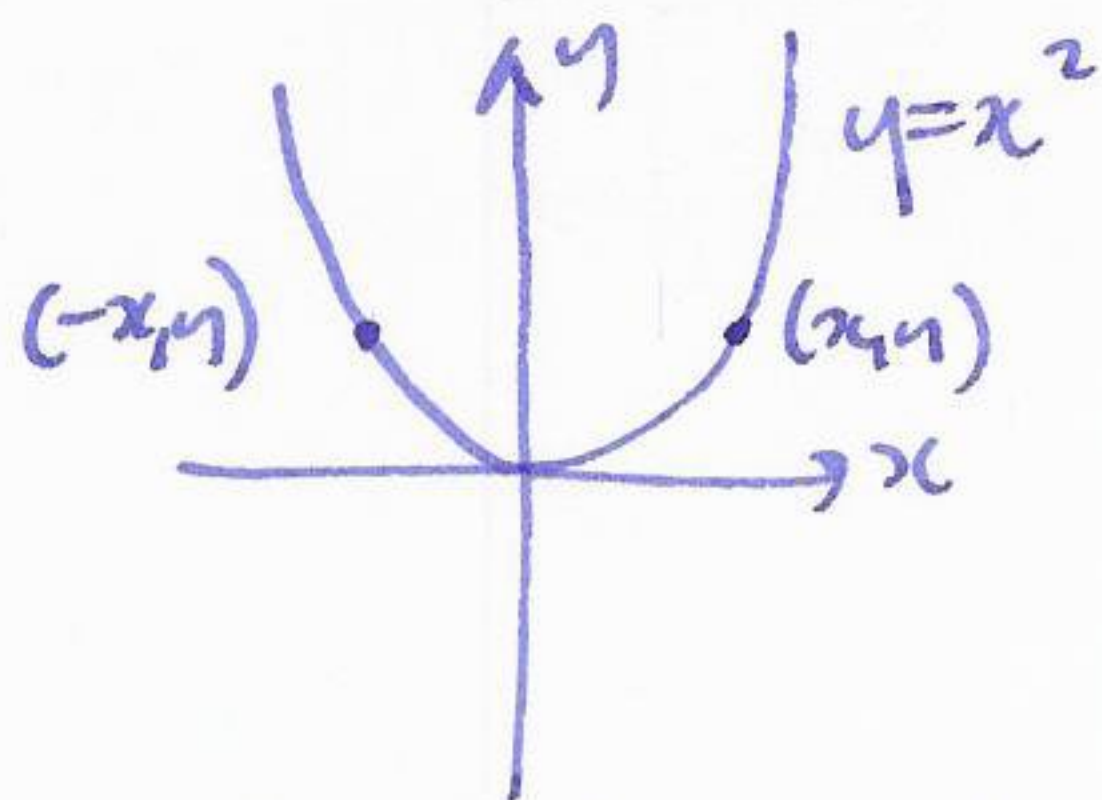


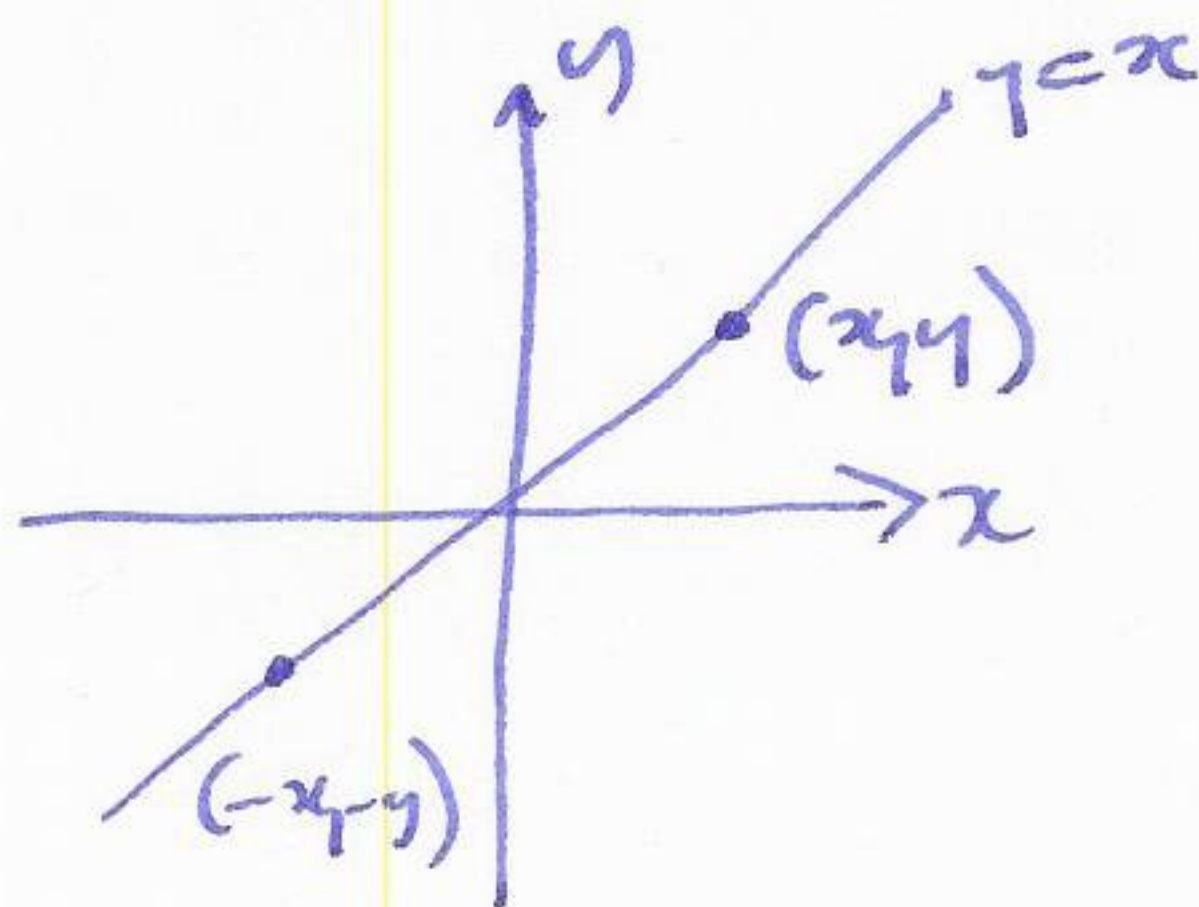
§2.4 Transformations and symmetry

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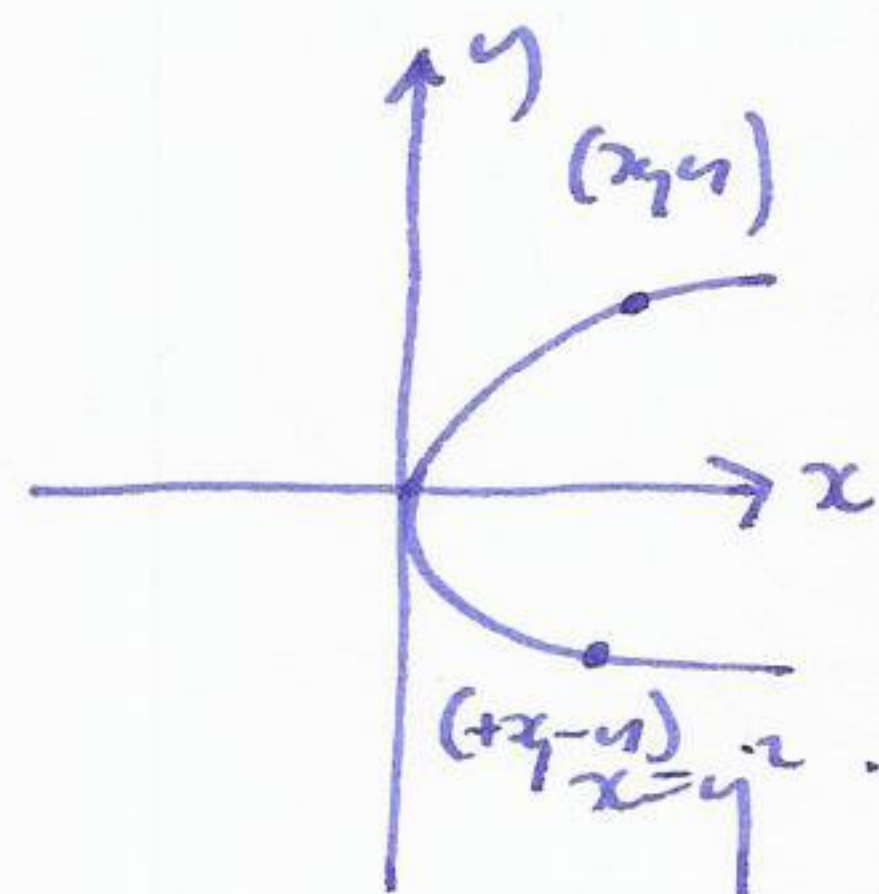
Examples



reflectional symmetry
about y -axis



$\frac{1}{2}$ turn symmetry
about origin.



