Math 130 Pre-Calculus

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- math tutoring: 1S-214
- students with disabilities

Text: PreCalculus: graphs + models BBEP

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§1.2 Functions and graphs

a <u>function</u> consists of 3 things:

$\{\text{domain}\} \longrightarrow (\text{range})$

- a set called the domain
- a set called the range.
- each element in the domain corresponds to <u>exactly one</u> element in the range

<u>Example</u>

domain range.

$\{1, 2\}$ $\{2, 4\}$

input output

$1 \longmapsto 2$

$2 \longmapsto 4$.

<u>Not functions</u>.

$$\begin{array}{ccc} 1 & \longmapsto & 2 \\ 2 & & 4 \end{array} \qquad \begin{array}{ccc} 1 & \to & 2 \\ & \searrow & \\ 2 & \to & 4 \end{array}$$

this definition is very general: $\overset{\text{domain}}{\{\text{apple, banana}\}}$ $\{\text{car, bicycle}\}$.

apple $\longmapsto$ car
banana $\longmapsto$ bicycle.

but we'll mainly be interested in the case where the domain and range are sets of numbers.

examples

$\mathbb{R}$ = all numbers $\ldots$ -1 0 1

$\mathbb{Z}$ whole numbers.

$[0,1]$ all numbers between 0 and inclusive

$[0,1)$ all numbers between 0 and 1, including zero but not 1.

Describing functions

example

$$\underset{}{\overset{\text{domain}}{\{0,1,2\}}} \longrightarrow \overset{\text{range}}{\{-2,-1,0\}}$$

$$0 \longmapsto 0$$
$$1 \longmapsto -1$$
$$2 \longmapsto -2.$$

suppose I want to describe a function where

domain = $\mathbb{Z}$
range = $\mathbb{Z}$ } infinite sets – don't want to write down infinitely many arrows!

instead we can describe the function:

$$\overset{\text{domain}}{\mathbb{Z}} \longrightarrow \overset{\text{range}}{\mathbb{Z}}$$

$$n \longmapsto -n$$

examples: $0 \mapsto 0$, $1 \mapsto -1$, $-1 \mapsto 1$, $100 \mapsto -100$.

How does this actually describe a function?

check : for each $\overset{\text{input}}{n \in \mathbb{Z}}$, there is a unique output $\underset{\in \mathbb{Z}}{-n}$ ✓.

<u>example</u>

domain range.

$\mathbb{R} \longrightarrow \mathbb{R}$

input output

$x \longmapsto 2x$

<u>examples</u>

$0 \mapsto 0$ $\quad -1 \mapsto -2$

$1 \mapsto 2$ $\quad -2 \mapsto -2$

$2 \mapsto 4$ $\quad 107 \mapsto 214$

check : for each input $x \in \mathbb{R}$, there is a unique output $2x \in \mathbb{R}$

<u>What can go wrong?</u>

domain range

$\mathbb{Z} \longrightarrow \mathbb{Z}$

input output

$n \longmapsto \frac{n}{2}$

$1 \mapsto \frac{1}{2}$ but $\frac{1}{2} \notin \mathbb{Z}$.

<u>not</u> a function $\mathbb{Z} \to \mathbb{Z}$ because output does not lie in $\mathbb{Z}$.

domain range.

$\mathbb{R} \longrightarrow \mathbb{R}$

input output.

$x \longmapsto \frac{1}{x}$

$0 \mapsto \frac{1}{0}$ not defined!

<u>not</u> a function because not defined at $0 \in \mathbb{R}$.

<u>Names for functions</u>

domain range

$f : \mathbb{R} \xrightarrow{f} \mathbb{R}$

$x \longmapsto 2x$

$x \longmapsto f(x)$

$f(x) = 2x$

domain range

$g : \mathbb{R} \xrightarrow{g} \mathbb{R}$

$x \longmapsto x+1$

$x \longmapsto g(x)$.

$g(x) = x+1$.

"the value of f at x" or "f of x".

evaluating functions:

$$f(0) = 2.0 = 0$$
$$f(1) = 2.1 = 2$$
$$f(-3) = 2.-3 = -6.$$

<u>warning</u> lazy people try and define functions like this:

$f(x) = 2x+1$ what's the domain and range?

you're meant to work out the domain and range.

here we can choose

$$\begin{array}{ccc} \text{domain} & & \text{range} \\ \mathbb{R} & \xrightarrow{f} & \mathbb{R} \\ x & \longmapsto & 2x+1. \end{array}$$

<u>example</u> $f(x) = \frac{1}{x}$ what's the domain and range?

