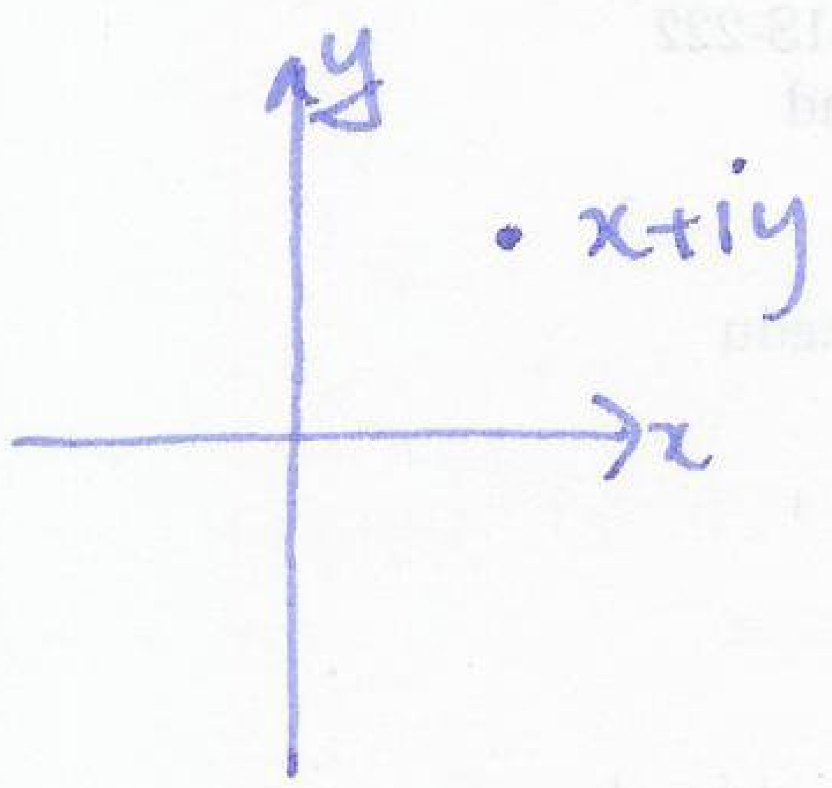
division. $\frac{1}{i}$ trick $\frac{1}{i}\,\frac{i}{i} = \frac{i}{(i)^2} = \frac{i}{-1} = -i$

$$\frac{1}{1+i} = \frac{1}{1+i}\,\frac{1-i}{1-i} = \frac{1-i}{1+i-i+1} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i.$$

complex numbers can be thought of as 2-d numbers.



§4.1 Polynomials $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

examples. $p(x) = 2$ (constant).

$p(x) = 3x - 1$ (linear)

$p(x) = x^2 + x + 1$ (quadratic).

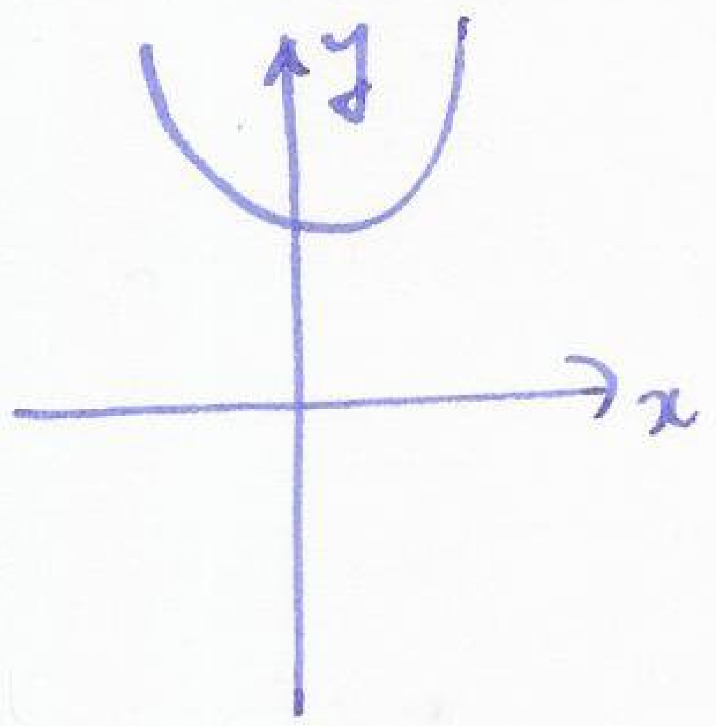
$p(x) = x^3 - x^2 - x + 1$ (cubic) quartic quintic.