symmetric
about x -axis

algebraic
tests:

$$y = x^2$$

y -axis symmetry: replace
 x by $-x$.

$$y = (-x)^2 = x^2 \checkmark$$

not

$$y = x$$

replace x by $-x$.

$$y = (-x) \quad \text{different not } y\text{-axis symmetric}$$

$\frac{1}{2}$ turn symmetry: replace x by $-x$ and y by $-y$

$$\textcircled{1} \quad y = x^2 \rightsquigarrow (-y) = (-x)^2$$

$$-y = x^2 \quad \text{not symmetric}$$

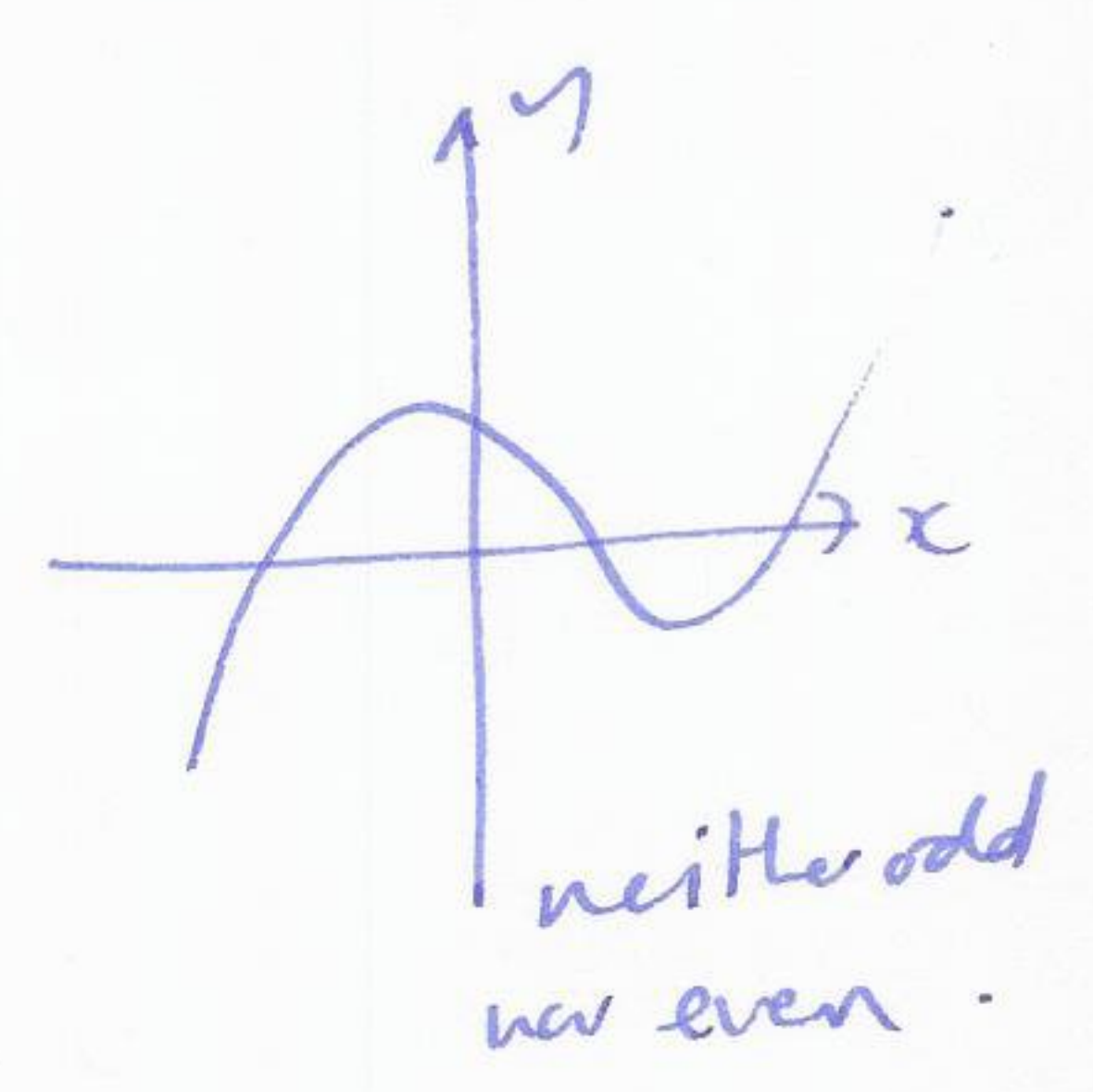
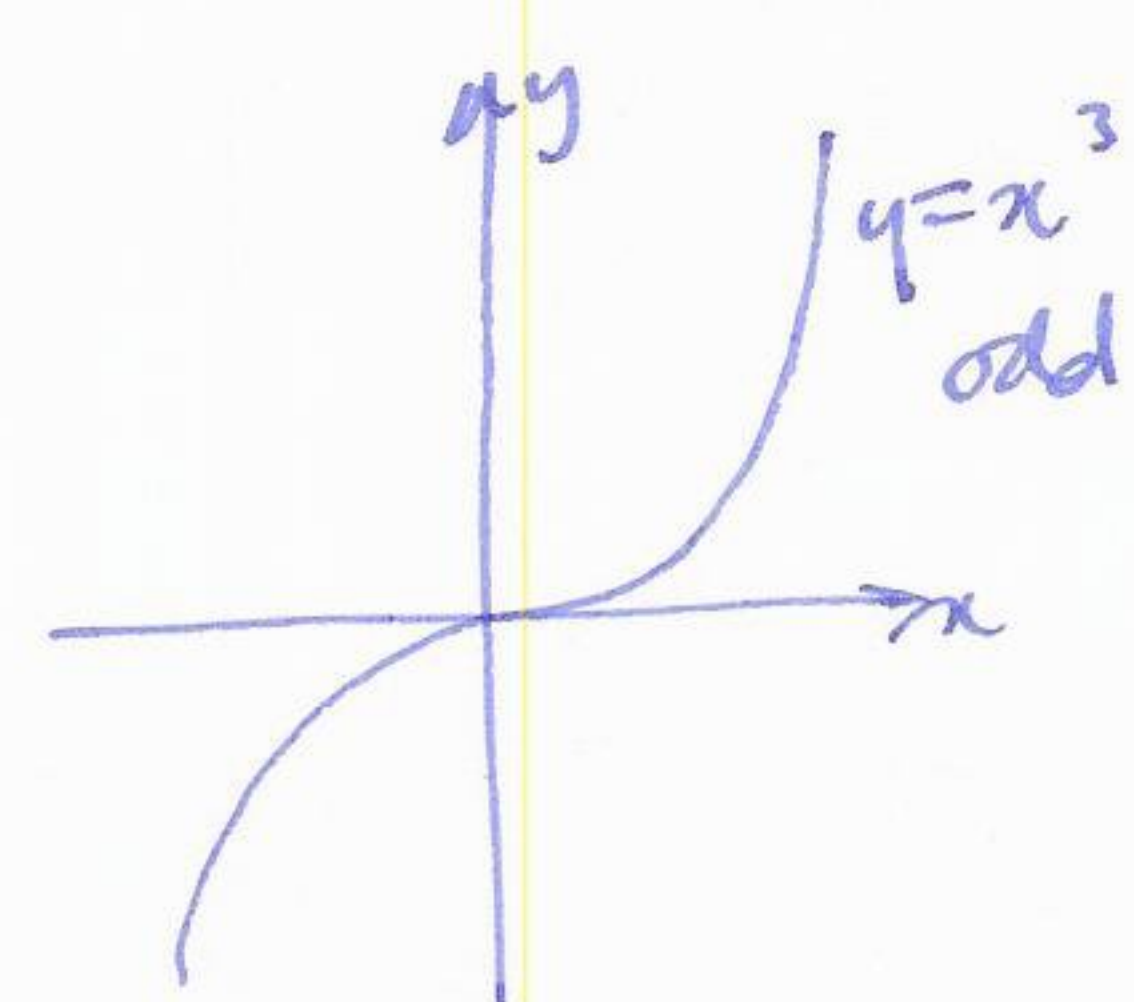
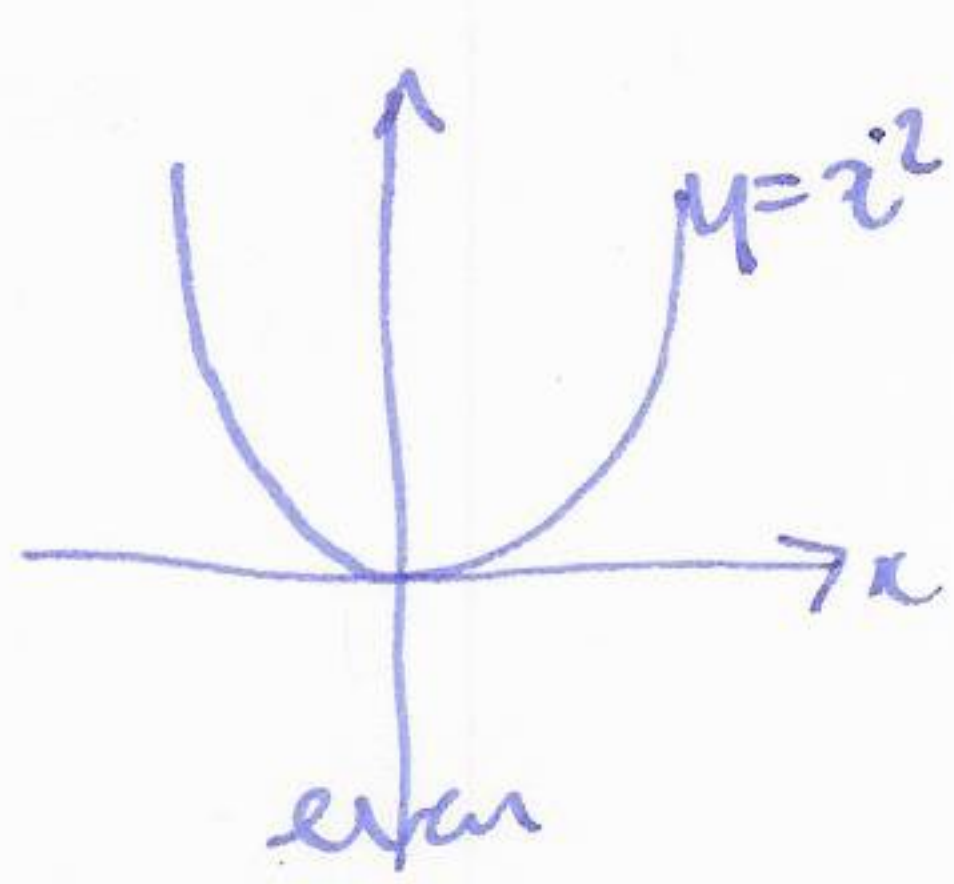
~~x -axis~~ $\textcircled{2} \quad y = x \rightsquigarrow (-y) = (-x) \quad \text{symmetric.}$

