<u>Phase shift</u> $y = A\sin(Bx + C) + D$

write as $y = A \sin\left(B\left(x + \frac{C}{B}\right)\right) + D$

phase shift is $-\frac{C}{B}$.

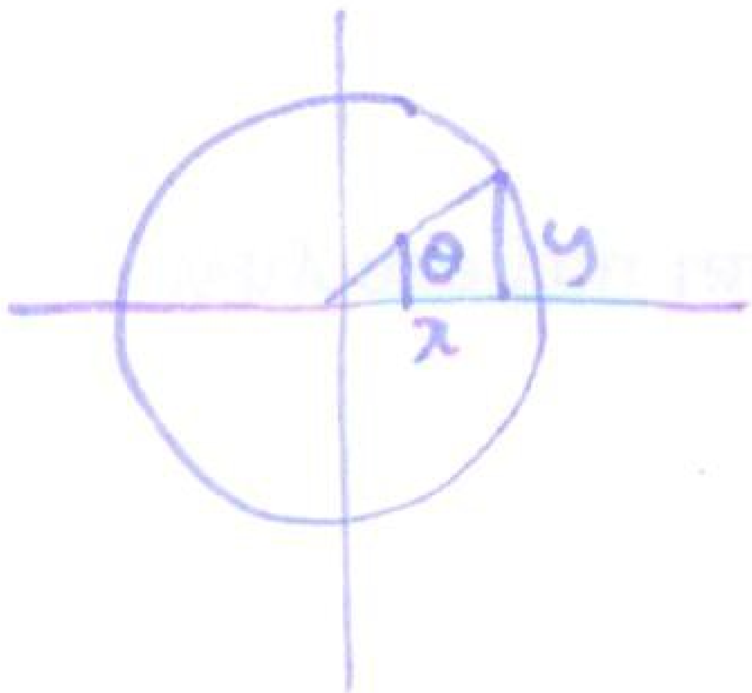
<u>note</u> $\sin(x)$ is a phase shift of $\cos(x)$ and vice versa.

§7.1 Trig identities

<u>recall</u> $\sin x = \frac{1}{\operatorname{cosec} x}$ $\cos x = \frac{1}{\sec x}$ $\tan x = \frac{1}{\cot x}$

$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

$\sin(-x) = -\sin x$
$\cos(-x) = \cos(x)$
$\tan(-x) = -\tan x$.



pythagoras : $x^2 + y^2 = 1$

$\Rightarrow$ $\boxed{\cos^2\theta + \sin^2\theta = 1}$ ← important.

<u>notation</u> : $\sin^2 x = (\sin x)^2$
$\cos^2 x = (\cos x)^2$ etc.

divide by $\sin^2\theta$:

$\cot^2\theta + 1 = \csc^2\theta$.

divide by $\cos^2\theta$:

$1 + \tan^2\theta = \sec^2\theta$.

<u>Example</u> simplify $\sin^2 x \cos^2 x + \cos^4 x$

$= \cos^2 x(\sin^2 x + \cos^2 x) = \cos^2 x$.

Example $\frac{\cot x}{\operatorname{cosec} x} = \frac{\frac{\cancel{\sin x}}{\cancel{\cos x}} \frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \cos x.$

$$\frac{2\sin^2 x + \sin x - 3}{1 - \cos^2 x - \sin x} = \frac{2\sin^2 x + \sin x - 3}{\sin^2 x - \sin x} = \frac{(2\sin x + 3)(\sin x - 1)}{\sin x(\sin x - 1)}$$

$$= \frac{2\sin x + 3}{\sin x} = 2 + \frac{3}{\sin x} = 2 + 3\operatorname{cosec} x.$$

Double angle formulae

$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

Sum formulae

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

useful trick:

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$
$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$\sin(A+B) + \sin(A-B) = 2\sin A \cos B.$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$