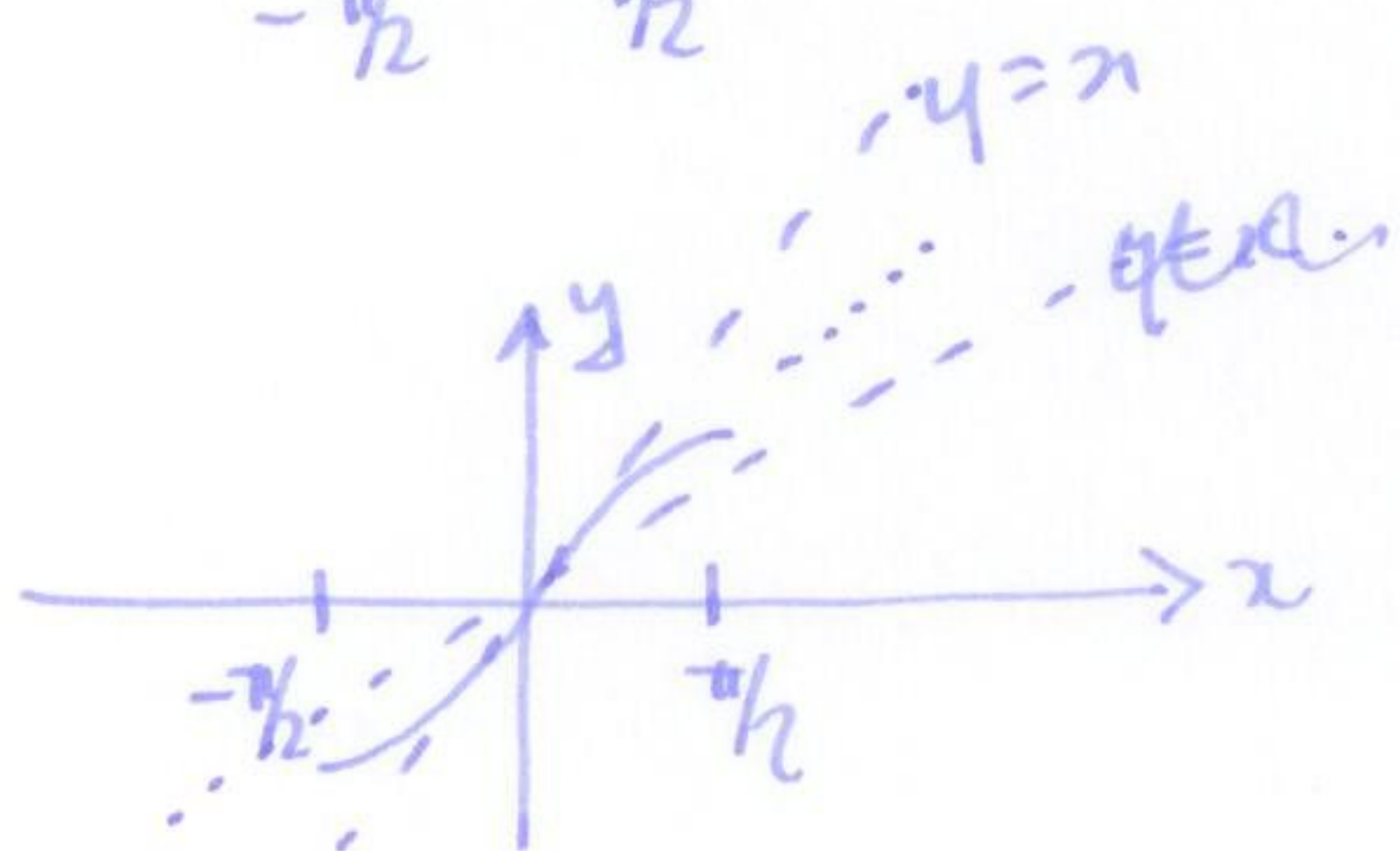
§7.4 Trig function inverses



no inverse!

unless: restrict domain.

$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$



invers