x -axis symmetry: replace y by $-y$

$$x = y^2 \rightsquigarrow x = (-y)^2 \quad \text{symmetric.}$$

A function is even if it is symmetric wrt y-axis i.e. $f(x) = f(-x)$.
odd has $\frac{1}{2}$ turn symmetry i.e. $f(x) = -f(-x)$.

Examples



Algebraic check

even does $f(x) = f(-x)$?

odd does $f(x) = -f(-x)$?

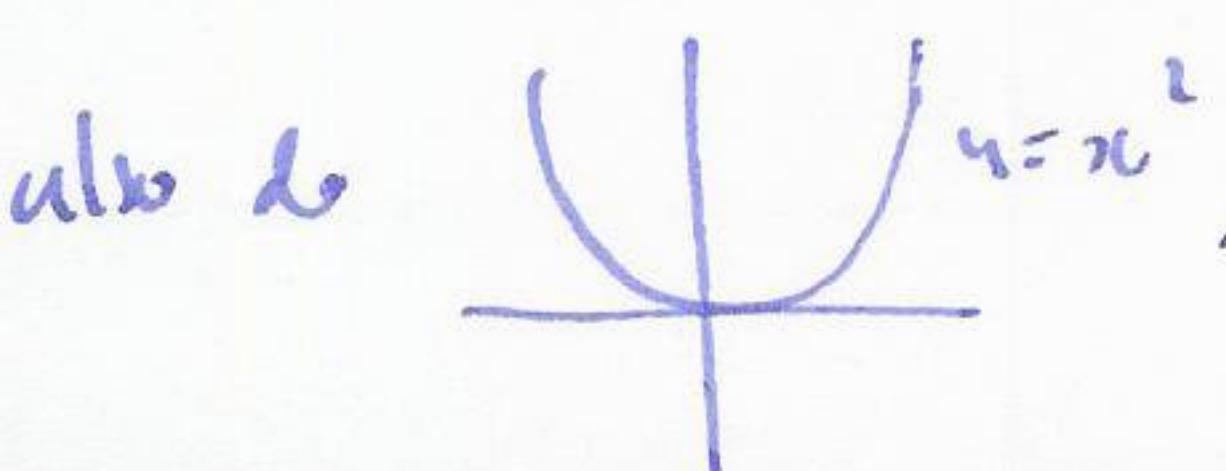
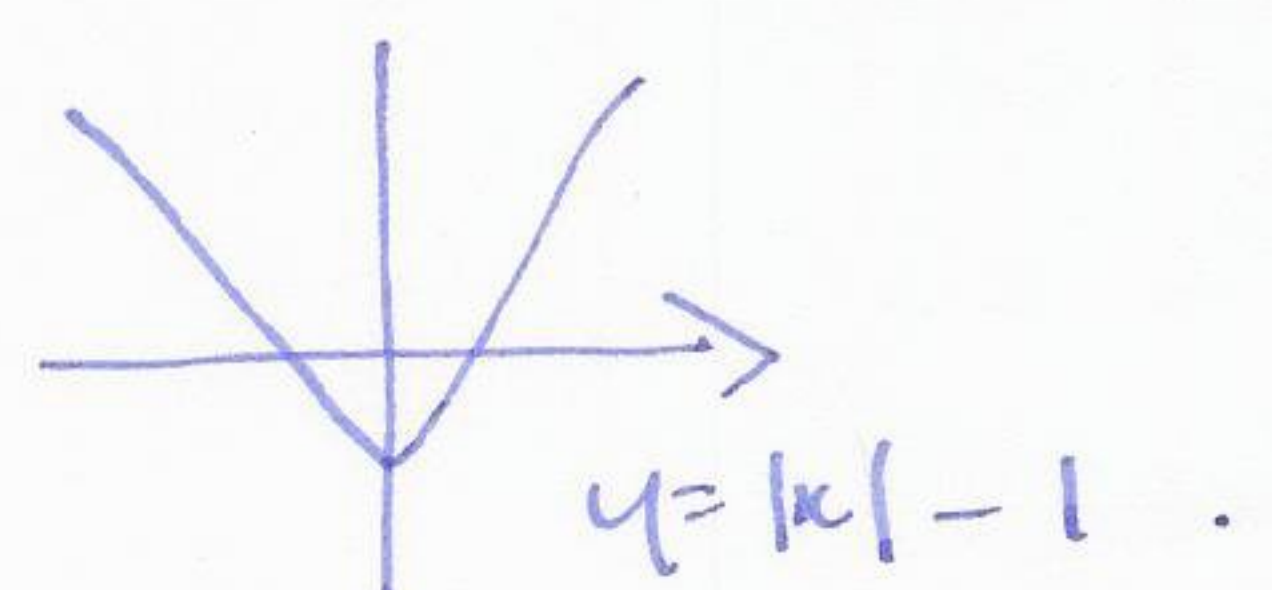
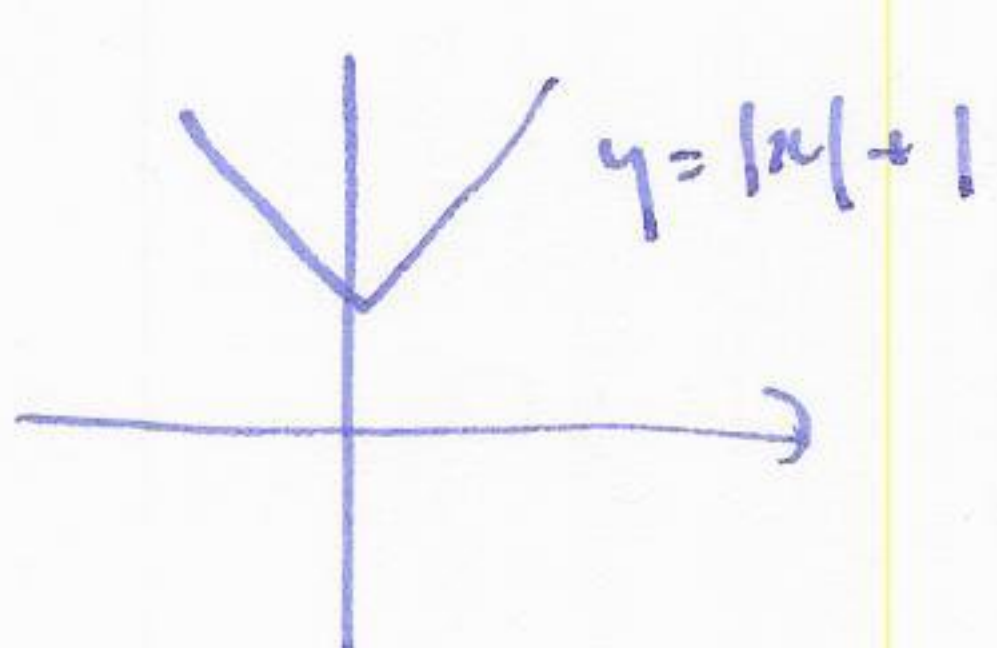
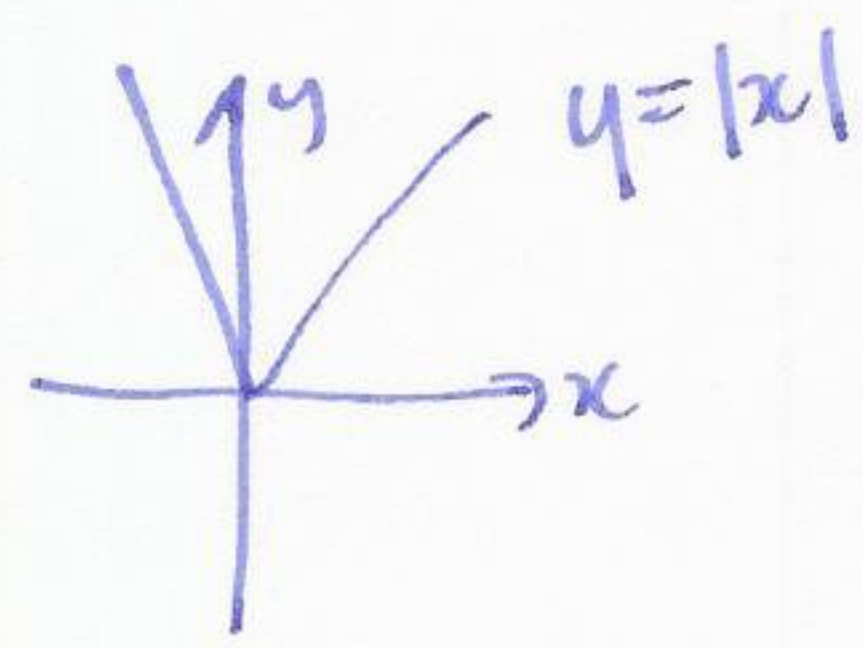
Example

