<u>note</u> $P(x)$ polynomial

$d(x)$ polynomial of degree $\leq P(x)$

then $P(x) = d(x) \cdot Q(x) + R(x)$ and degree of $R(x) <$ deg of d.

$\uparrow$ divisor $\uparrow$ quotient $\uparrow$ remainder.

<u>Thm</u> The remainder theorem.

$P(x)$ polynomial, divide by $x-c$, then

$$P(x) = \cancel{d(x)}\,(x-c)\,Q(x) + R(x).$$

evaluate at c: $P(c) = \underbrace{(c-c)}_{0} Q(c) + R(c)$

$= R(c)$

if you divide $P(x)$ by $(x-c)$, then the remainder is $\cancel{R(c)}$ $P(c)$.

This gives a quick check that you did long division correctly...

<u>Thm</u> The factor theorem

If $P(x)$ is a polynomial and $P(c) = 0$, then $x-c$ divides $P(x)$.

<u>Proof</u> long division: $P(x) = (x-c)\,Q(x) + R(x)$.

<u>Q</u>: what is the remainder? $P(c) = \underbrace{(c-c)}_{0}\,Q(c) + R(c)$

$0 = P(c) = R(c)$ so $P(x) = (x-c)\,Q(x) + 0$. $\square$

Last time polynomials

Long division $p(x)$ polynomial

$d(x)$ polynomial of degree $\leq p(x)$.

then
$$d(x)\overline{\big)\, p(x)\,}^{\,q(x)}$$

$$\boxed{p(x) = d(x)q(x) + r(x).}$$

$r(x)$ remainder degree $r(x) <$ degree $d(x)$.

Remainder theorem

if $d(x)$ is a linear polynomial $d(x) = x - c$

then $R(x) = p(c)$.

check
$$p(x) = d(x)\,q(x) + r(x)$$
$$p(x) = (x-c)\,q(x) + r(x)$$
$$p(c) = 0\,q(c) + r(c) \quad \square.$$

so $f(c) = 0 \iff (x-c)$ is a factor of $p(x)$.

Example x^2+1 $\qquad (i)^2+1 = -1+1 = 0.$

$$\begin{array}{r|l} & x + i \\ \hline x - i & x^2 + 0x + 1. \\ & x^2 - ix \\ \hline & ix + 1 \\ & ix - i^2 \end{array}$$

$$\begin{array}{r} ix + 1 \\ ix + 1 \\ \hline 0. \end{array}$$

so $\qquad x^2+1 = (x+i)(x-i).$

check $\qquad = x^2 + ix - ix - i^2 = x^2+1.$

§4.4 Zeros of polynomials

the coefficients of a polynomial are the numbers in front of the x's.

eg $\qquad x^2+x+1$ integer coefficients.

$\frac{1}{2}x^2+\frac{1}{3}x-\frac{2}{7}$ rational coefficients

$\pi x^2+ex-\sqrt{3}$ real coefficients.

$ix^2+(1+i)x-3-i$ complex coefficients

Fundamental theorem of algebra

Every polynomial of degree n with $n \geq 1$ has at least one ~~comp~~ root (possibly a complex number).

Corollary every polynomial of degree n can be factored into n linear factors $f(x) = a_n(x-c_1)(x-c_2)\dots(x-c_n).$