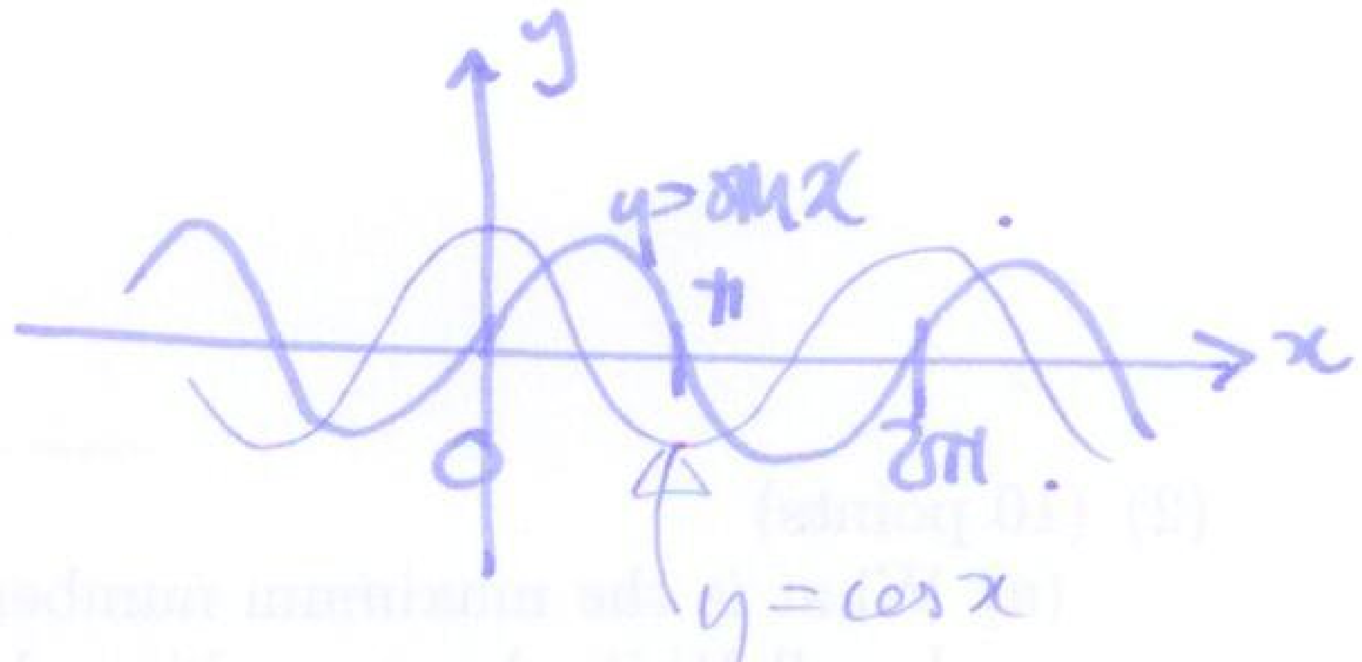
§7.2 Cofunction identities:



$$\sin\left(\frac{\pi}{2}-x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2}-x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2}-x\right) = \cot x \qquad \cot\left(\frac{\pi}{2}-x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2}-x\right) = \operatorname{cosec} x \qquad \operatorname{cosec}\left(\frac{\pi}{2}-x\right) = \sec x.$$

Example show: $\sin\left(x+\frac{\pi}{2}\right) = \cos x$

$$\sin\left(x+\frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2} \quad \text{(addition formula)}.$$

$$= \sin x \cdot 0 + \cos x \cdot 1$$

$$= \cos x.$$

so: $\sin\left(x+\frac{\pi}{2}\right) = \cos x \qquad \cos\left(x+\frac{\pi}{2}\right) = -\sin x$

$\sin\left(x-\frac{\pi}{2}\right) = -\cos x \qquad \cos\left(x-\frac{\pi}{2}\right) = +\sin x.$

Example $\tan\left(x+\frac{\pi}{2}\right) = \dfrac{\sin\left(x+\frac{\pi}{2}\right)}{\cos\left(x+\frac{\pi}{2}\right)} = \dfrac{\cos x}{-\sin x} = -\cot x.$

Double angle $\sin 2\theta = 2\sin\theta\cos\theta \qquad \tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}.$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

Half angle identities

start with $\cos 2x = \cos^2 x - \sin^2 x$

$$= 1 - \sin^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x.$$

so $\sin^2 x = \dfrac{1 - \cos 2x}{2}.$

replace x by $\frac{x}{2}$:

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}.$$

take square root:

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

so:

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

sums and products.