$f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x) = -x^3 - x \neq f(x)$ not even.

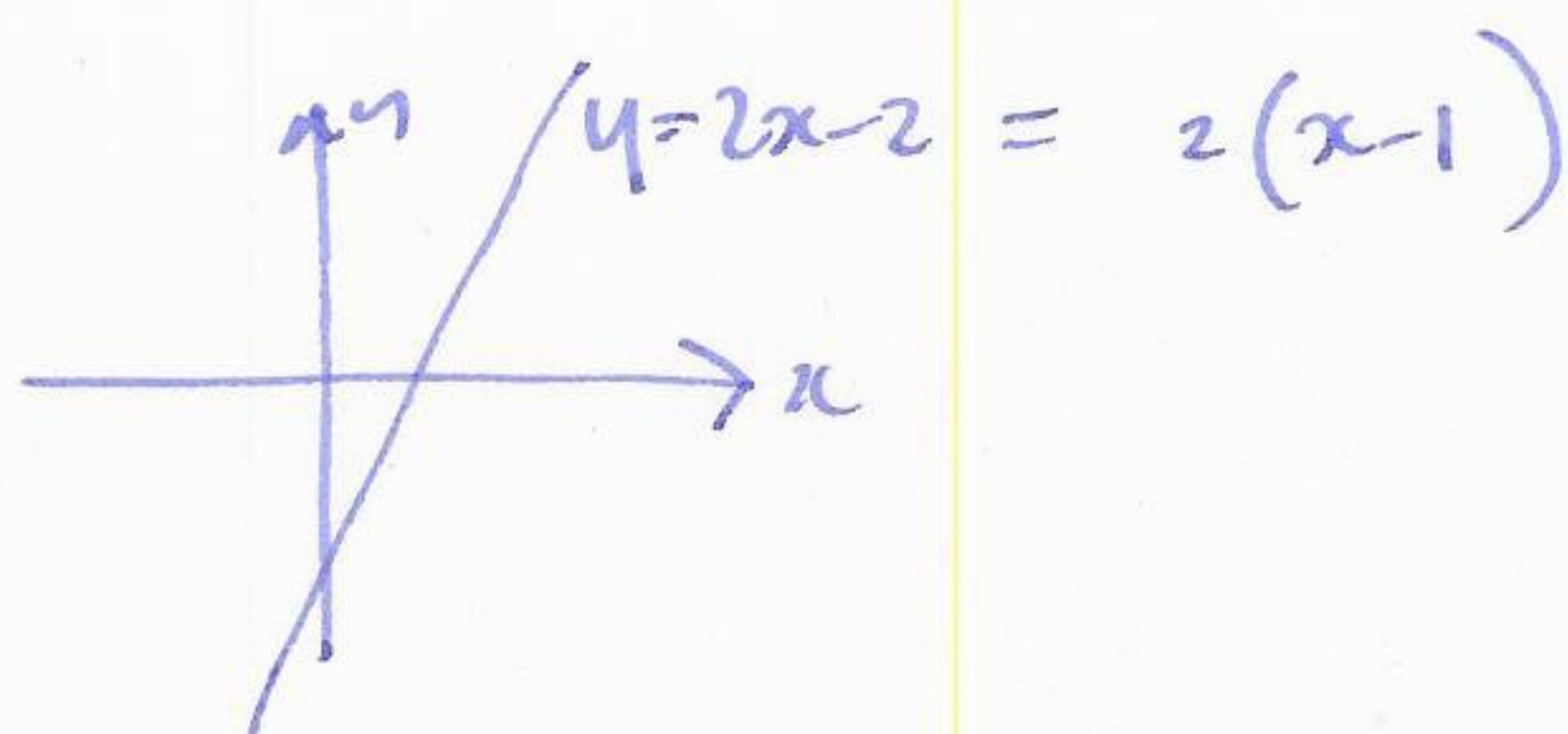
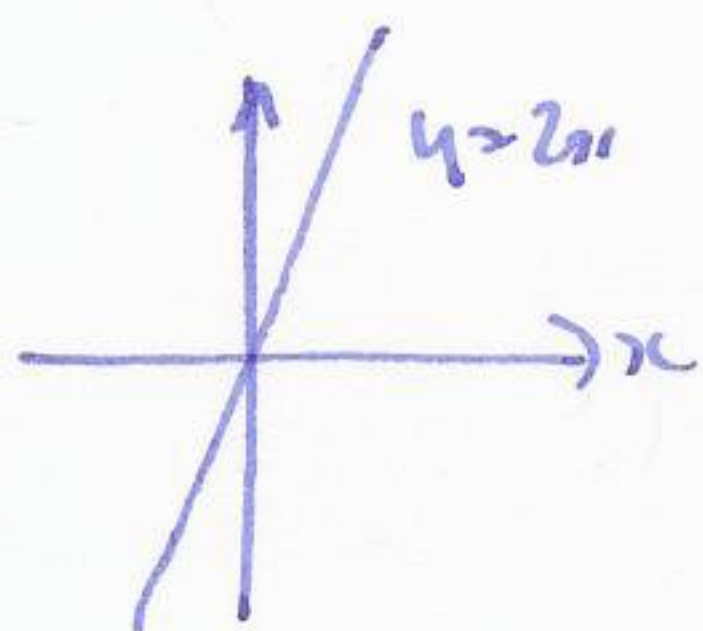
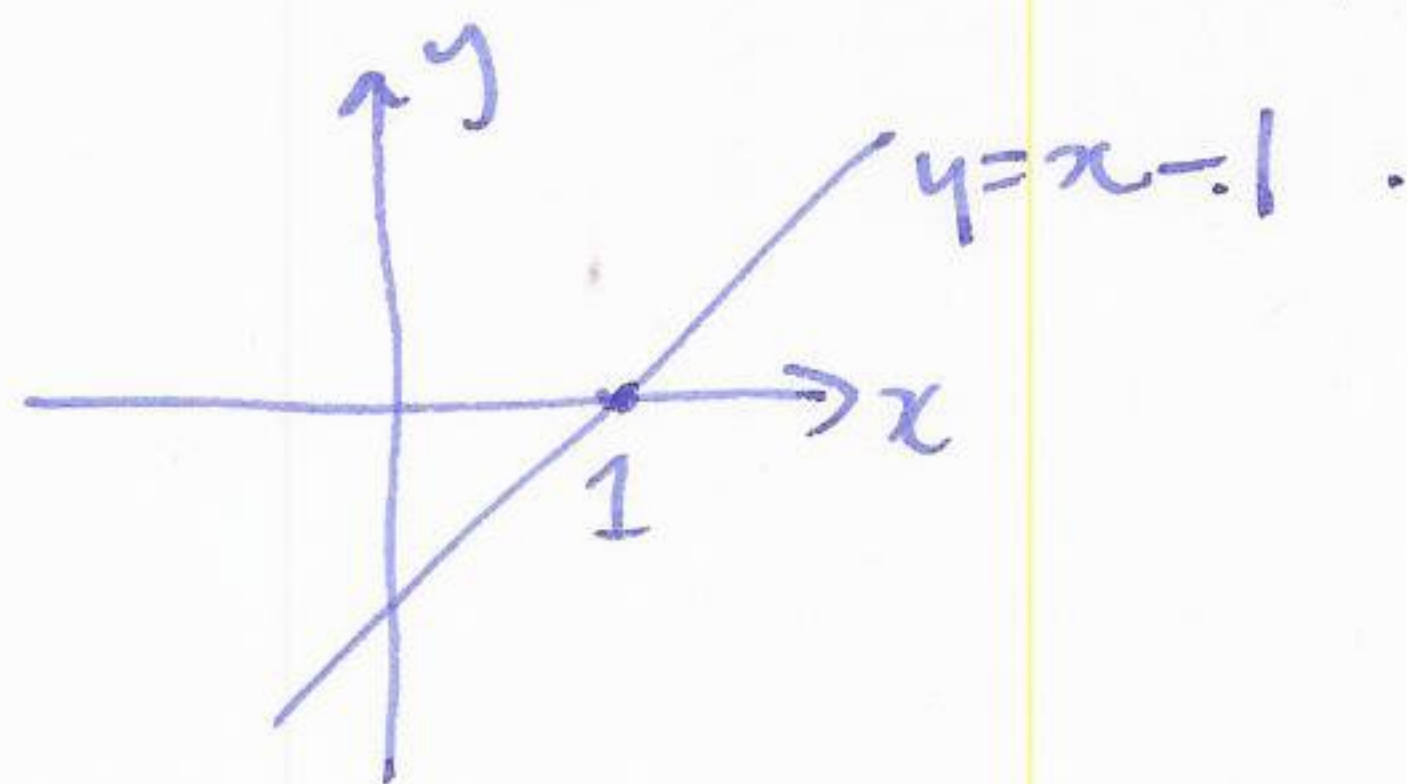
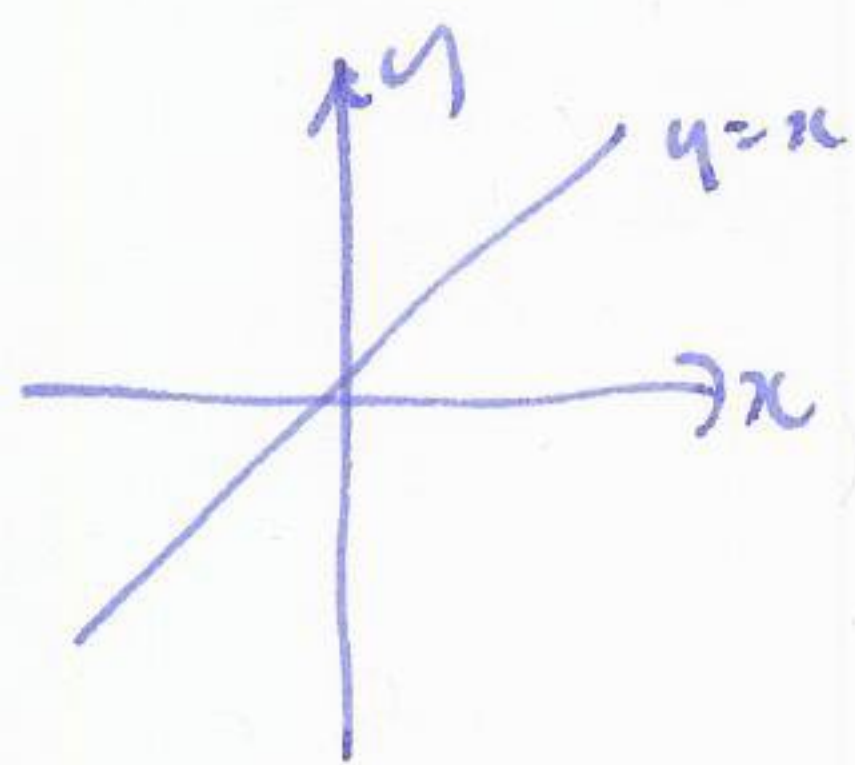
$-f(-x) = -[(-x)^3 + (-x)] = -(-x^3 - x) = x^3 + x = f(x)$
odd.

