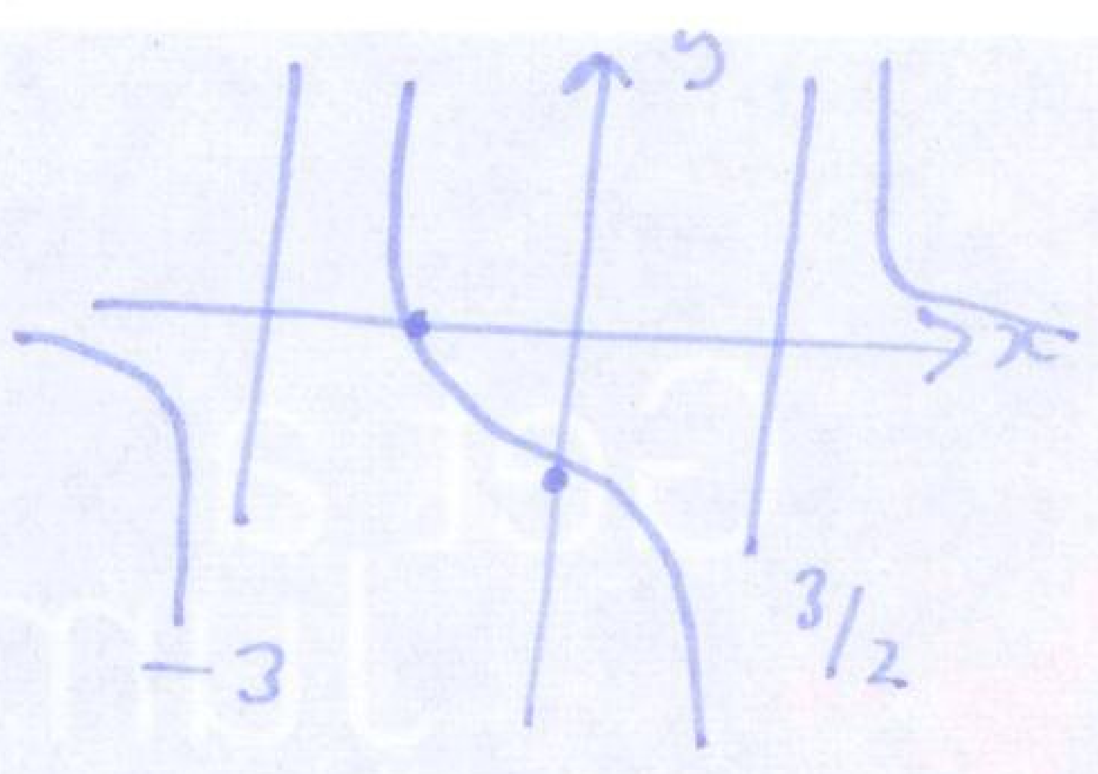
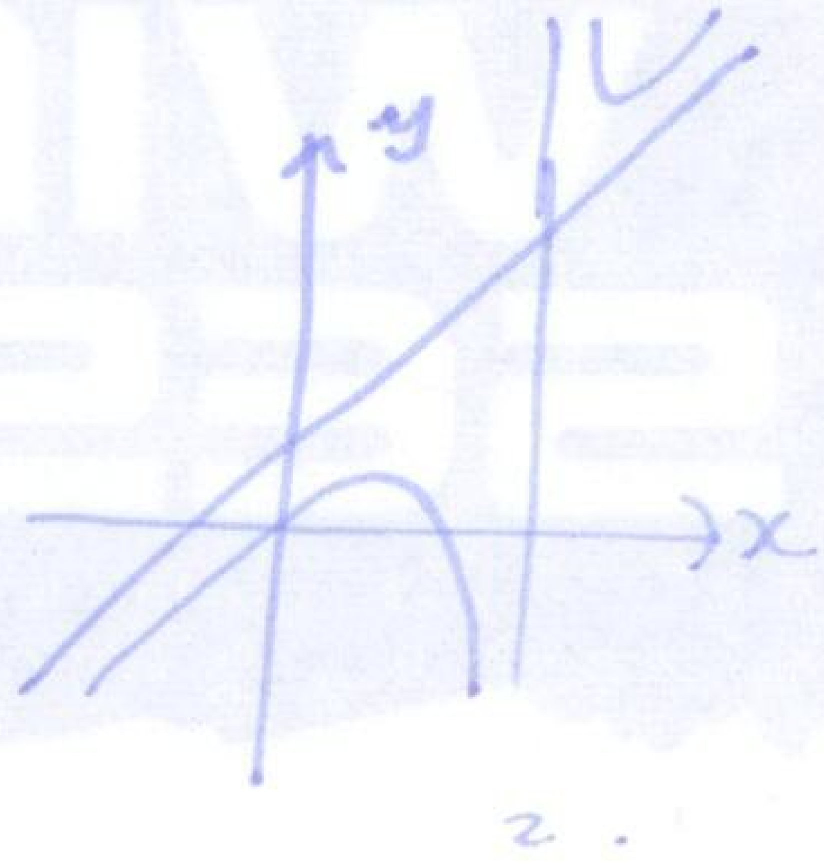
Example $\dfrac{2x+3}{3x^2+7x-6} = \dfrac{2x+3}{(3x-2)(x+3)}$