<u>recall</u> $\sin(x+y) = \sin x \cos y + \sin y \cos x$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

so $\sin(x-y) = \sin x \cos y - \sin y \cos x$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

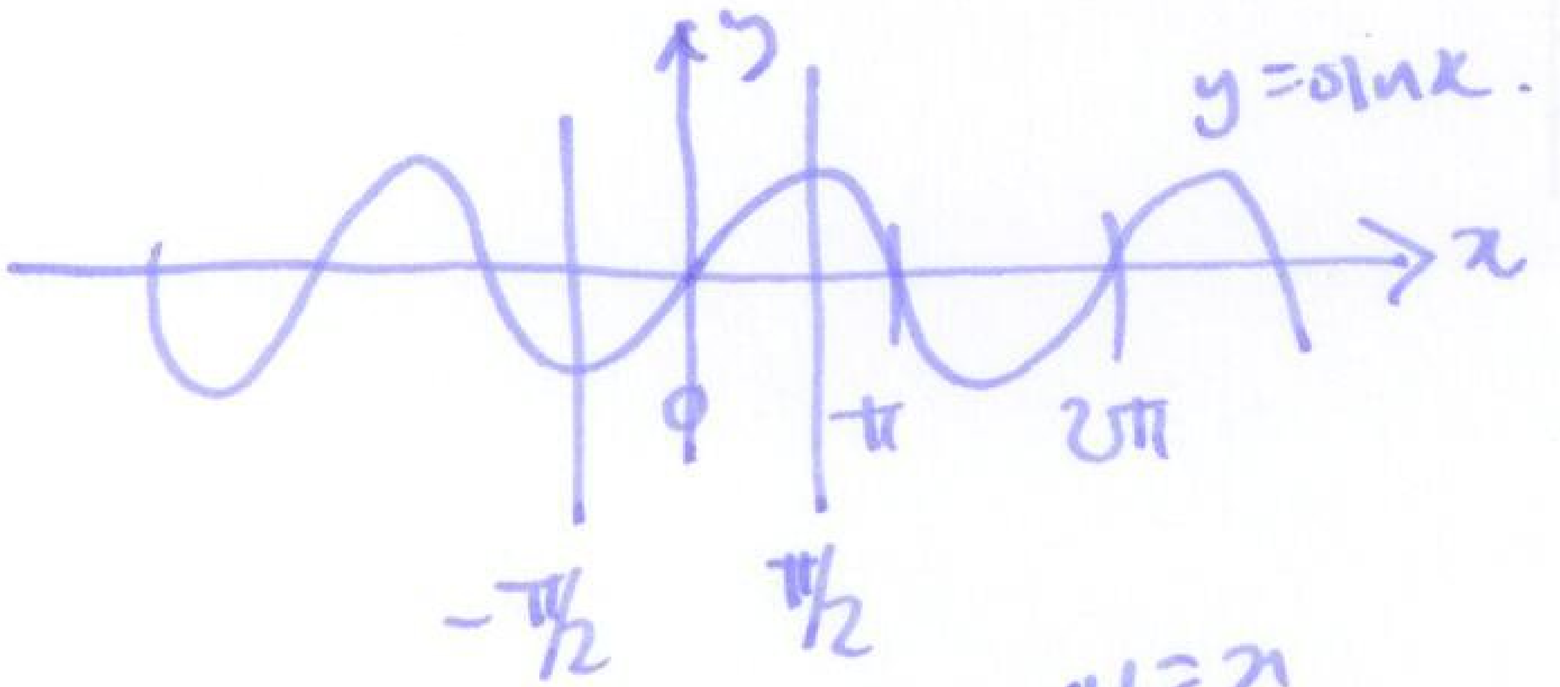
$\sin(x+y) + \sin(x-y) = 2 \sin x \cos y$

$\sin(x+y) - \sin(x-y) = 2 \sin y \cos x$

$\cos(x+y) + \cos(x-y) = 2 \cos x \cos y$

$\cos(x-y) - \cos(x+y) = 2 \sin x \sin y$.