vertical translation

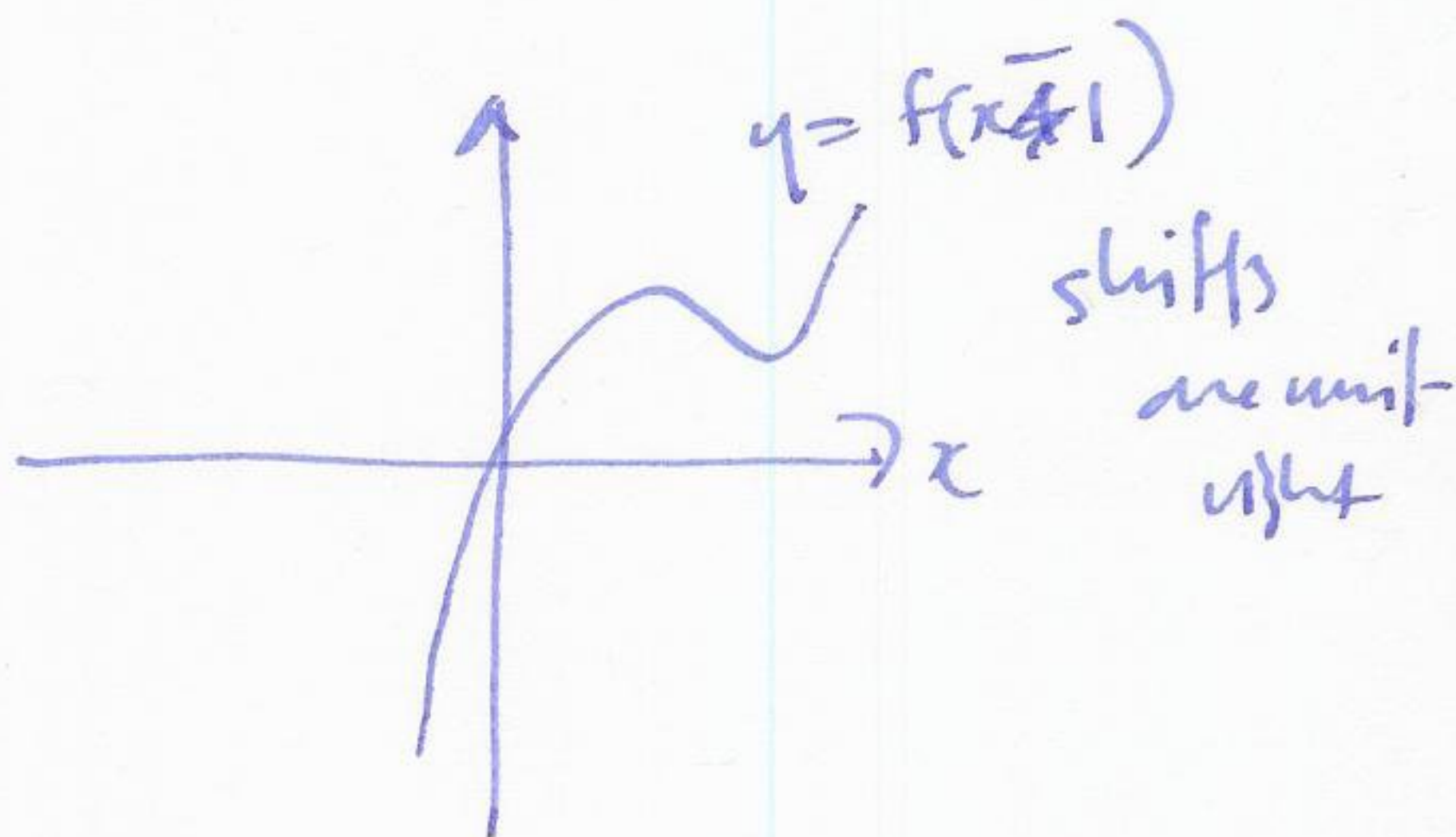
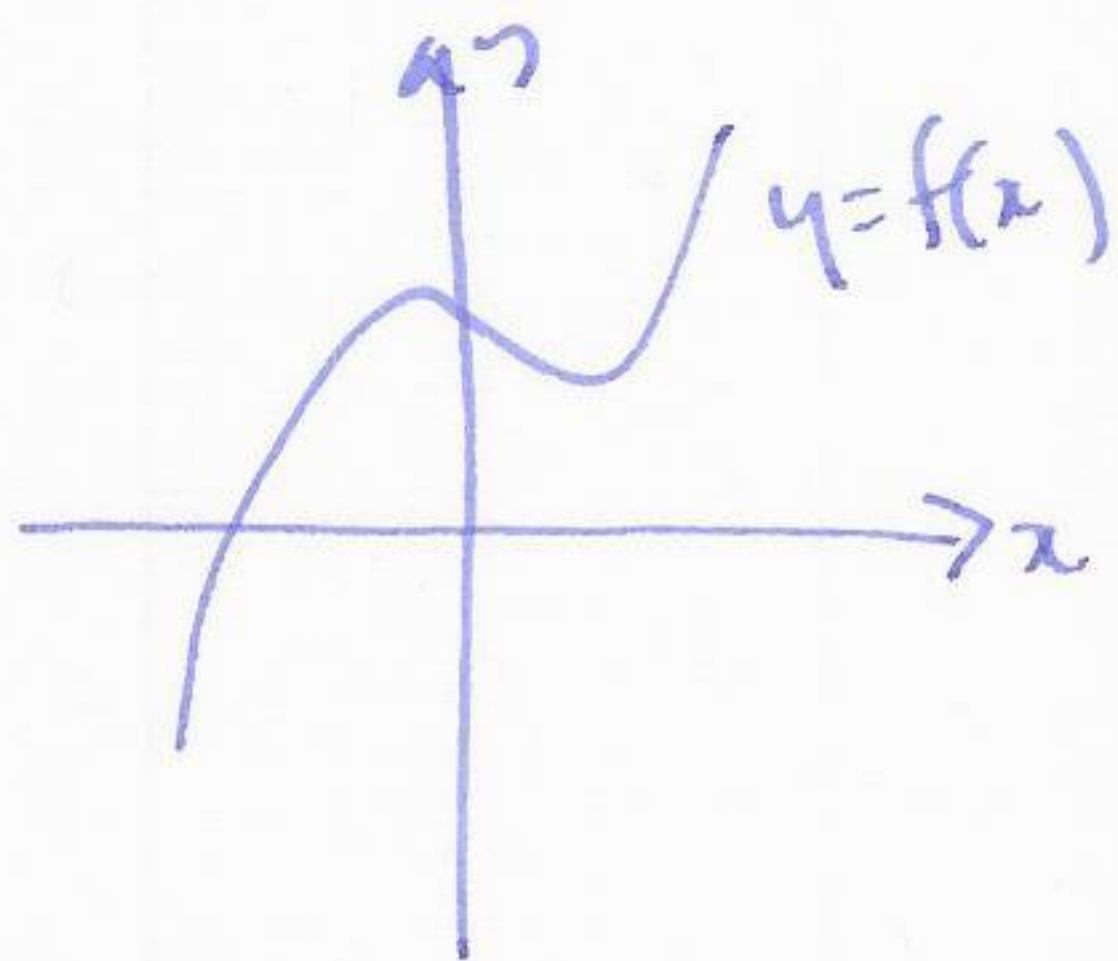