degree : largest power of x

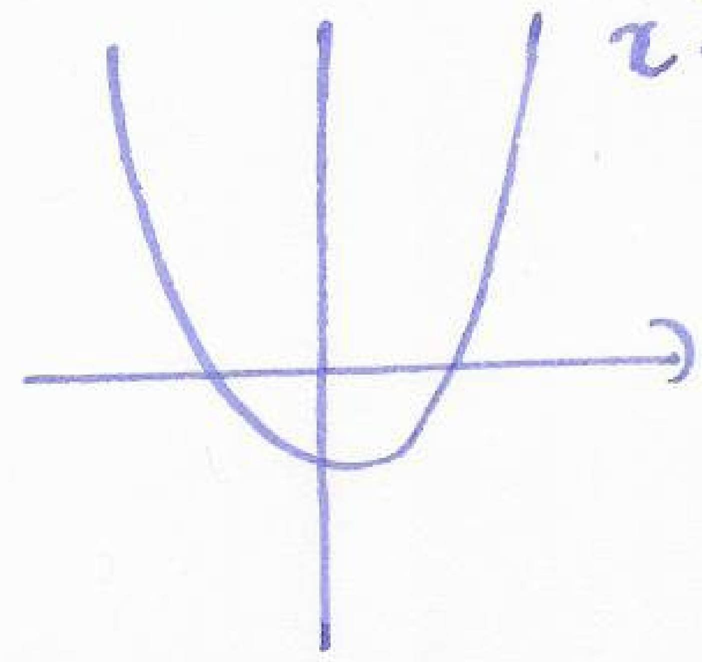
		degree	leading term	leading coeff
examples :	2	0	2	2
	$4x+1$	1	$4x$	4
	$6x^2 - 3x - 4$	$2.$	$6x^2$	6
	$-x^3 - x$	$3.$	$-x^3$	-1

not polynomials $\quad x+\frac{1}{x}$, $x^2+\sqrt{x}$ etc.

