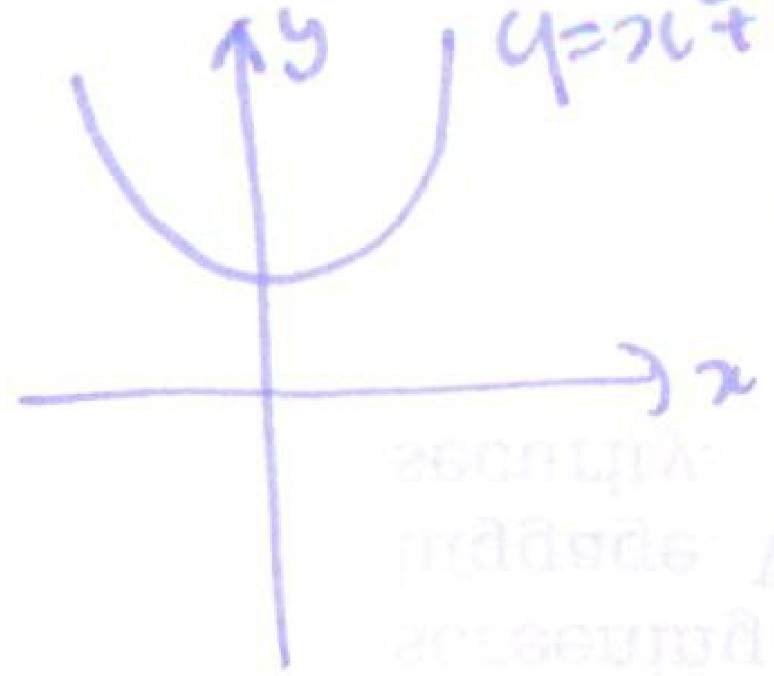
§3.1 Complex numbers.



$y=x^2+1$ $\sqrt{-1}$ is not a real number.

(no solutions).

However, it makes sense and is useful, to consider a new number i such that $i^2=-1$.

$\{1,2,3,\ldots\}$ $\mathbb{N}$. i is not a real number we say it is a complex number

$\{0,1,2,3,\ldots$ $\mathbb{N}\cup\{0\}$.

$\mathbb{Z}$ $\{\ldots-1,0,1,\ldots\}$.

$\mathbb{Q}$ a/b

$\mathbb{R}$.

Rules for complex numbers.

multiples of i: ~~4i~~ $2i$

$(2i)^2 = 4\cdot -1 = -4$ so $\sqrt{-4}=2i$.

$(-2i)^2 = (-2)^2(i)^2 = 4\cdot -1 = -4$

so $-2i$ also a square root of -4.

~~e.g.~~ arbitrary multiple bi $(bi)^2 = (b)^2(i)^2 = b^2 \cdot -1 = -b^2$.

can add real/complex numbers: $\underline{a+bi}$ arbitrary complex number.

e.g. $0+i=i$

$1+i$

$3+7i$

$\underbrace{a}_{\text{real part}}+\underbrace{bi}_{\text{imaginary part}}$

if $b=0$ say number is real

if $a=0$ say number is purely imaginary

addition $(a+bi)+(c+di) = (a+c)+(b+d)i$

example $(1+i)+(1-i) = 2$

$(1+i)+(-1+i) = 2i$

multiplication $(a+bi)(c+di) = ac + adi + bci + bd(i)^2$

$= ac + i(ad+bc) + bd.-1.$

$= ac-bd + i(ad+bc).$

example $(1+i)(1-i) = 1 - i + i + i.(-i)$

$= 1 \cancel{-i+i} + 1 = 2.$

$(i)^2 = -1$

$(i)^3 = (i)^2.i = -i$

$(i)^4 = (i^2)(i^2) = (-1)(-1) = 1$ etc.

$(i)^5 = i$

complex conjugates

the conjugate of $a+bi$ is $a-bi$

example $3+2i$ has conjugate $3-2i$

$1-i$ $\quad$ $1+i$

$-1-i$ $\quad$ $-1+i$

$(3+2i)(3-2i) = 9-6i+6i-4.(-1) = 9+4 = \underbrace{13}_{\text{real}}$!

$(a+bi)(a-bi) = a^2 - abi + abi - b^2(i)^2$

$= a^2+b^2 + i(ab-ab) = a^2+b^2$ real!