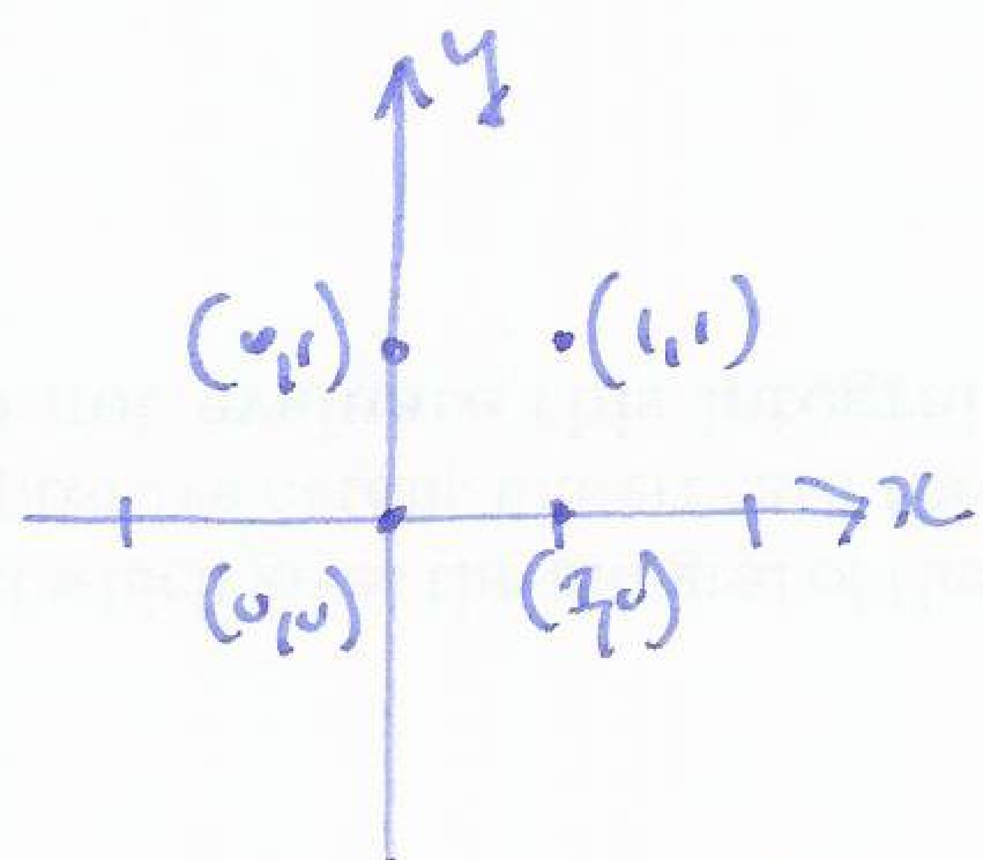
here we can choose

$$\begin{array}{ccc} \text{domain} & & \text{range} \\ \mathbb{R}\setminus 0 & \xrightarrow{f} & \mathbb{R} \end{array}$$

(all numbers <u>except</u> zero).

we're often interested in functions from $\mathbb{R}$ to $\mathbb{R}$. For these functions we can draw pictures of the function called <u>graphs</u>.

<u>recall</u> 2d coordinates:

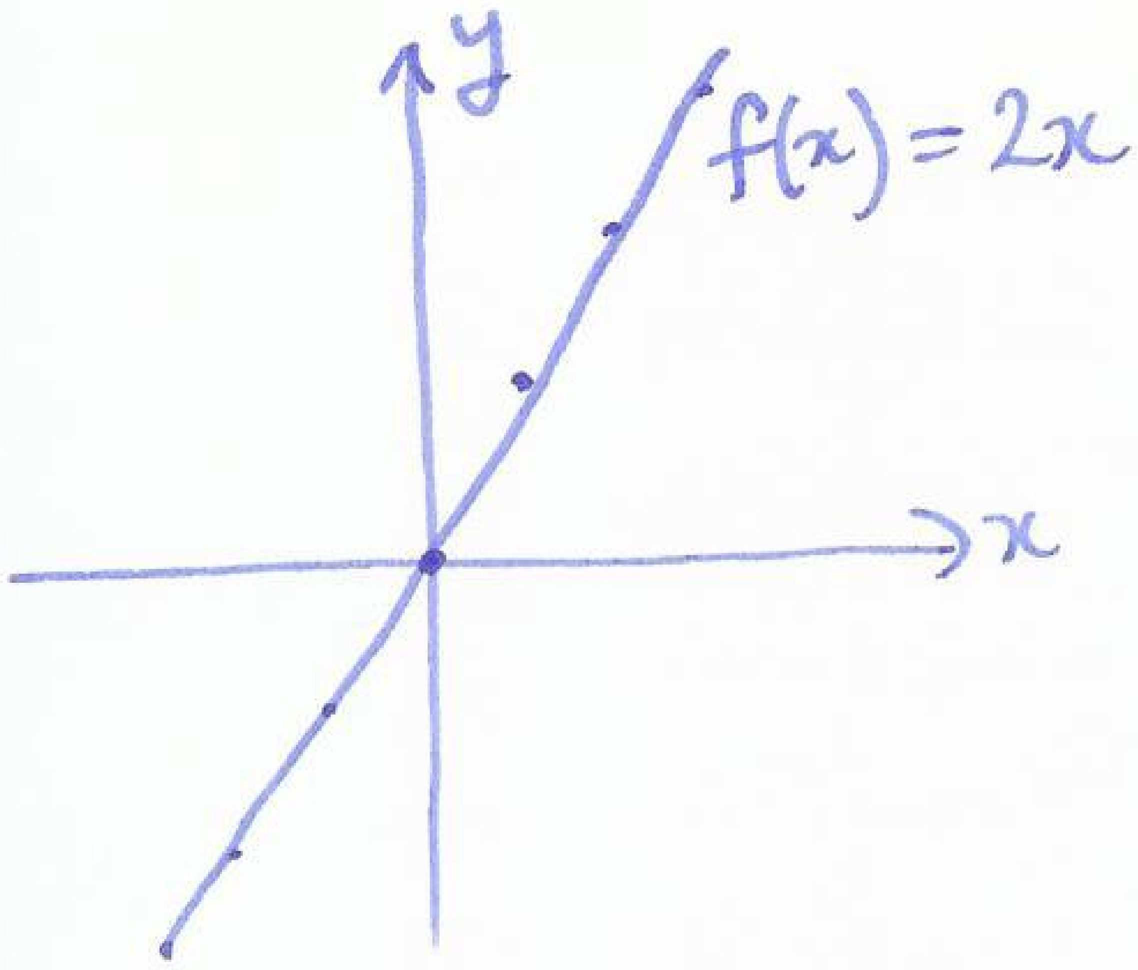


every point in the plane can be described by a pair of numbers (x,y).