Example (diagonal asymptote)

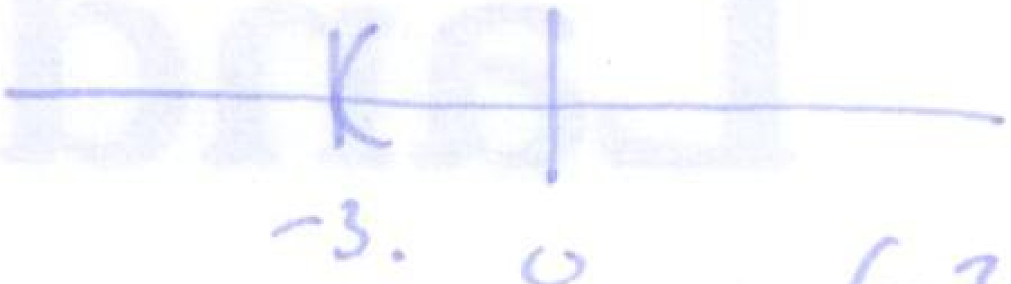
$\dfrac{2x^2-3x-1}{x-2} = 2x+1+\dfrac{1}{x-2}$



HW 12, 13, 14 due Wed 10^{th}
HW 16, 17 Sun 14^{th}
and §4.5 54, 64, 70, 80, 82. Tue 16^{th} in class

§4.6 Polynomial and rational inequalities

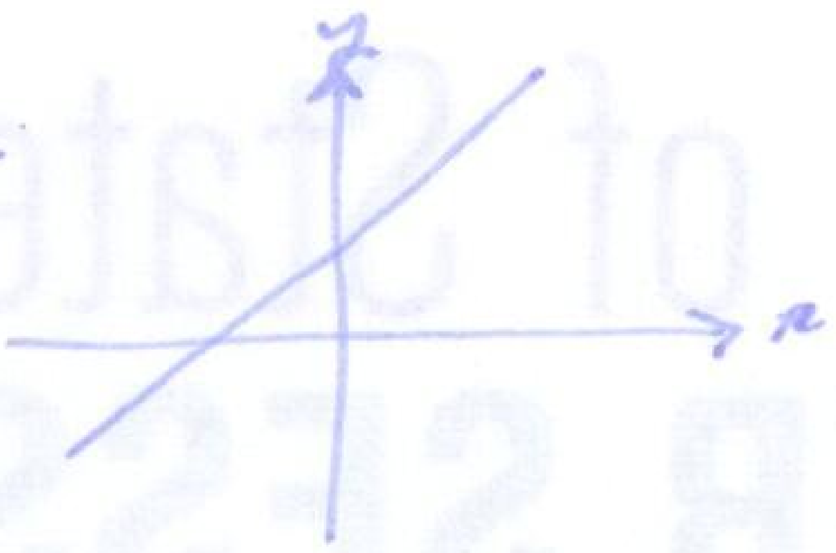
inequalities examples $x > -3$.



$(-3, \infty)$.

consider function $f(x) = x+3$.