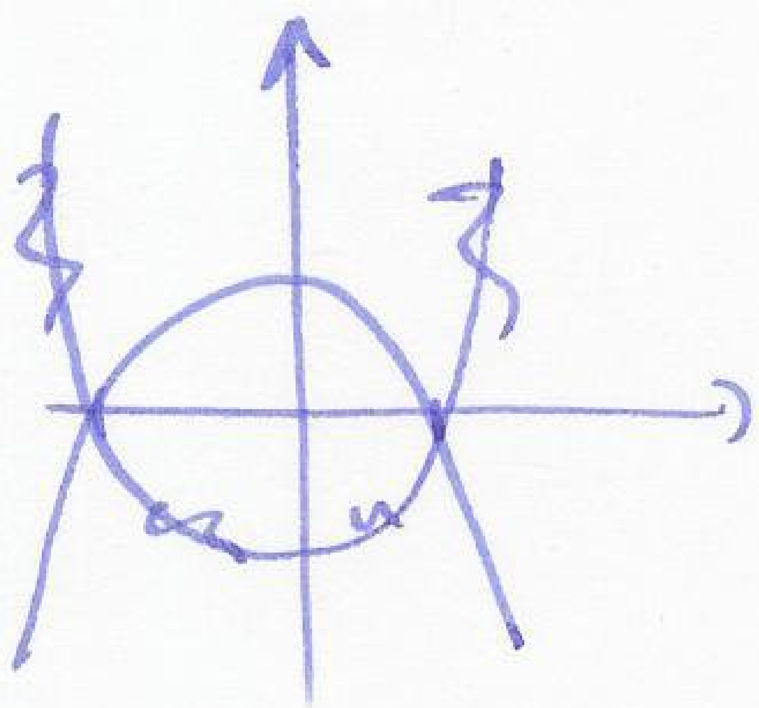
sketching polynomials



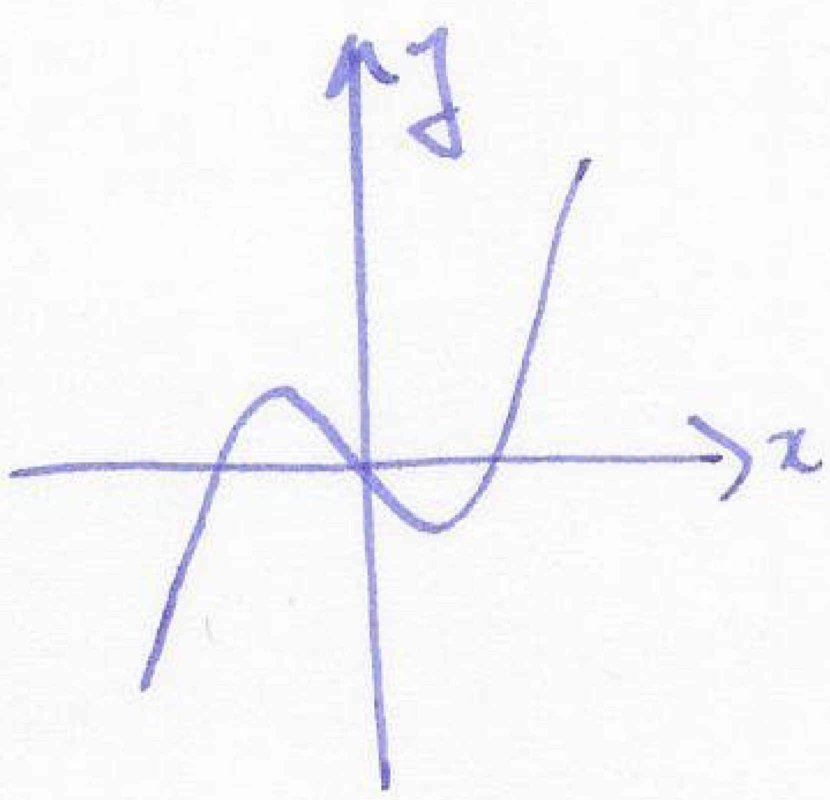
x^2+4



$x^2-4 = (x-2)(x+2)$