we want to draw a picture of a function,

example $f(x) = 2x$ $\qquad$ $f : \mathbb{R} \longrightarrow \mathbb{R}$, $\; x \longmapsto 2x$.

we can think of this as a collection of pairs of numbers $(x, f(x))$ (input, output), i.e a subset of the plane.



$$\left.\begin{array}{l} \text{draw } (x, f(x)) \\ \phantom{\text{draw }} (x, y) \end{array}\right\} \begin{array}{l} \text{you'll see people} \\ \text{write } y = f(x). \end{array}$$

i.e. draw all pairs $(x, f(x))$.

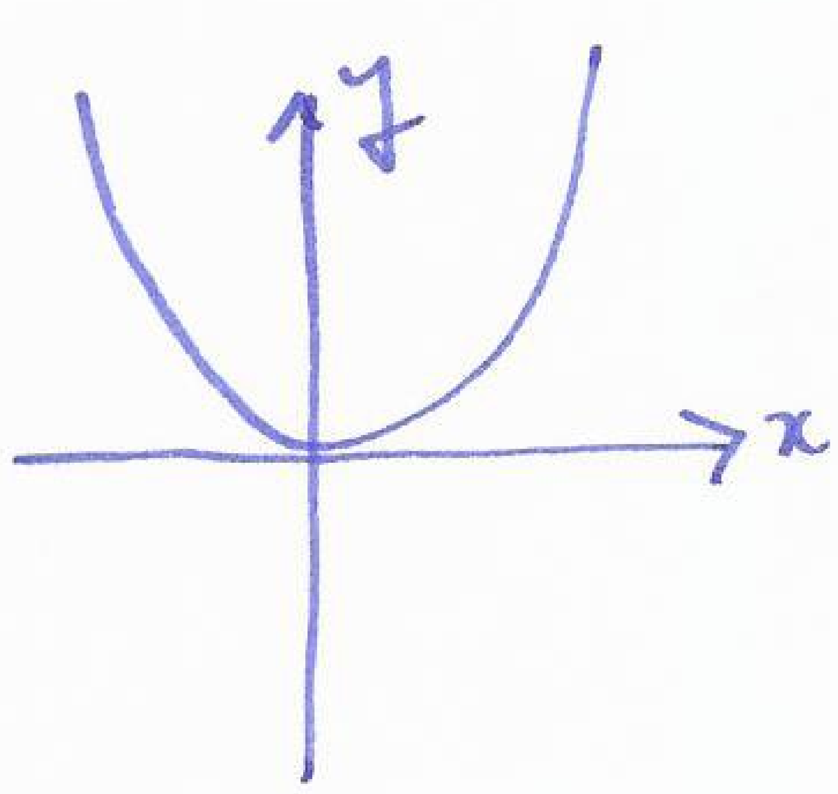
in this case all pairs $(x, 2x)$.

e.g. $(0,0)$ $(1,2)$ $(2,4)$, ...

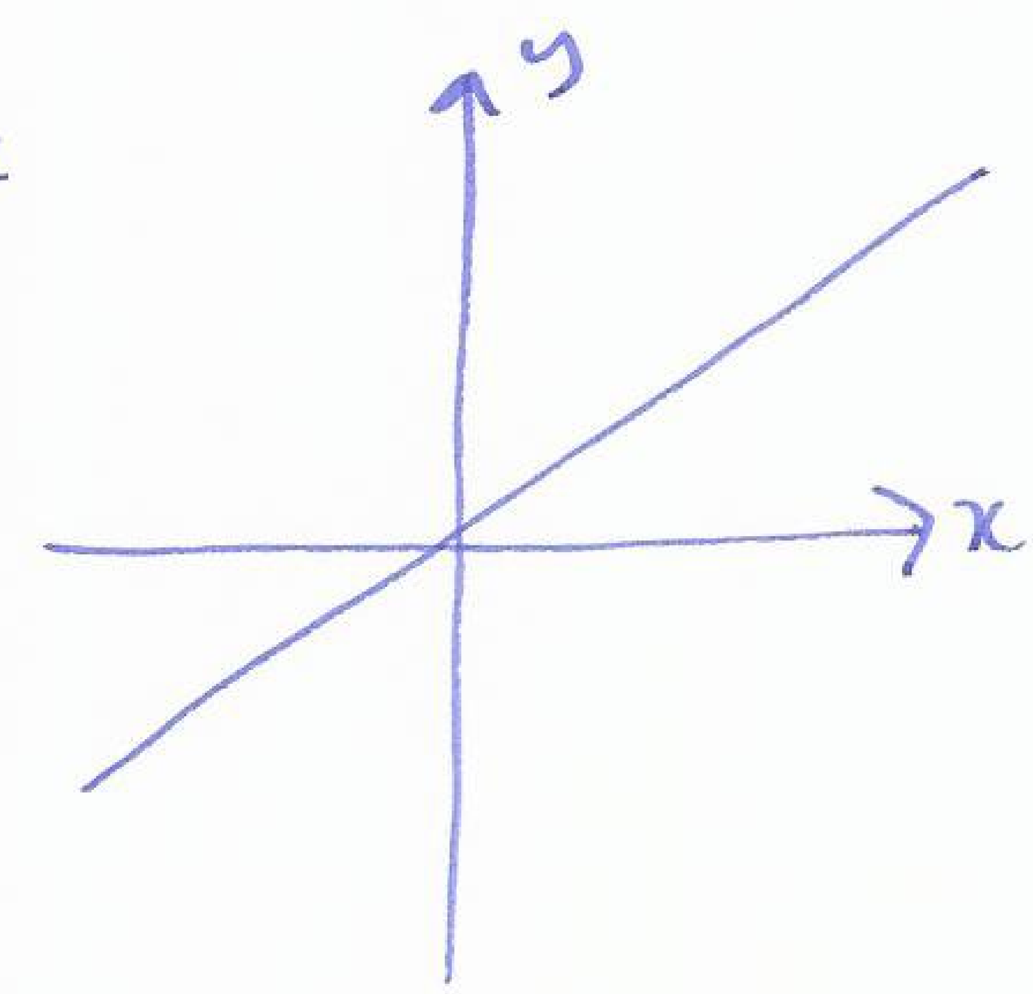
$(-1,-2)$ $(\frac{1}{2}, 1)$ ...