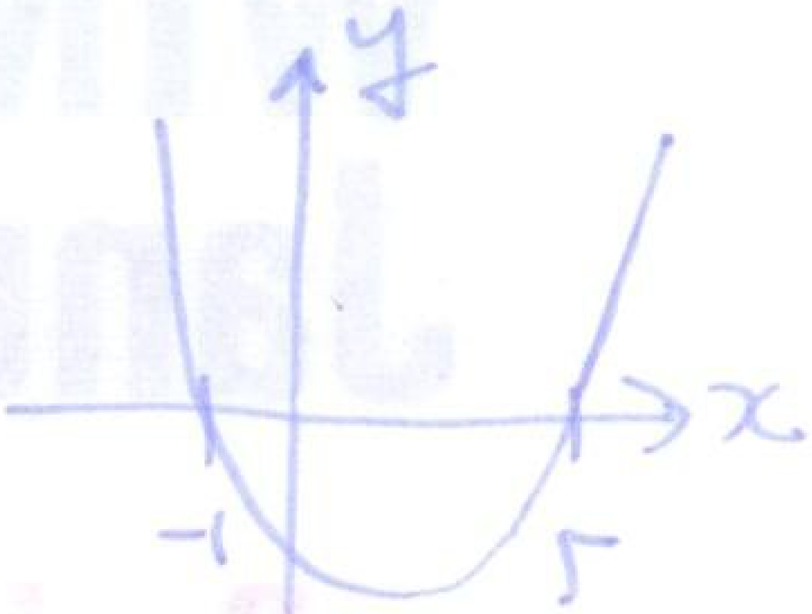
and look for $f(x) > 0$



Example $x^2 > 4x+5$

$x^2-4x-5 > 0$ consider $f(x) = x^2-4x-5$

$(x-5)(x+1)$

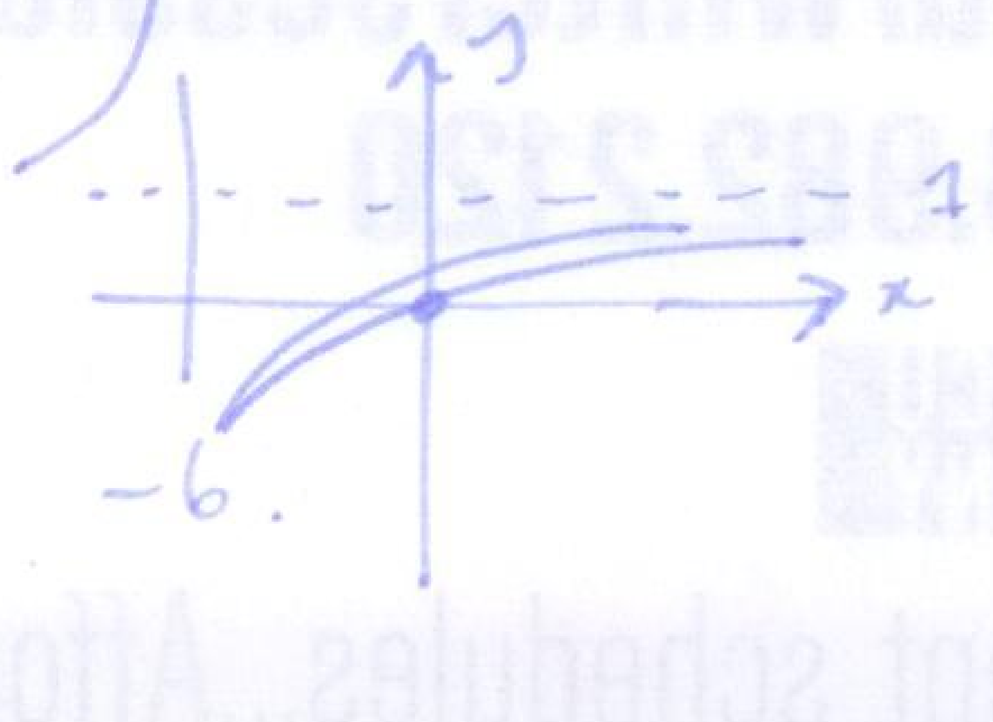


$(5, \infty) \cup (-\infty, -1)$

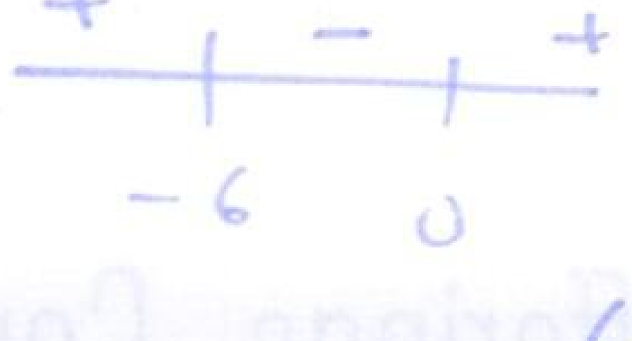
Example $\dfrac{x}{x+6} > 0$

Q: why don't we multiply by $x+6$?