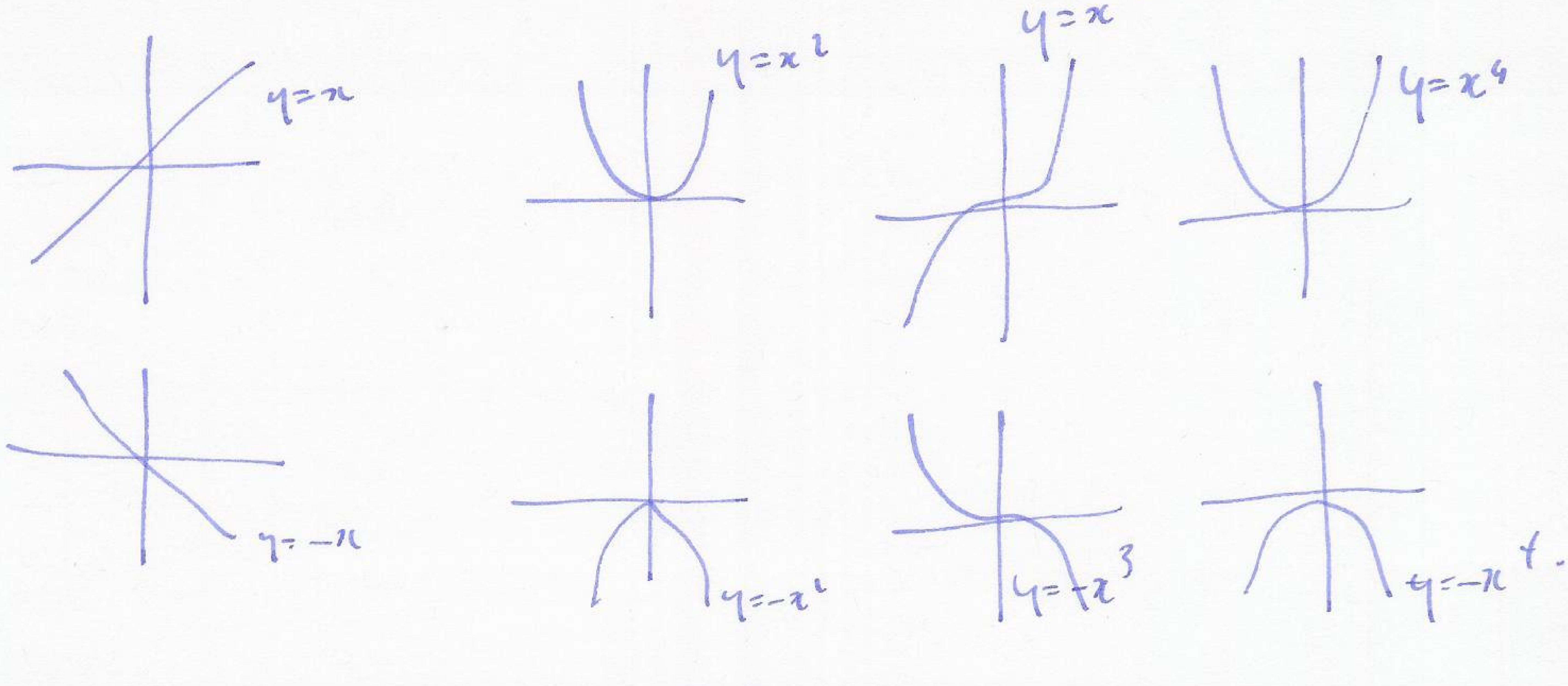


$(1-x)(x+2) = -x^2-x+2$

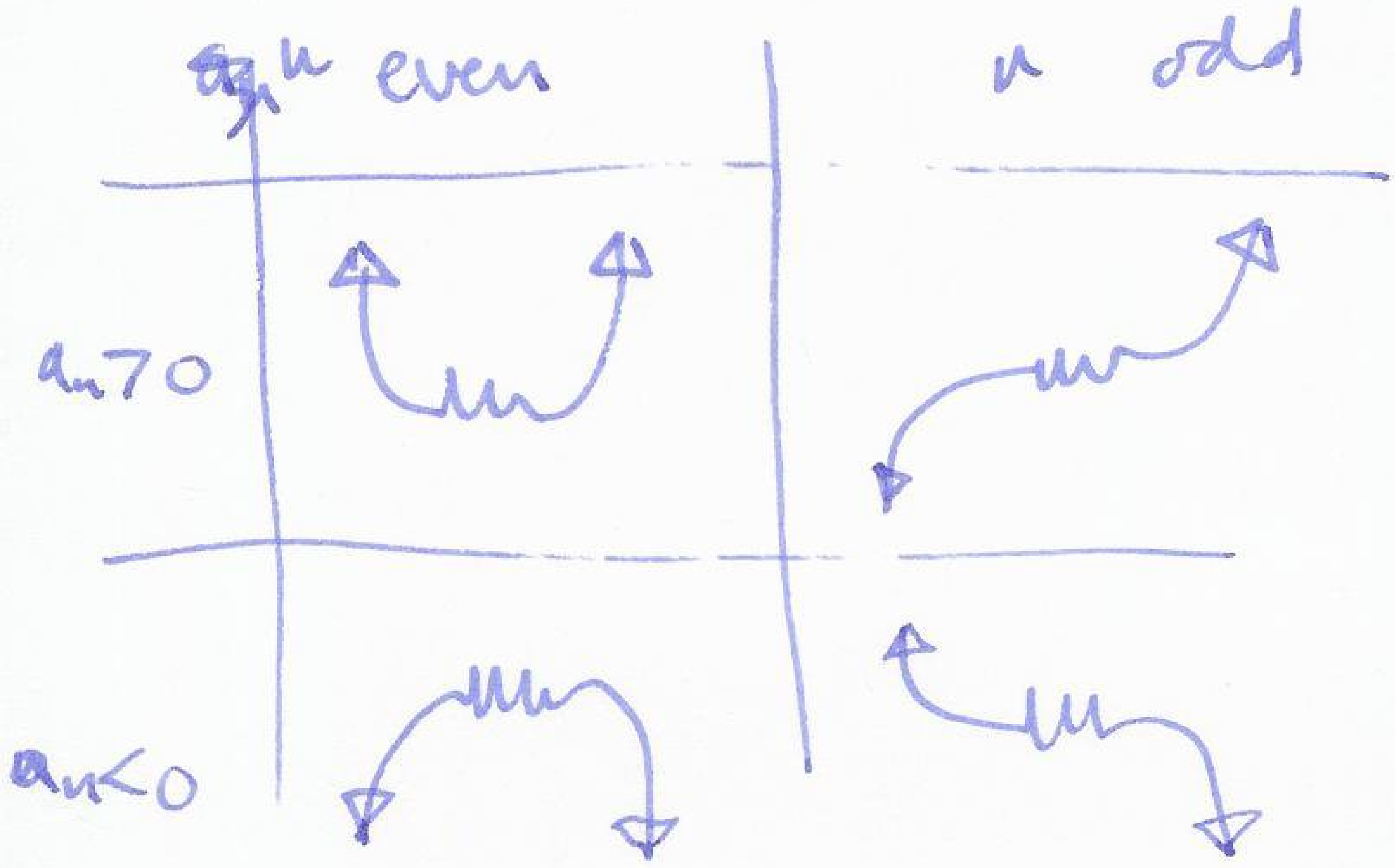


$$x(x^2-1) = x^3-x$$
$$= x(x+1)(x-1)$$

consider



leading term test