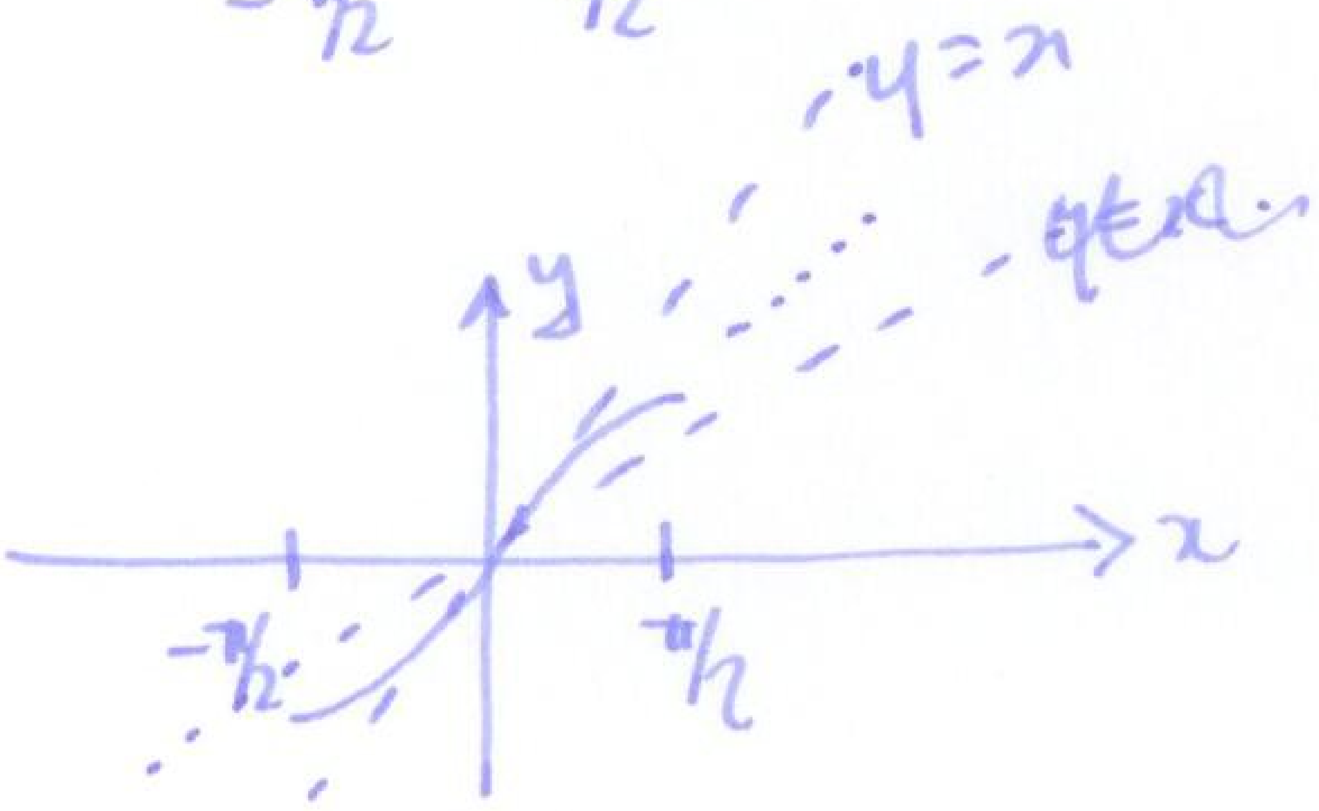
§7.4 Trig function inverses



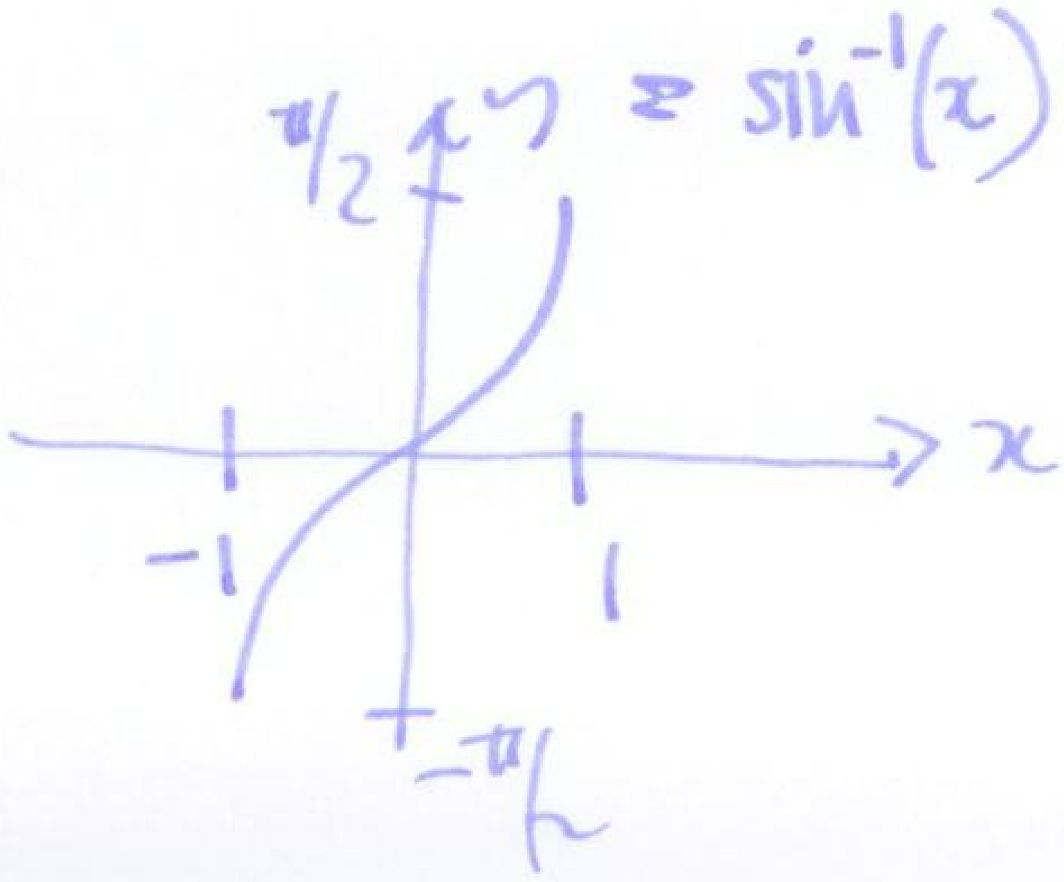
no inverse!