$\dfrac{x}{x+6} = 1 - \dfrac{5}{x+6}$



$\dfrac{x}{x+6} = 0 \Rightarrow x = 0.$



$(-\infty, -6) \cup (0, \infty)$.

Example $\dfrac{x-3}{x+4} \geq \dfrac{x+2}{x-5}$

$\dfrac{x-3}{x+4} - \dfrac{x+2}{x-5} \geq 0 \qquad \dfrac{(x-3)(x-5)-(x+2)(x+4)}{(x+4)(x-5)} \geq 0$

$\dfrac{x^2-8x+15-x^2-6x-8}{(x+4)(x-5)} \geq 0 \qquad \dfrac{-14x-7}{(x+4)(x-5)} \geq 0 \qquad \dfrac{7(2x-1)}{(x+4)(x-5)} \geq 0.$

---

more graphs of rational functions

$\dfrac{2x-1}{x^2+x+6} = \dfrac{2x-1}{(x-2)(x+3)} \qquad \dfrac{2x^2+3x+1}{x^2-3x+2} = \dfrac{(x+1)(2x+1)}{(x-1)(x-2)}$

$\dfrac{x^3+x^2-4x-4}{x^2+x-12} = \dfrac{(x+1)(x-2)(x+2)}{(x-3)(x+4)}$

---

§2.3 Composition of functions

suppose we have two functions $f: \mathbb{R} \to \mathbb{R}$
$g: \mathbb{R} \to \mathbb{R}$.

previously: we can add/subtract them $f+g$, $f-g$
scale them cf
divide/ ~~divio~~ multiply fg f/g

we can also compose them $\mathbb{R} \xrightarrow{f} \mathbb{R} \xrightarrow{g} \mathbb{R}$
$x \mapsto f(x) \qquad g(f(x))$.

notation $\boxed{(g \circ f)(x) = g(f(x))}$ do f first!!

$$(f \circ g)(x) = f(g(x))$$

<u>Example</u> $f(x) = 2x - 1$ $\qquad g(x) = x^2 - 2x + 1$

$$(g \circ f)(x) = g(f(x)) = g(2x-1) = (2x-1)^2 - 2(2x-1) + 1$$

$$\begin{aligned} &= 4x^2 - 4x + 1 \\ &\quad\ \ - 4x + 2 \\ &\quad\qquad\ \ + 1 \\ &= 4x^2 - 8x + 4. \end{aligned}$$

$$(f \circ g)(x) = f(g(x)) = f(x^2 - 2x + 1) = 2(x^2 - 2x + 1) - 1$$

$$\begin{aligned} &= 2x^2 - 4x + 2 - 1 \\ &= 2x^2 - 4x + 1. \end{aligned}$$

<u>warning</u> ① $(f \circ g) \neq (g \circ f)$!!