in words

n even $a_n > 0$

$f(x) \to \infty$ as $x \to \infty$

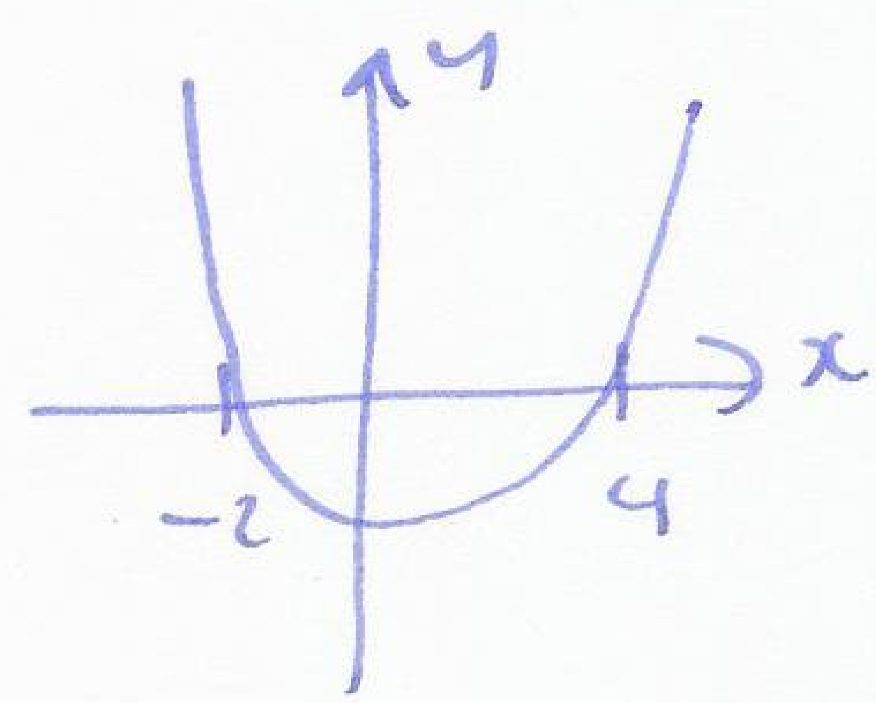
$f(x) \to \infty$ as $x \to -\infty$

etc.