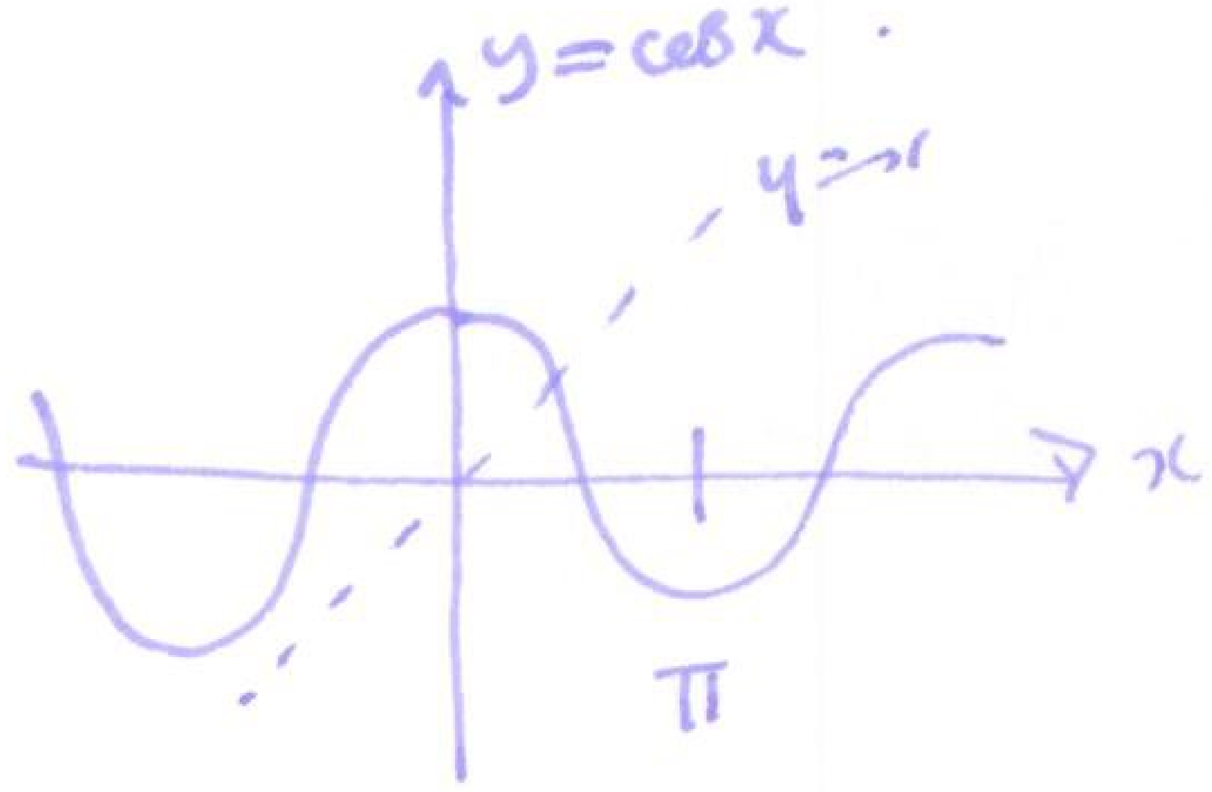
unless: restrict domain.