<u>Examples</u>

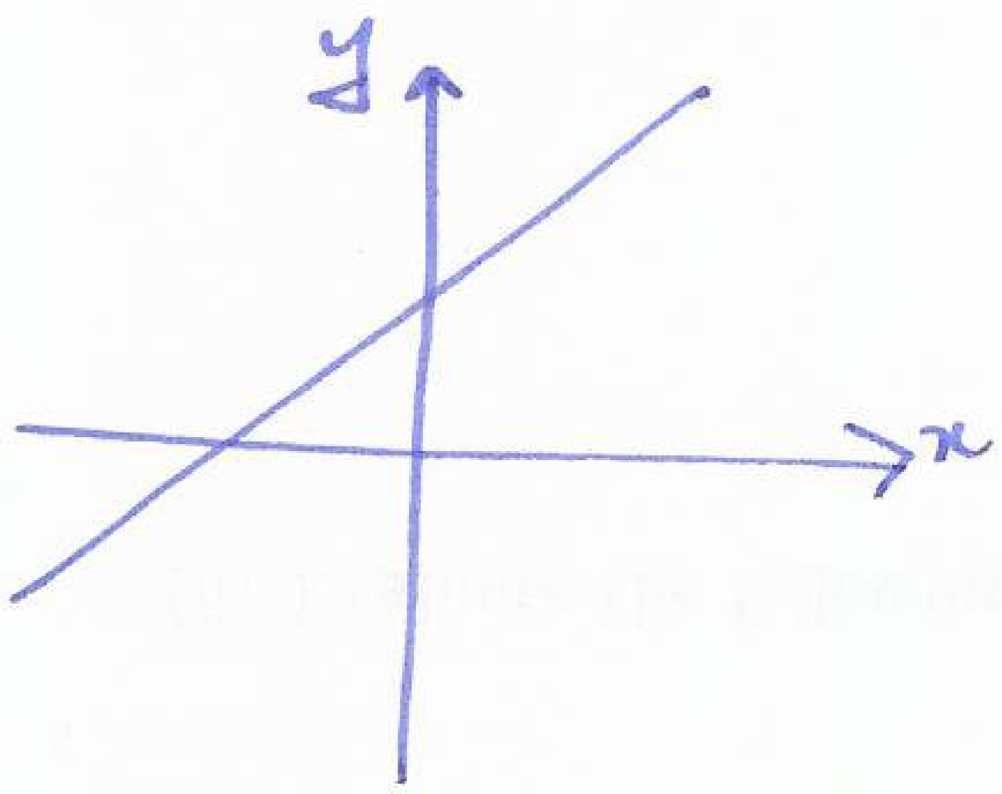
$f(x) = x^2$



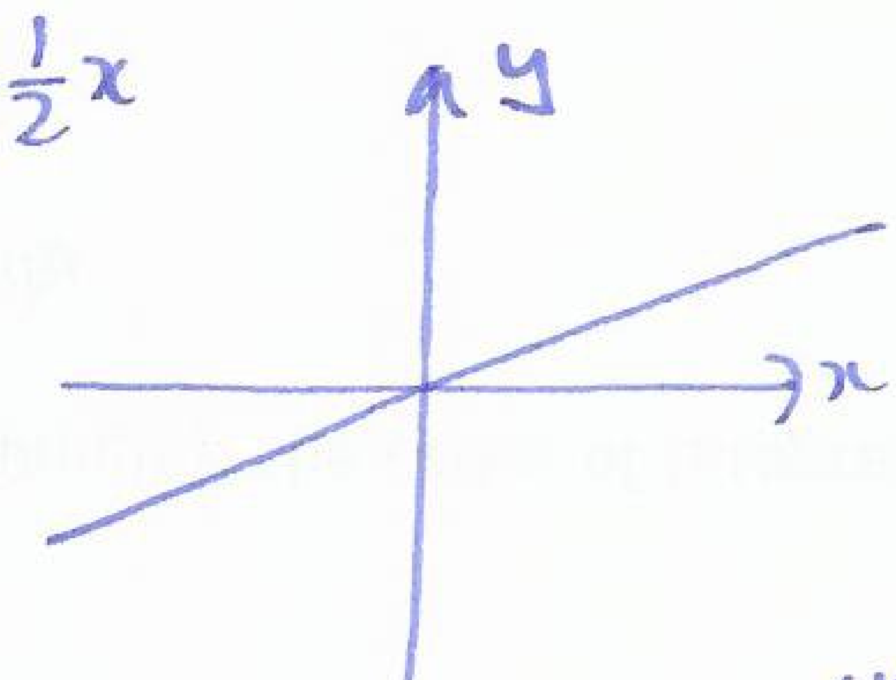
$f(x) = x$



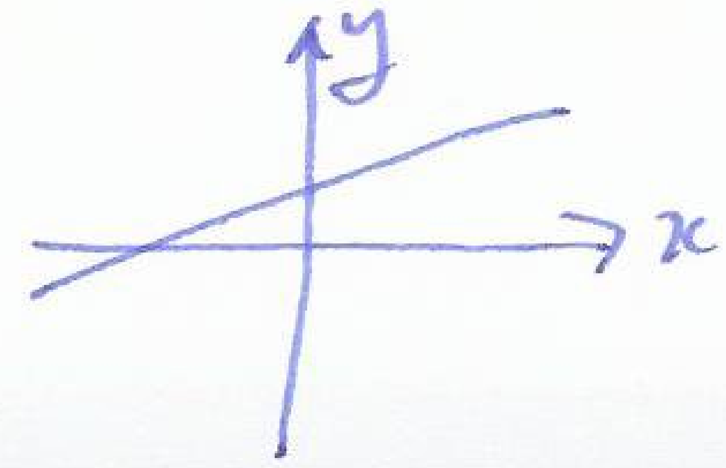
$f(x) = x + 1$



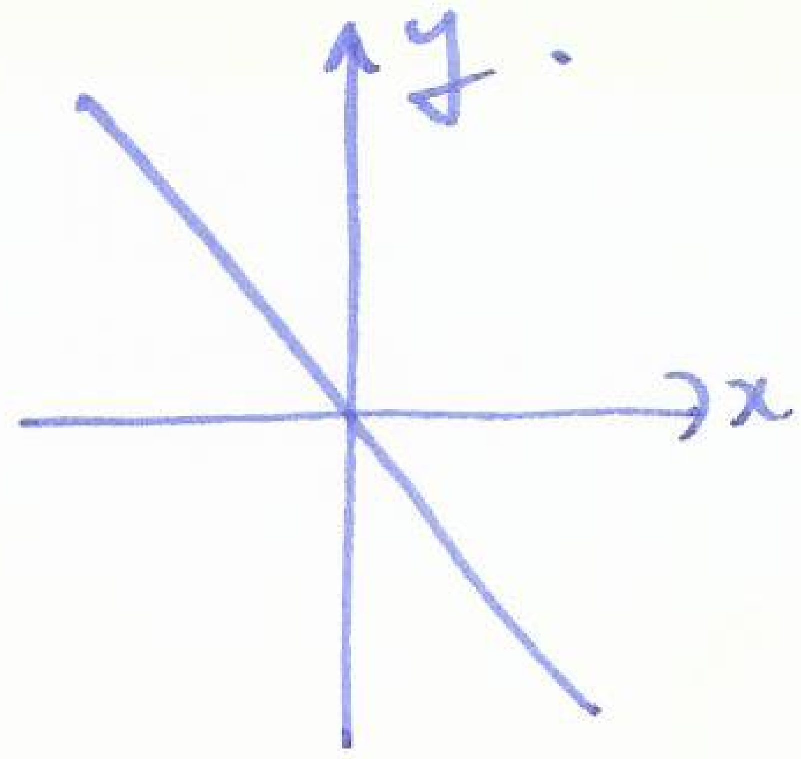