Finding zeros of factored polynomials

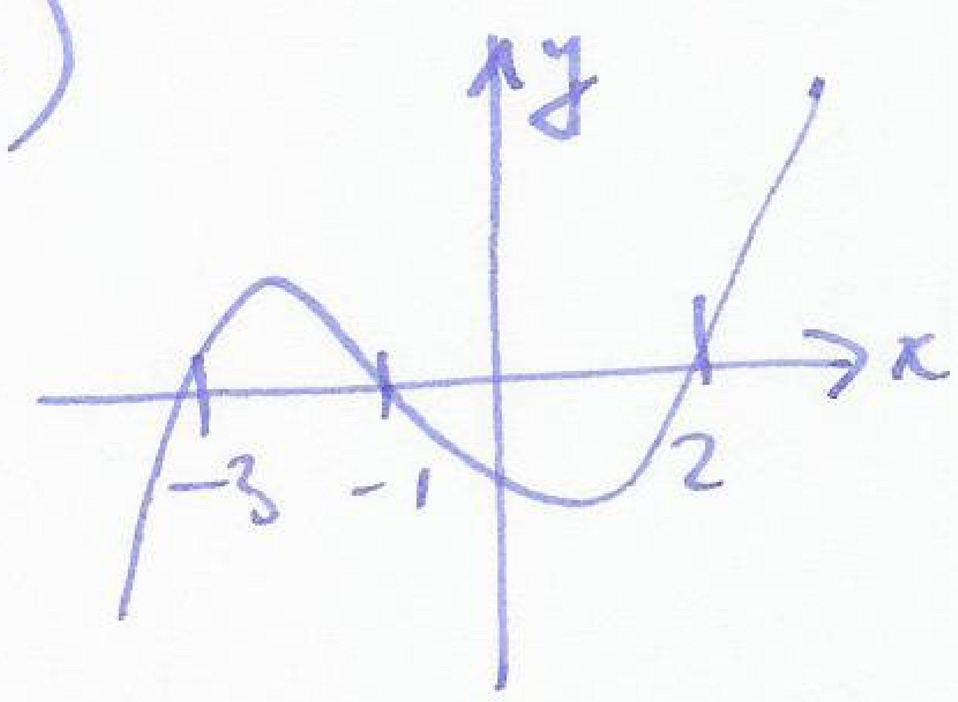
Examples

quadratic: $x^2 - 2x - 8 = (x+2)(x-4)$

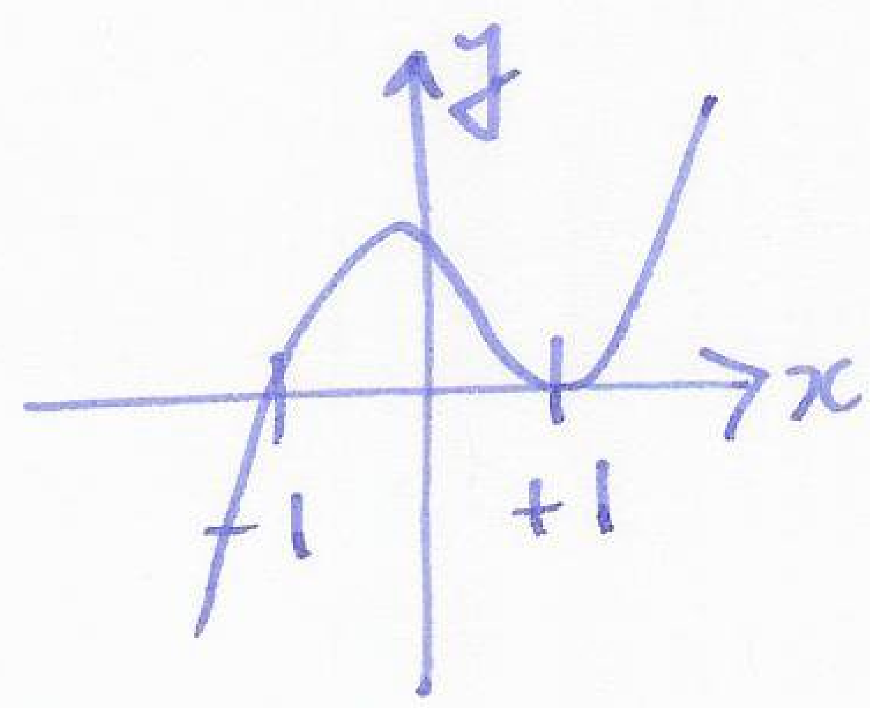


(formula: $ax^2 + bx + c = 0 \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.)

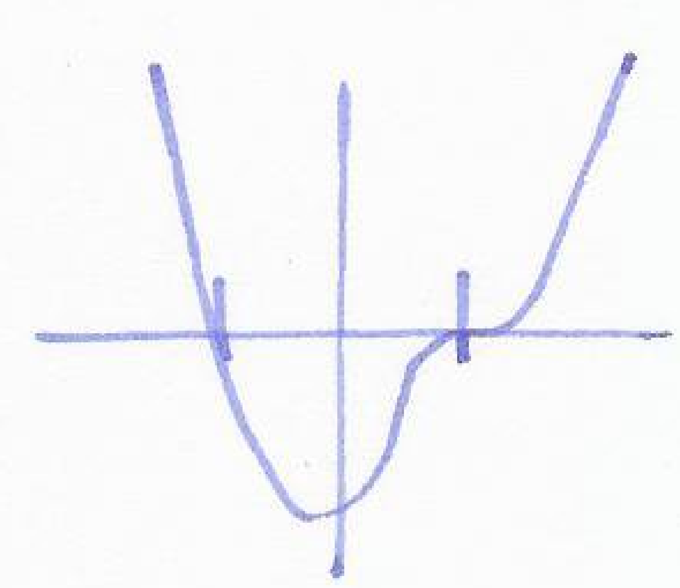
cubic: $x^3 + 2x^2 - 5x - 6 = (x+3)(x+1)(x-2)$