general case: $y = f(x)$ $y = f(x) + a$

Horizontal translation

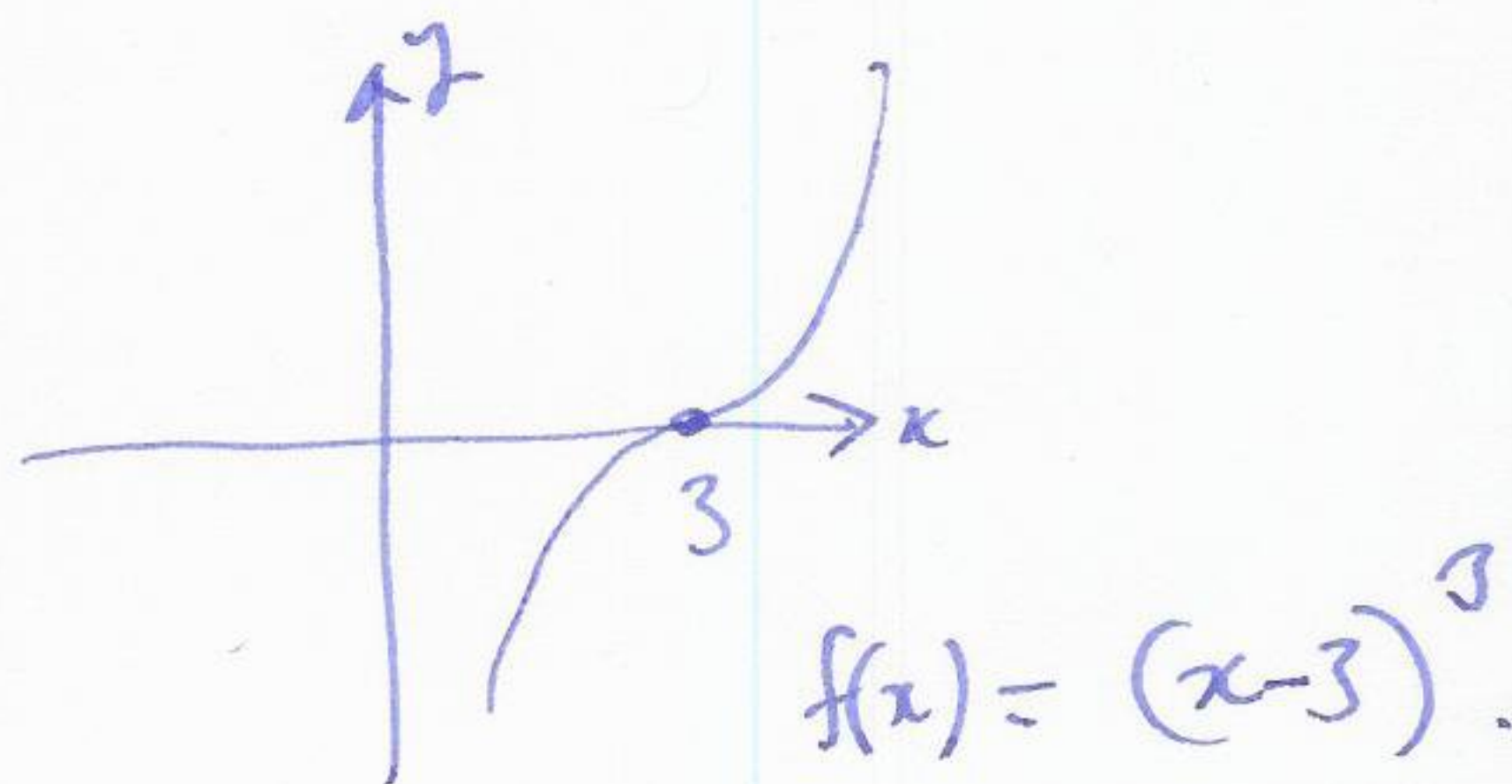
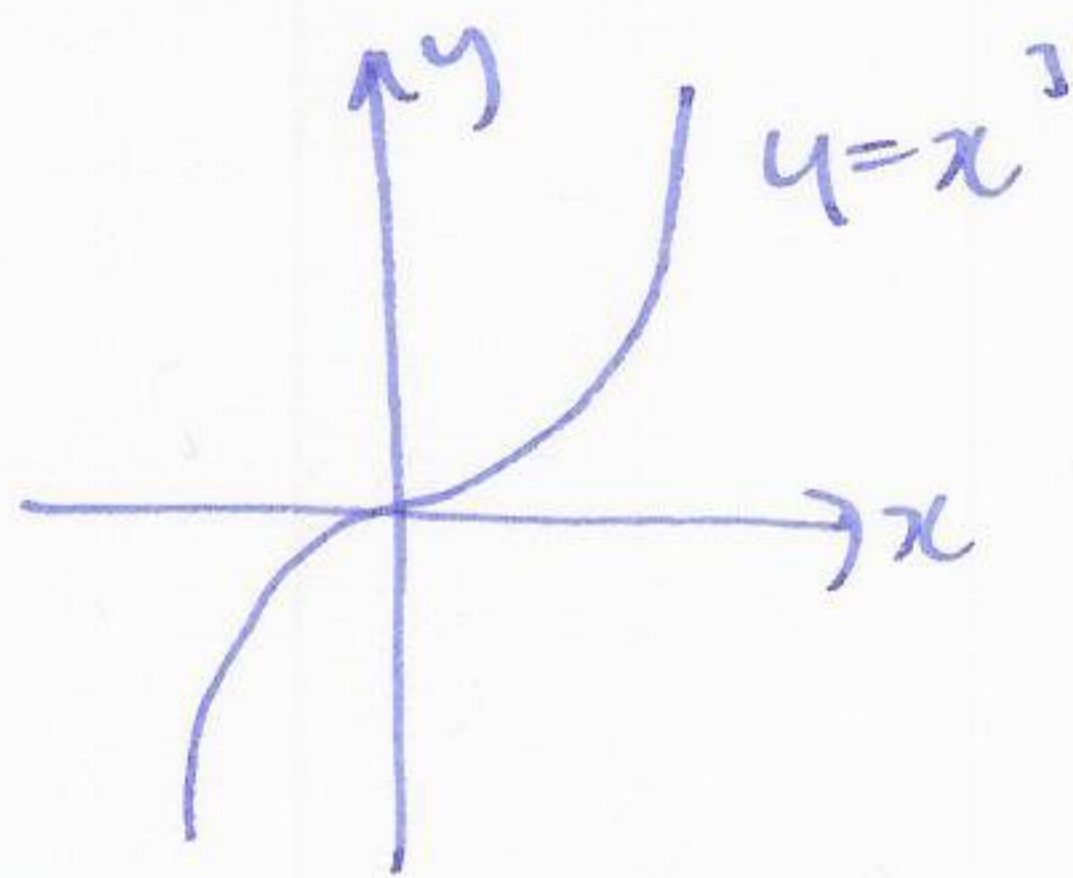


general case:

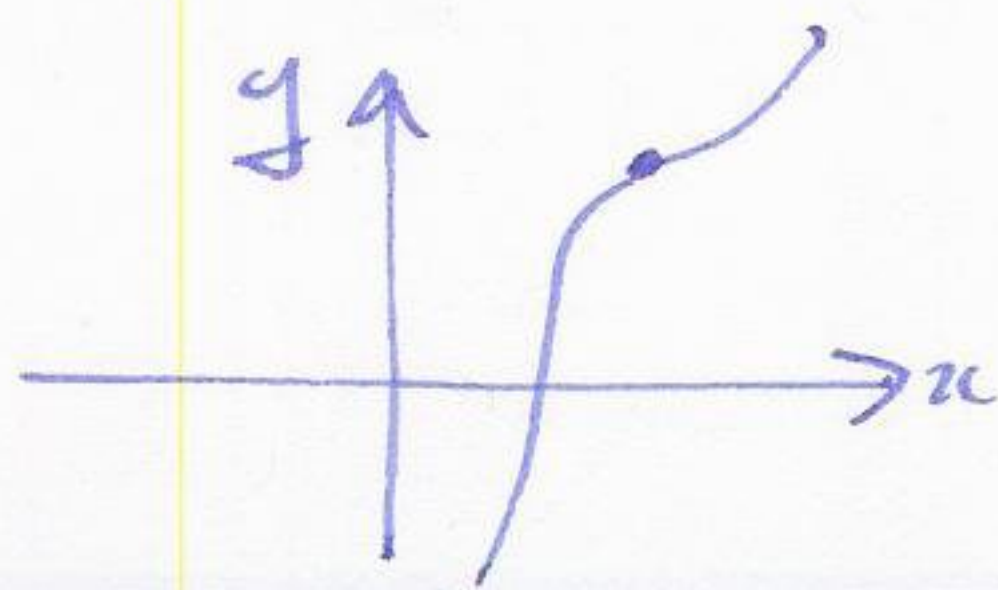


$y = f(x - d)$ shifts d units right.
 $f(x + 1)$ left.