$-\frac{\pi}{2} \le x \le \pi/2$



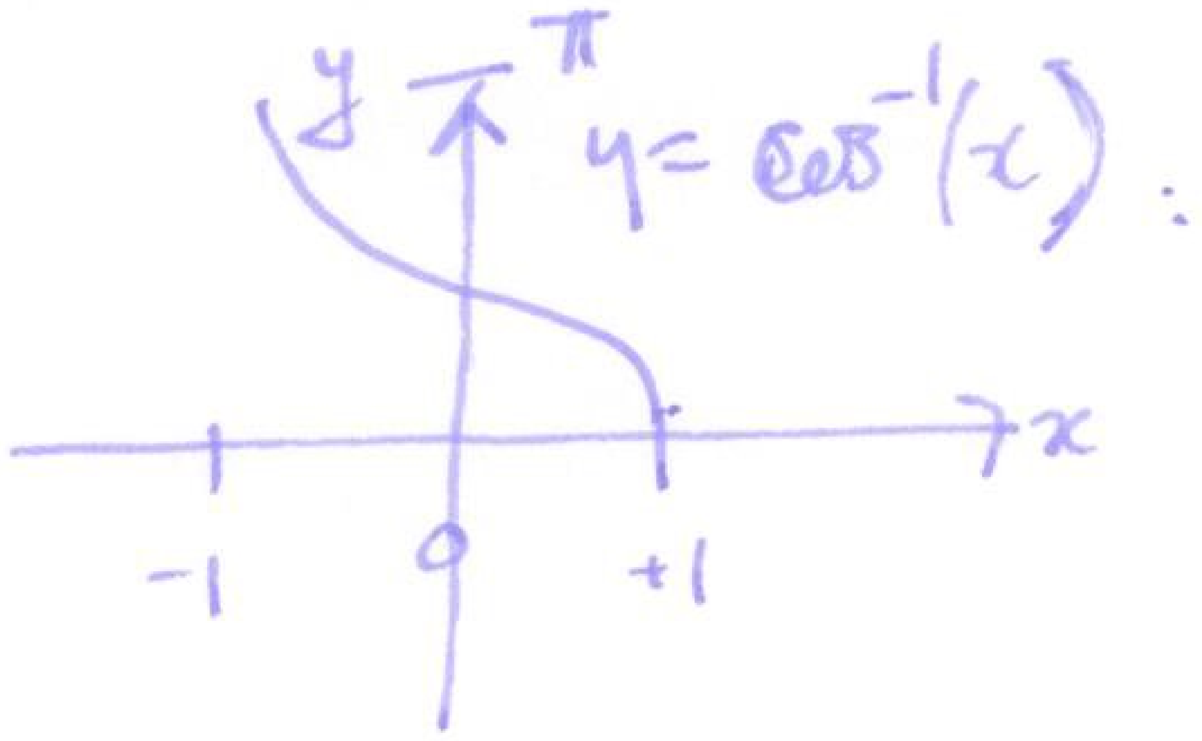
inverse

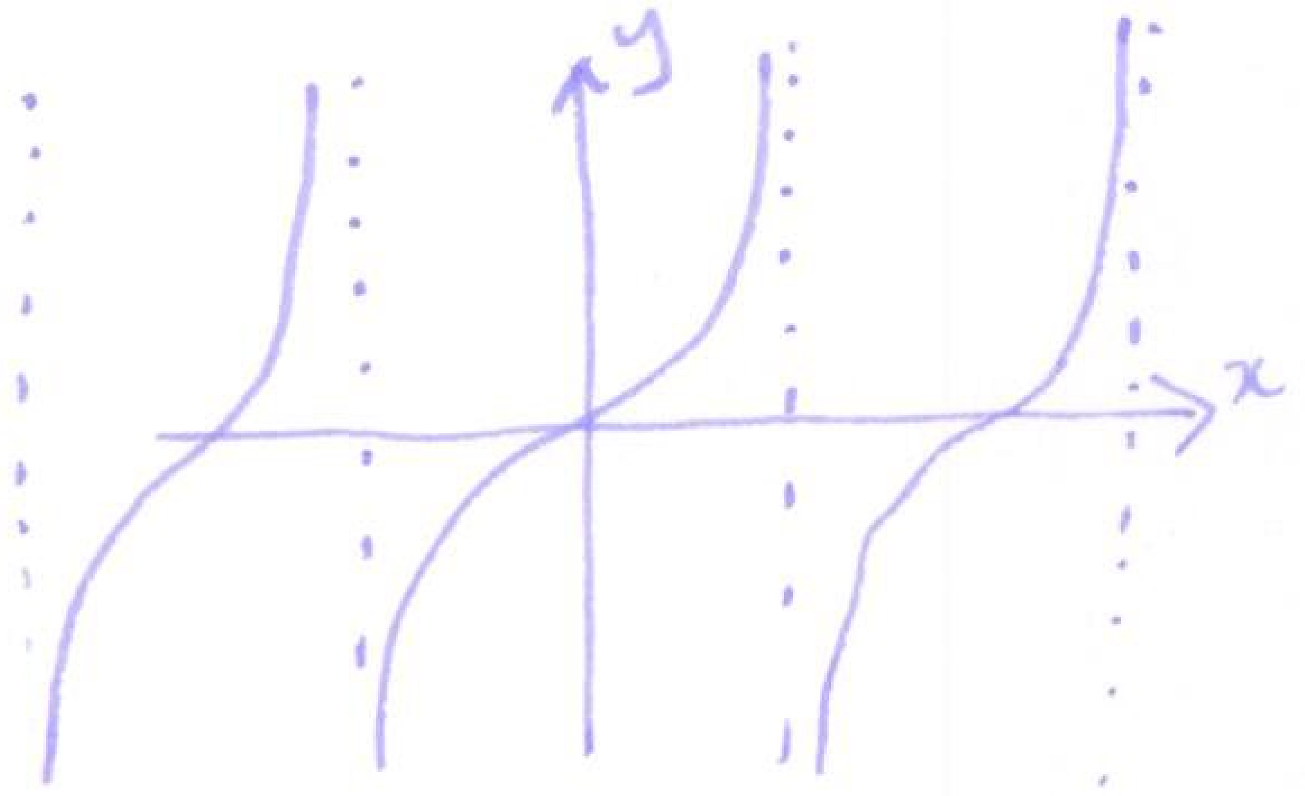




no inverse !

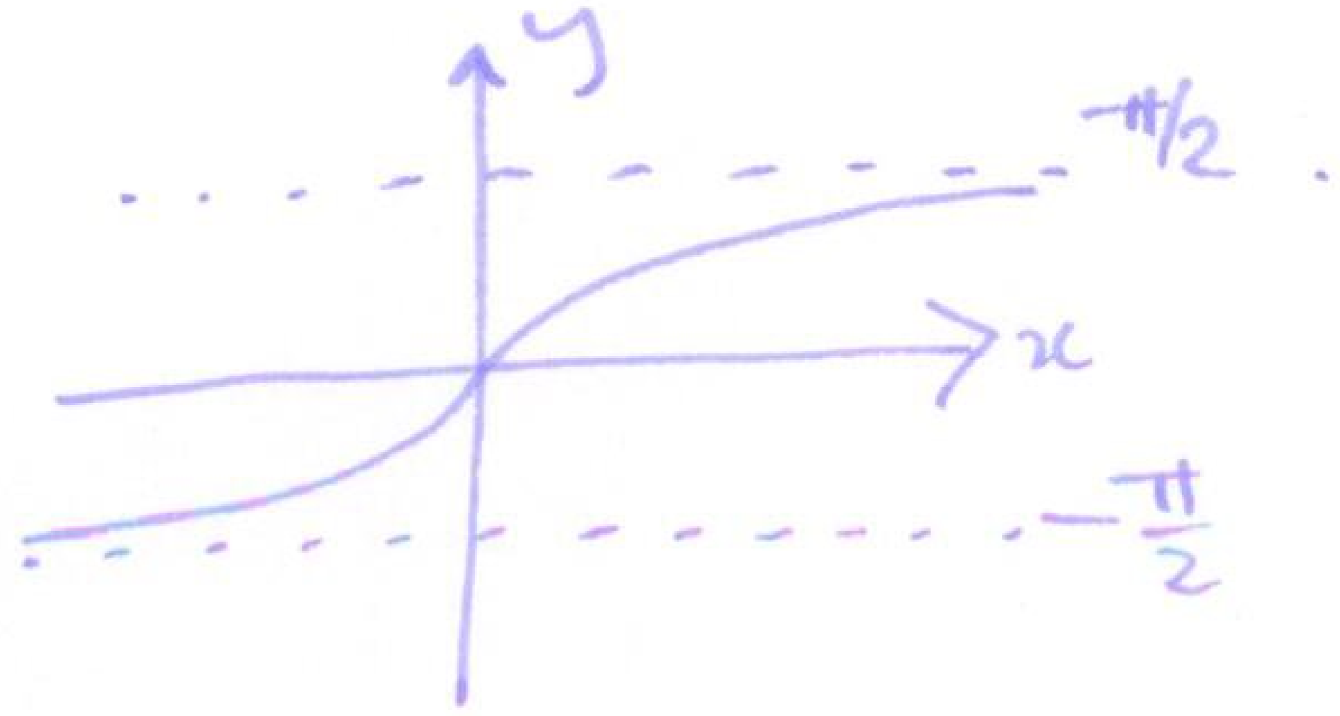
unless : restrict domain $0 \leq x \leq \pi$





no inverse !

unless : restrict domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$



<u>warning</u> $\sin^{-1}(x) \neq \frac{1}{\sin(x)}$

$\cos^{-1}(x) \neq \frac{1}{\cos(x)}$

$\tan^{-1}(x) \neq \frac{1}{\tan(x)}$

$\sin^{-1}(x) = \arcsin(x)$ inverse sin(x) function

$\cos^{-1}(x) = \arccos(x)$

$\tan^{-1}(x) = \arctan(x)$.