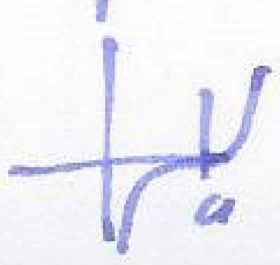
what about $(x-1)^2(x+1)$



$(x-1)^3(x+1)$



summary $(x-a)^n$ largest $(x-a)$ factor in $p(x)$

then n even  n odd 

factoring complicated polynomials (hard - can only do approximately in general).

techniques.

- look for an easy factor $x^3+x = x(x^2+1)$

$$x^4-x^3+x-1$$
$$x^3(x-1)+(x-1) = (x^3+1)(x-1).$$

- guess a root x^3+1 -1 works

$$= (x+1)(x^2-x+1).$$

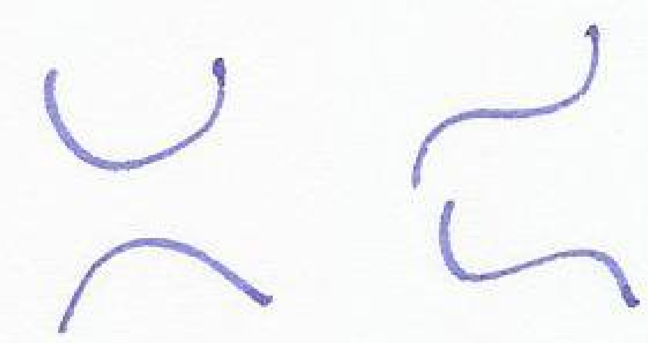
- x^4+2x^2+1 quadratic in x^2.

$$= (x^2+1)^2.$$

Application Ibuprofen in bloodstream. $M(t)=0.5t^4+3.45t^3-96.65t^2+347.7t$.

§4.2 Graphing polynomial functions

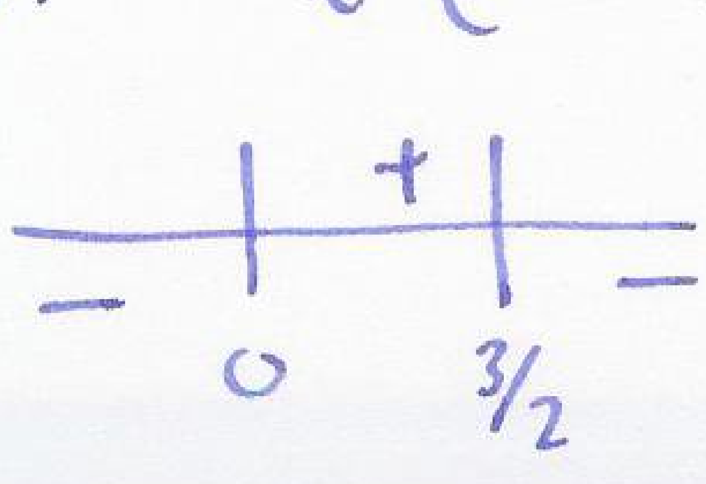
useful facts • large scale behavior even/odd etc.

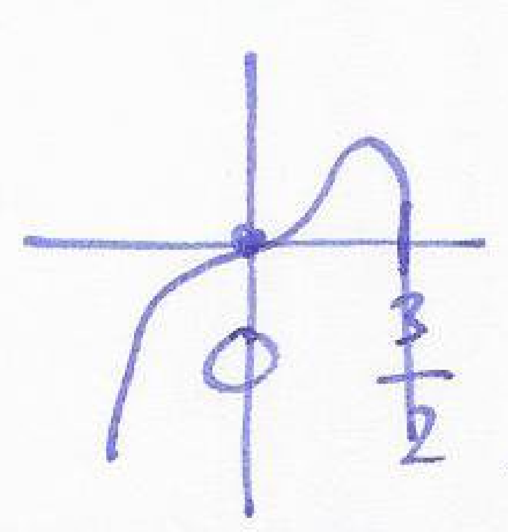


- degree n then • at most n zeros.
  - at most $n-1$ turning points.