$f(x) = \frac{1}{2}x$



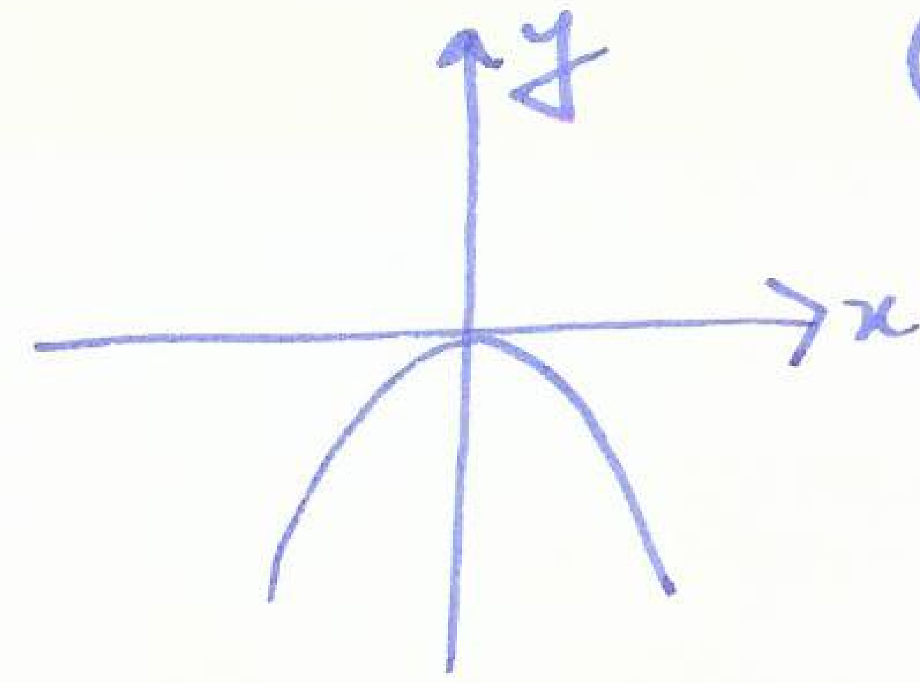
$f(x) = \frac{1}{2}x + 1$



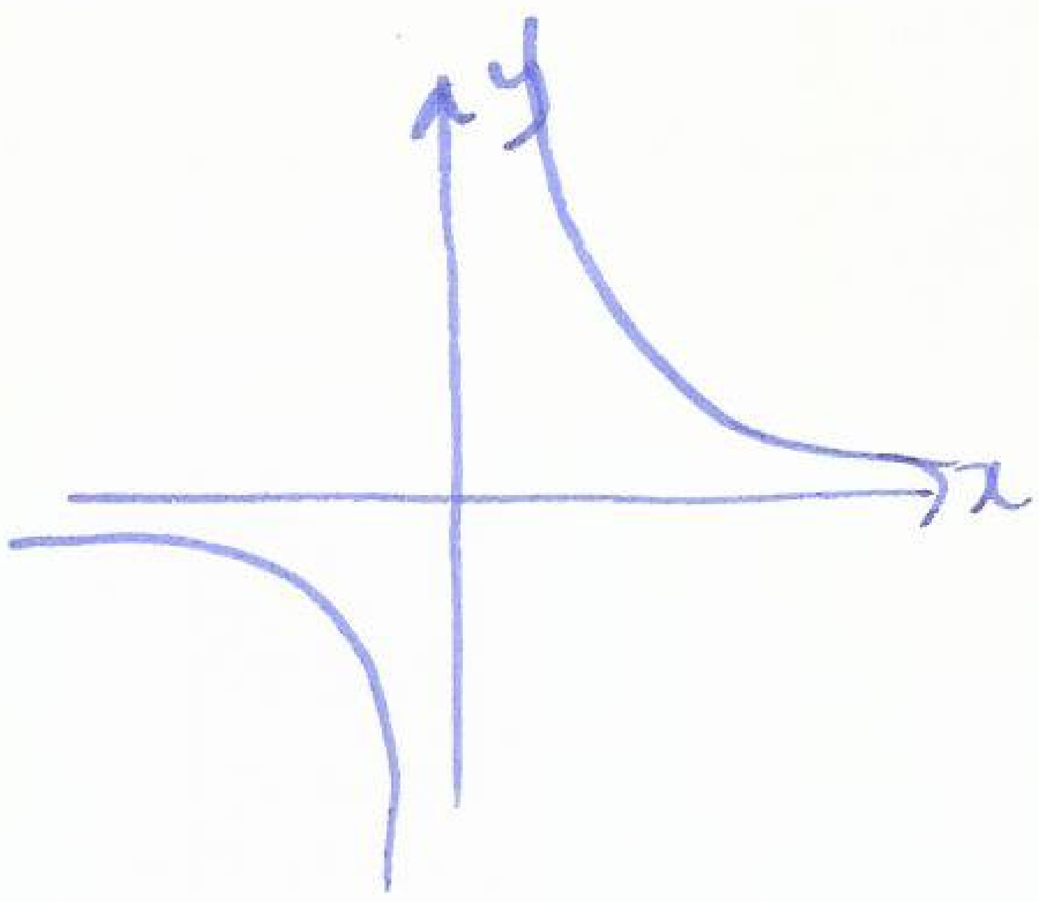
$f(x) = -x$



$f(x) = -x^2$



$f(x) = \frac{1}{x}$