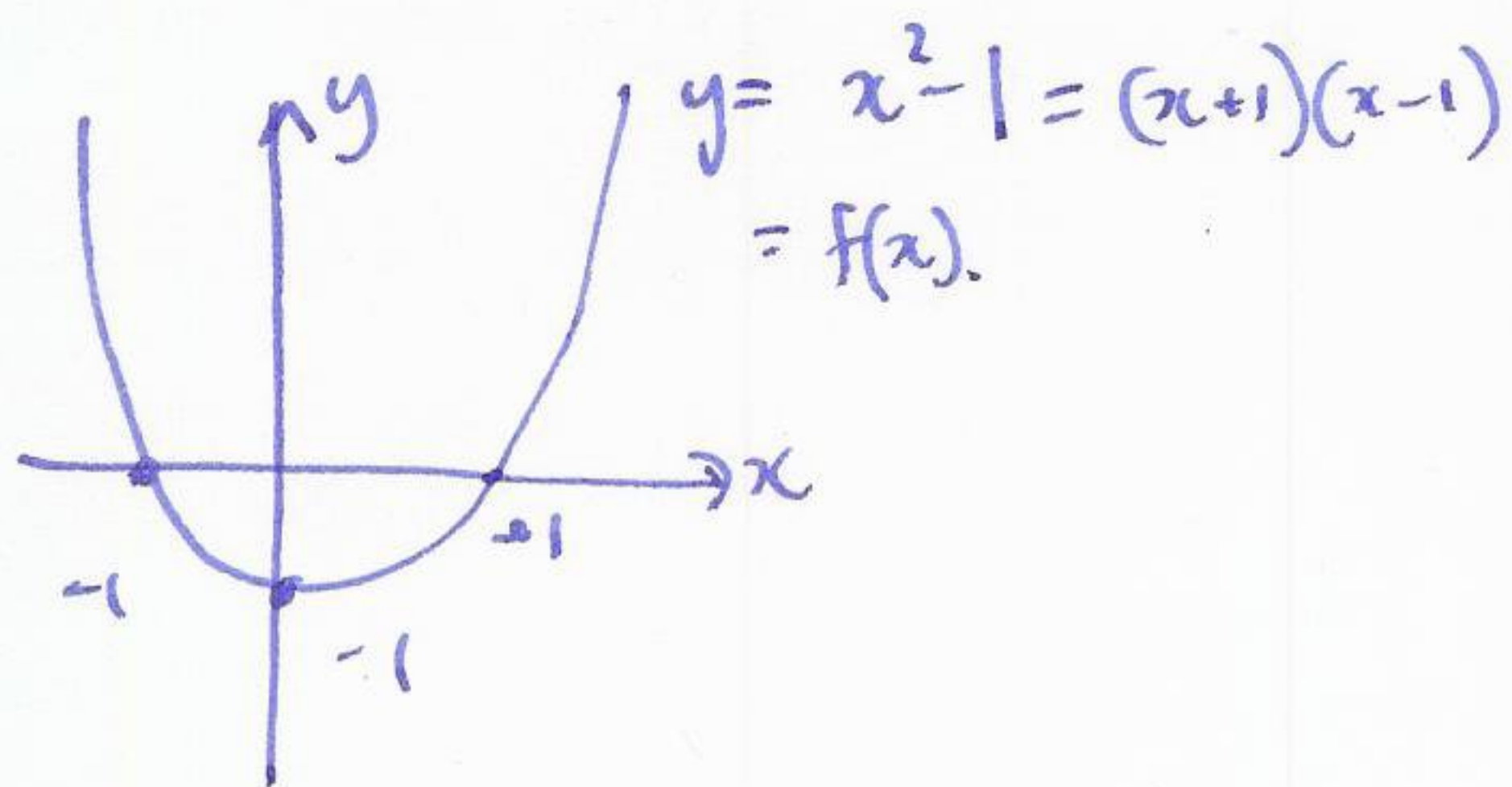
Example



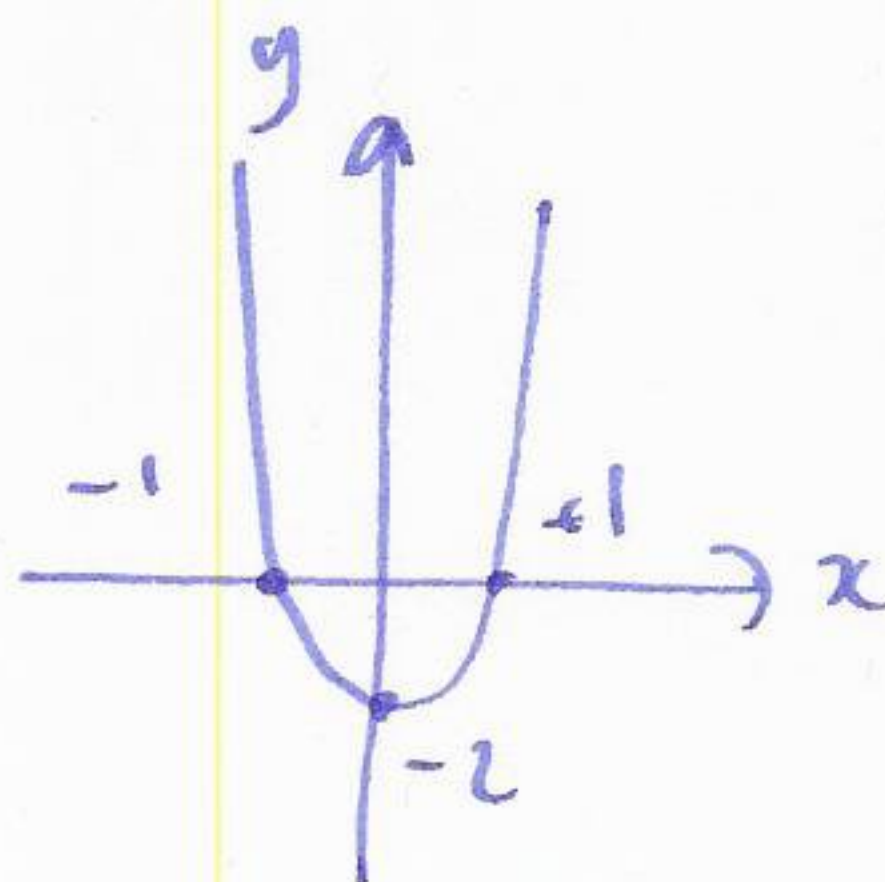
both at once! $f(x) = (x - 3)^3 + 4$



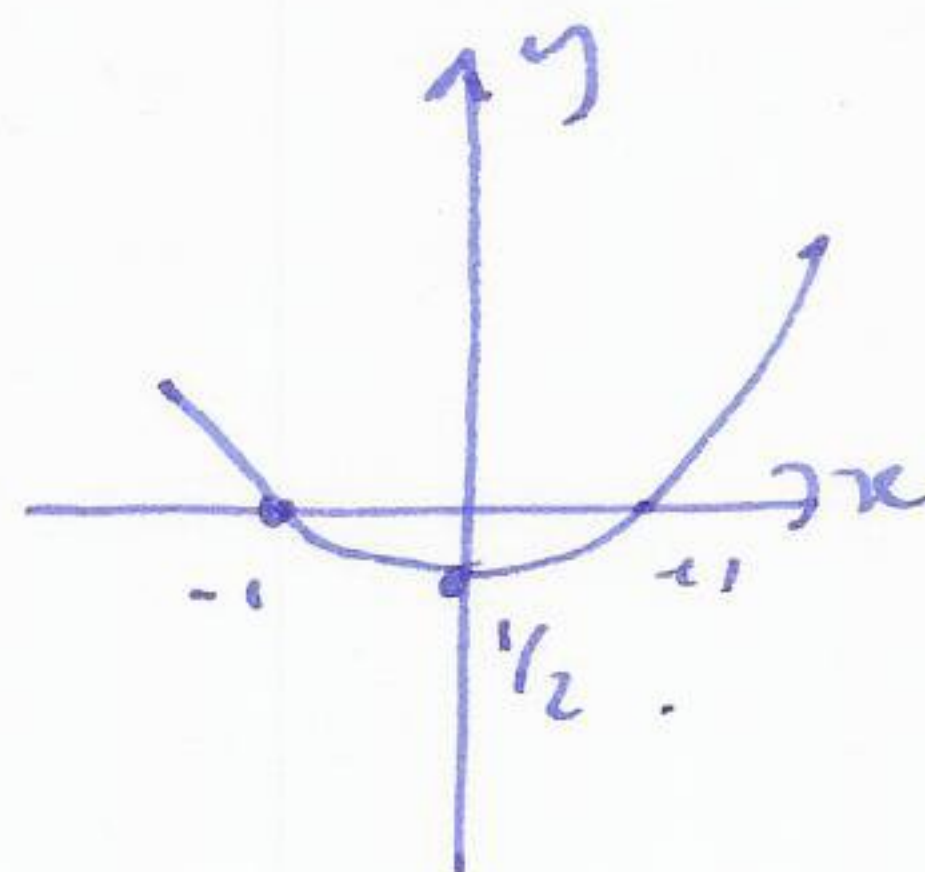
vertical/horizontal stretch/shrink



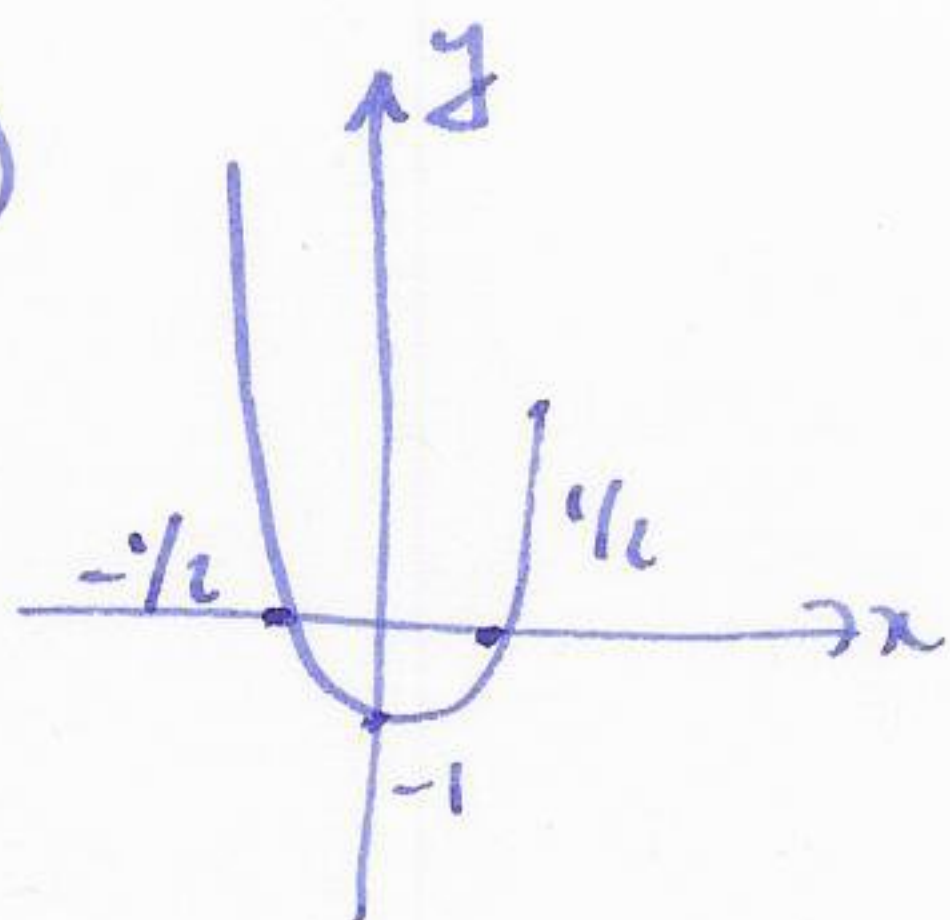
$y = 2f(x)$



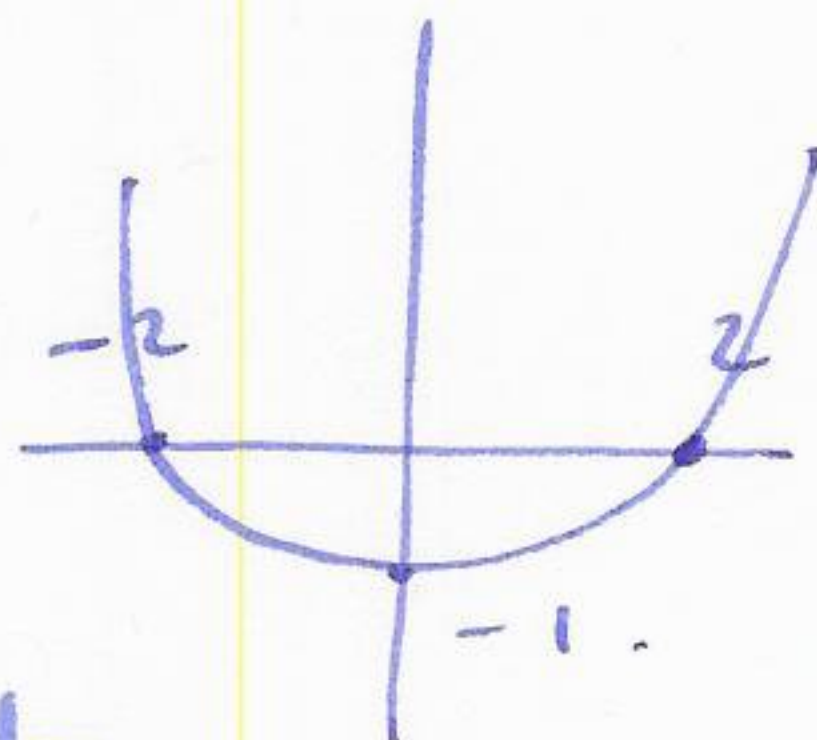
$y = \frac{1}{2}f(x)$



$y = f(2x)$



$y = f(\frac{1}{2}x)$



summary

$y = af(x)$ scale ^{vertically} by a
bigger if $a > 1$
smaller if $0 < a < 1$

$y = f(ax)$ scale horizontally by a
bigger if $0 < a < 1$
smaller if $a > 1$

Q: what about $-a$? (a or negative)