$\sin\left(\sin^{-1}(x)\right) = x \qquad -1 \leq x \leq 1$

$\cos\left(\cos^{-1}(x)\right) = x \qquad -1 \leq x \leq 1$

$\tan\left(\tan^{-1}(x)\right) = x \qquad$ all x .

<u>Q</u> what is

$\sin^{-1}\left(\sin(x + 100\pi)\right)$?

$-\pi/2 \leq x \leq \pi/2$

$\sin^{-1}\left(\sin(x)\right) = x \qquad -\pi/2 \leq x \leq \pi/2$

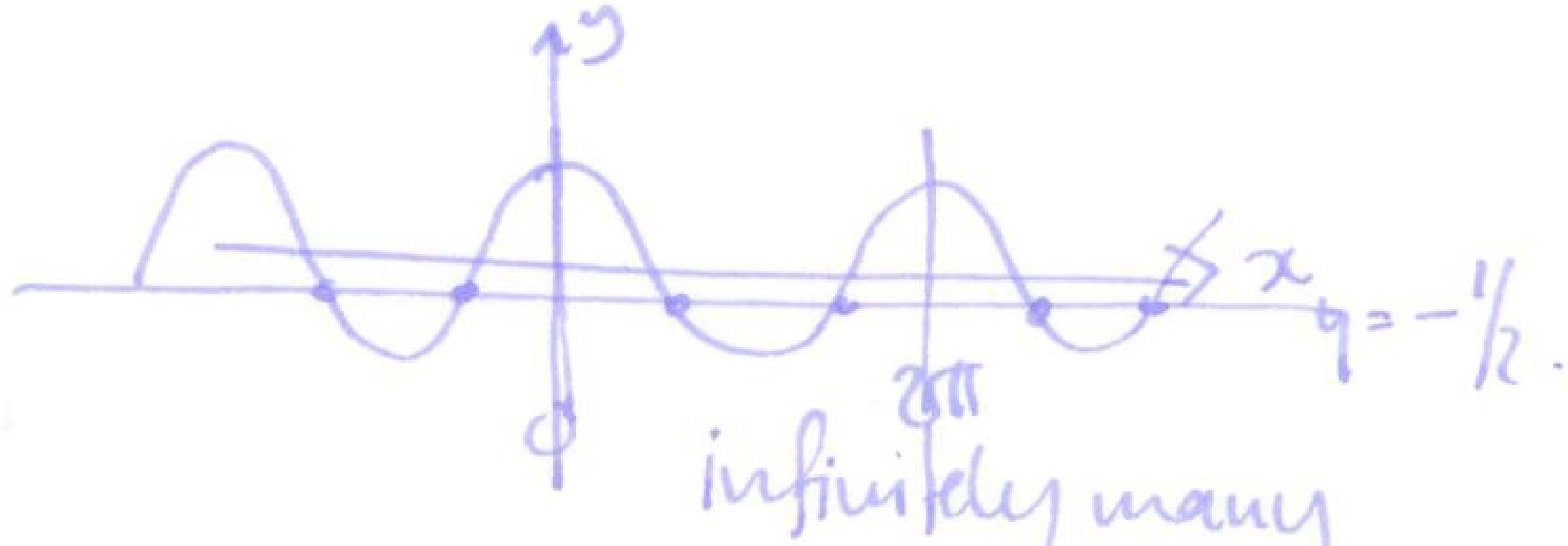
$\cos^{-1}\left(\cos(x)\right) = x \qquad 0 \leq x \leq \pi$

$\tan^{-1}\left(\tan(x)\right) = x \qquad -\frac{\pi}{2} \leq x \leq \pi/2$.