Q: what's the difference between a function and an equation?

function

domain range

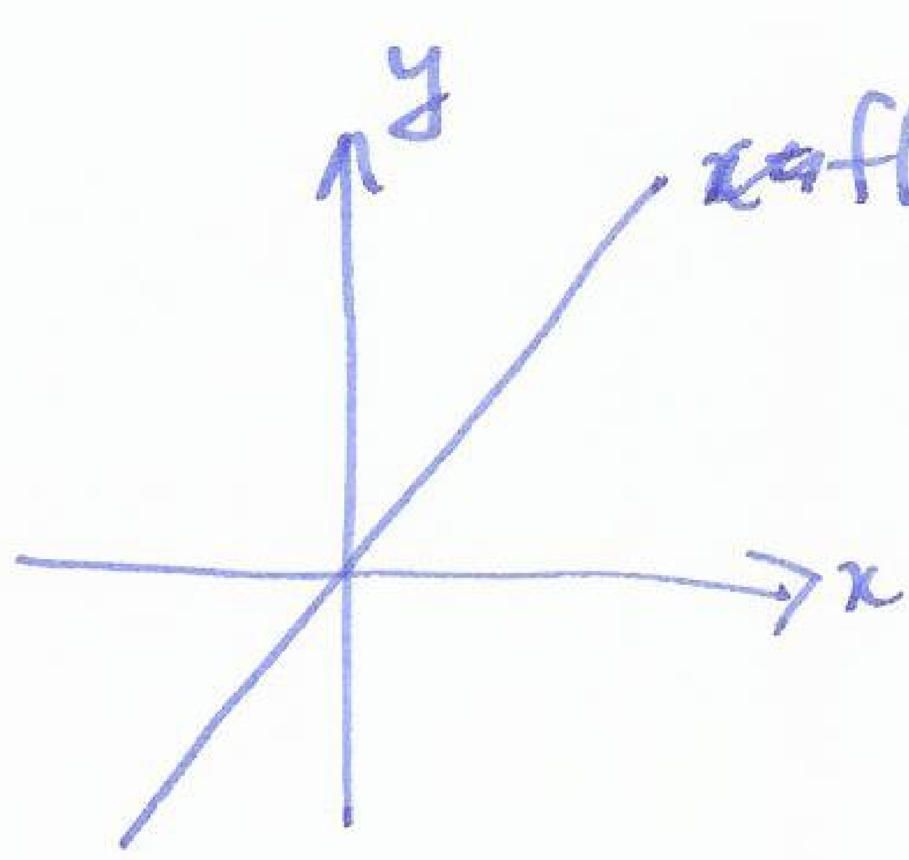
$\mathbb{R} \to \mathbb{R}$

$x \mapsto f(x)$

$x \mapsto 2x$.

equation: relation between some variables.