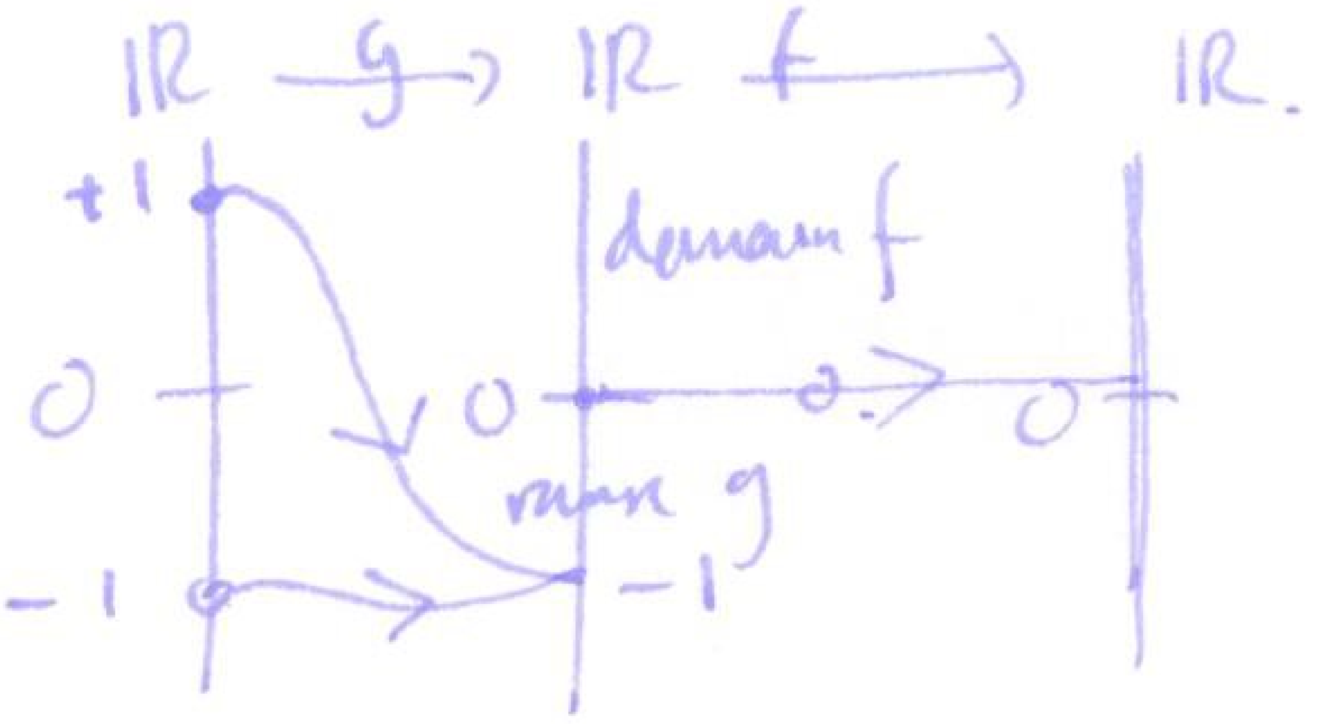
<u>warning</u> ② domain / range problems. ← normally have to work out domain range, so compound function may have smaller domain.

<u>Example</u> $f(x) = \sqrt{x}$ ← only defined for $x \geq 0$.

$g(x) = -x^2$ ← output always ≤ 0.

$(f \circ g)$, ~~$(g \circ f)$~~ : $\mathbb{R} \xrightarrow{g} \mathbb{R} \xrightarrow{f} \mathbb{R}$

$x \longmapsto -x^2 \longmapsto \sqrt{-x^2}$ only works for zero!



$$x \longmapsto -x^2 \longmapsto \sqrt{-x^2}.$$

$x \longmapsto \sqrt{x}$ ← confusing! write $y \longmapsto \sqrt{y}$

Example $f(x) = \dfrac{1}{x-2}$ $g(x) = \dfrac{3}{x}$ find $\left.\begin{matrix} f\circ g \\ g\circ f \end{matrix}\right\}$ and domains.

$$f\circ g(x) = f(g(x)) = f\left(\frac{3}{x}\right) = \frac{1}{\frac{3}{x}-2} = \frac{x}{3-2x}$$

domain : $3-2x \neq 0$ $x \neq 3/2 = (-\infty, 3/2) \cup (3/2, \infty)$

$$(g\circ f)(x) = g(f(x)) = g\left(\frac{1}{x-2}\right) = \frac{3}{1/x-2} = 3x-6$$

domain : $\mathbb{R} = (-\infty, \infty)$

Decomposing functions as compositions

given $h(x) = (3x+1)^4$ we can write it as.

$h(x) = g(f(x))$ where $f(x) = 3x+1$
$g(x) = x^4$

check : $g(f(x)) = g(3x+1) = (3x+1)^4$.

often you can decompose in many ways, as

$h(x) = g(f(x))$ where $f(x) = (3x+1)^2$
$g(x) = x^2$.

check : $g(f(x)) = g((3x+1)^2) = ((3x+1)^2)^2 = (3x+1)^4$.