§7.5 Solving trig equations

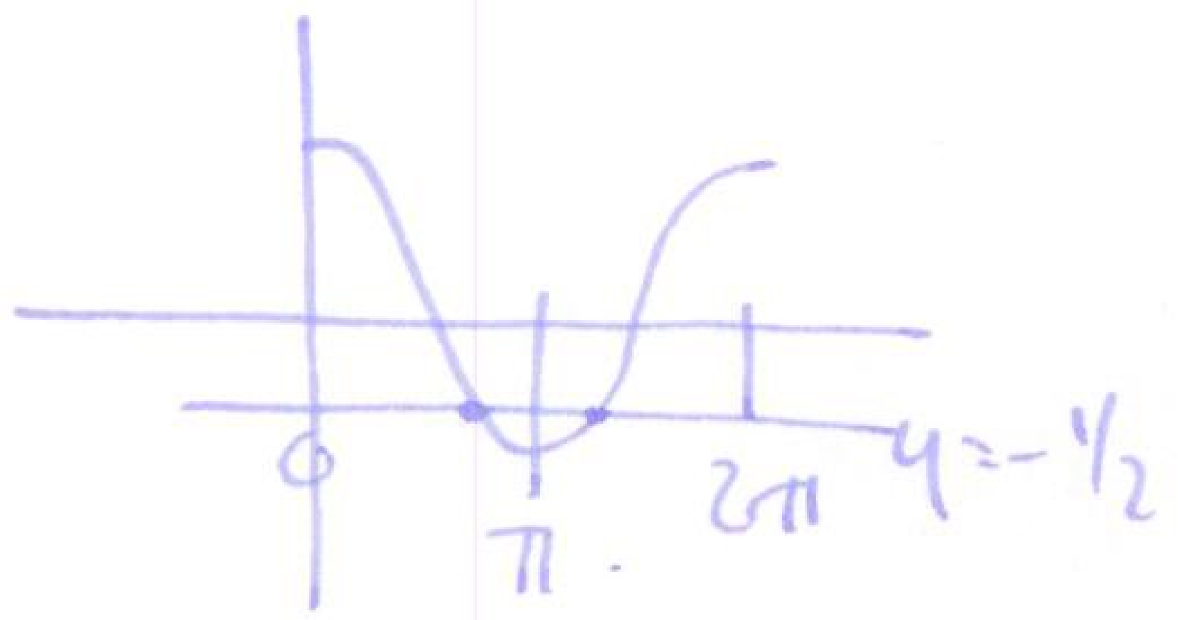
Example $2\cos x = -1$

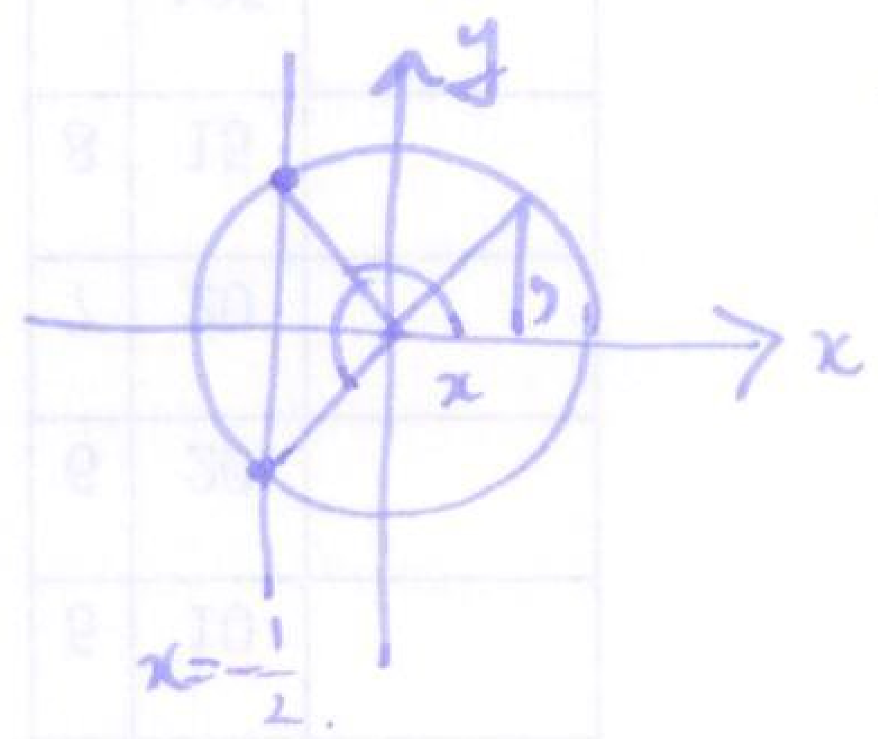
$\cos x = -\frac{1}{2}$



infinitely many solutions!

however $\cos x$ is 2π-periodic, so find all solutions $0 \le x \le 2\pi$ and add on all multiples of 2π.





$x = \cos\theta$

$y = \sin\theta$.

solutions $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$.

all solutions: $\frac{2\pi}{3} + 2\pi n, \quad \frac{4\pi}{3} + 2\pi n$.