Example $-2x^4+3x^3$. leading term:

zeroes: $x^3(-2x+3)$





summary

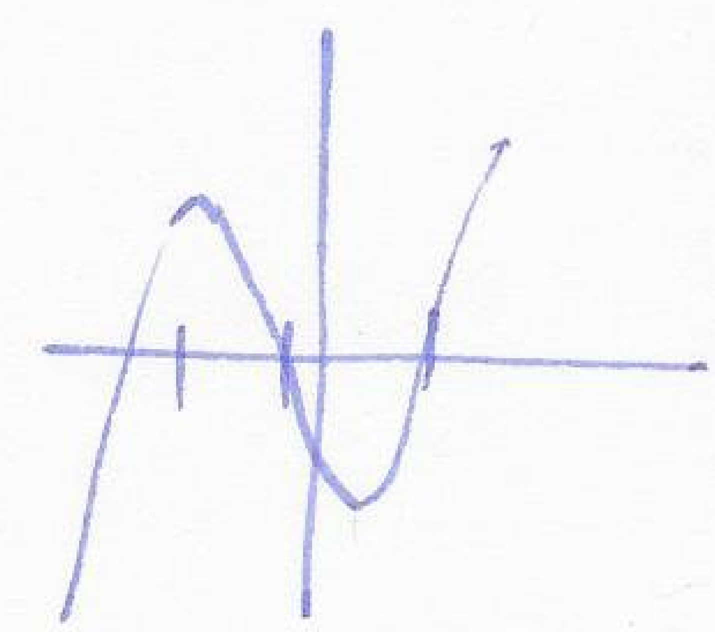
- use leading term to give large scale behavior.
- find zeroes (solve $f(x)=0$) (factor / calculator approx).
- use zeros and test for +/- between zeroes.
- find $f(0)$ (y-intercept)
- find some extra values.
- check at most n zeros / at most n-1 turning points.

Example $f(x) = 2x^3 + x^2 - 8x - 4$.

$x^2(2x+1) - 4(2x+1)$

$(2x+1)(x^2-4)$

$(2x+1)(x+2)(x-2)$



§4.3 Division, remainders, factors

example $p(x) = x^3 + 2x^2 - 5x - 6 = (x+3)(x+1)(x-2)$

factors are $x+3$, $x+1$, $x-2$

zeros are -3, -1, 2

~~So~~ we can multiply the factors to get a polynomial, can we divide by a factor?

Long division

$$x+1 \overline{\big)\, x^3 + 2x^2 - 5x - 6}$$

$$\frac{x^3 + 2x^2 + 5x - 6}{x+1}$$

$$
\begin{array}{r|l}
 & x^2 + x - 6 \quad \leftarrow \text{quotient} \\
\hline
x+1 & x^3 + 2x^2 - 5x - 6 \\
 & \underline{x^3 + x^2} \\
 & \qquad x^2 - 5x - 6 \\
 & \qquad \underline{x^2 + x} \\
 & \qquad\qquad -6x - 6 \\
 & \qquad\qquad \underline{-6x - 6} \\
 & \qquad\qquad\qquad 0 \quad \leftarrow \text{remainder}
\end{array}
$$

so $\dfrac{x^3+2x+5x-6}{x+1} = x^2+x-6$

what about: $\dfrac{x^3+2x^2+5x-6}{x-1}$? $= x^2+3x-2+\dfrac{-8}{x-1}$

$$
\begin{array}{r|l}
 & x^2 + 3x - 2 \\
\hline
x-1 & x^3 + 2x^2 - 5x - 6 \\
 & \underline{x^3 - x^2} \\
 & \quad 3x^2 - 5x - 6 \\
 & \quad \underline{3x^2 - 3x} \\
 & \qquad -2x - 6 \\
 & \qquad \underline{-2x + 2} \\
 & \qquad\quad -8 \quad \leftarrow \text{remainder}
\end{array}
$$

$x-1$ not a factor of $p(x)$.