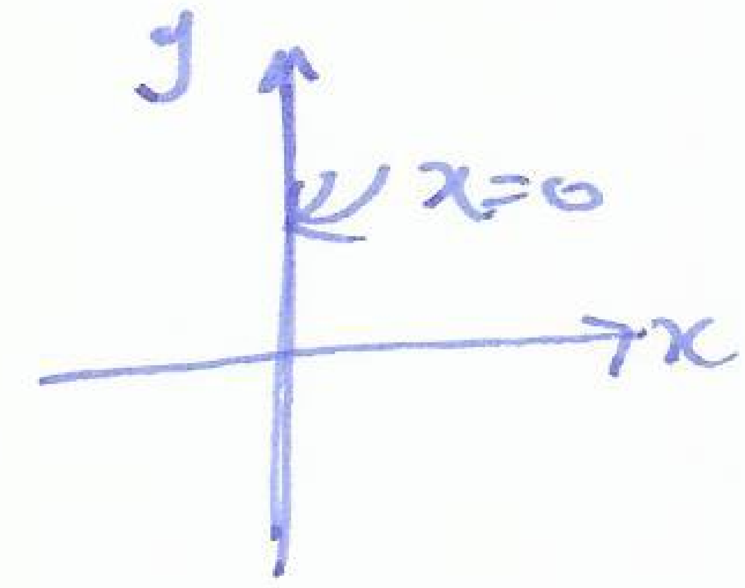
$y = 2x$.



$\cancel{x=}\, f(x) = 2x$

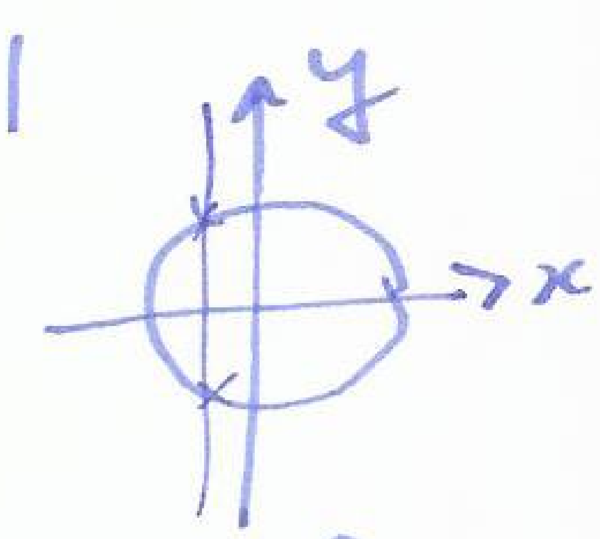
$y = 2x$.

$x = 0$.



not a function of x!

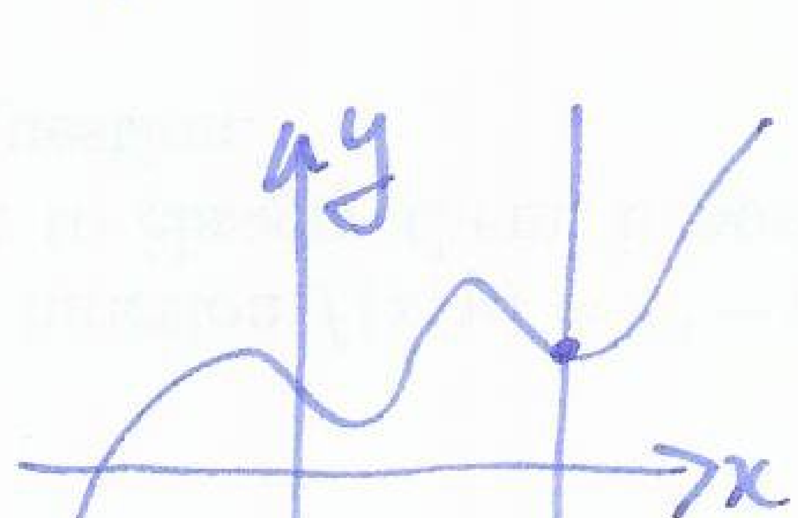
$x^2 + y^2 = 1$



not a function

Q: how do we tell if an equation describes a function?

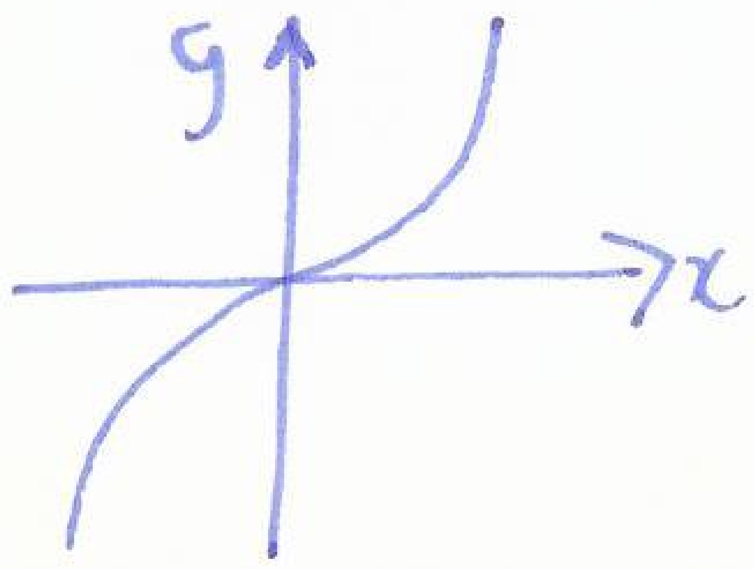
A the vertical line test:



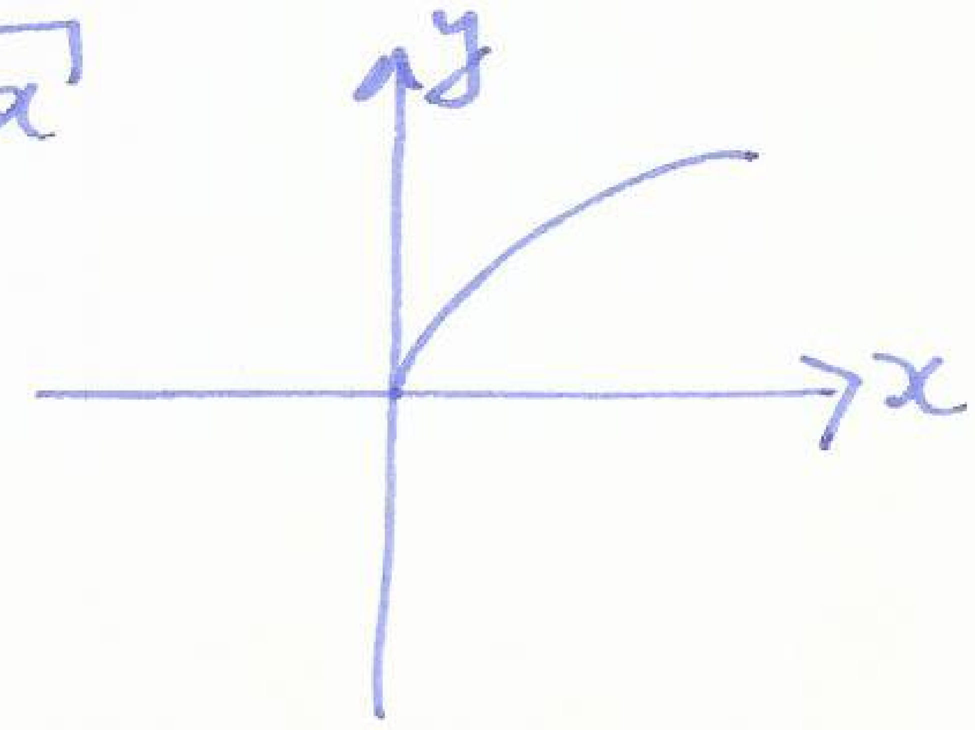
every vertical line intersects the solution set in exactly one point.

more examples of functions

$f(x) = x^3$

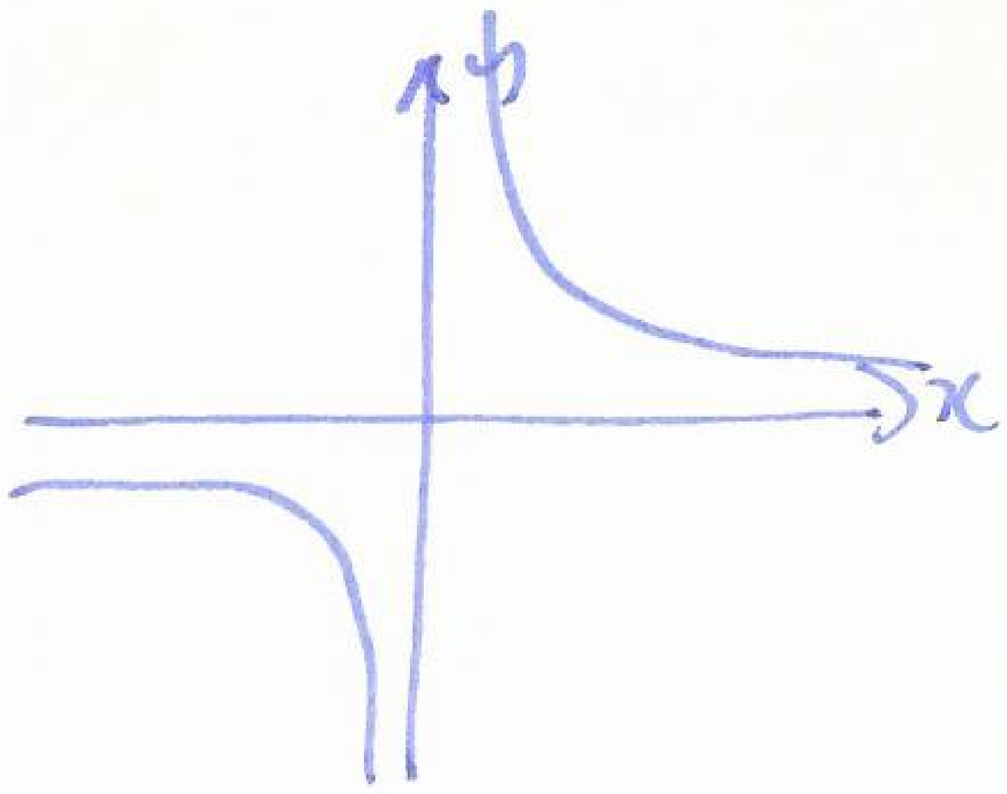


$f(x) = \sqrt{x}$



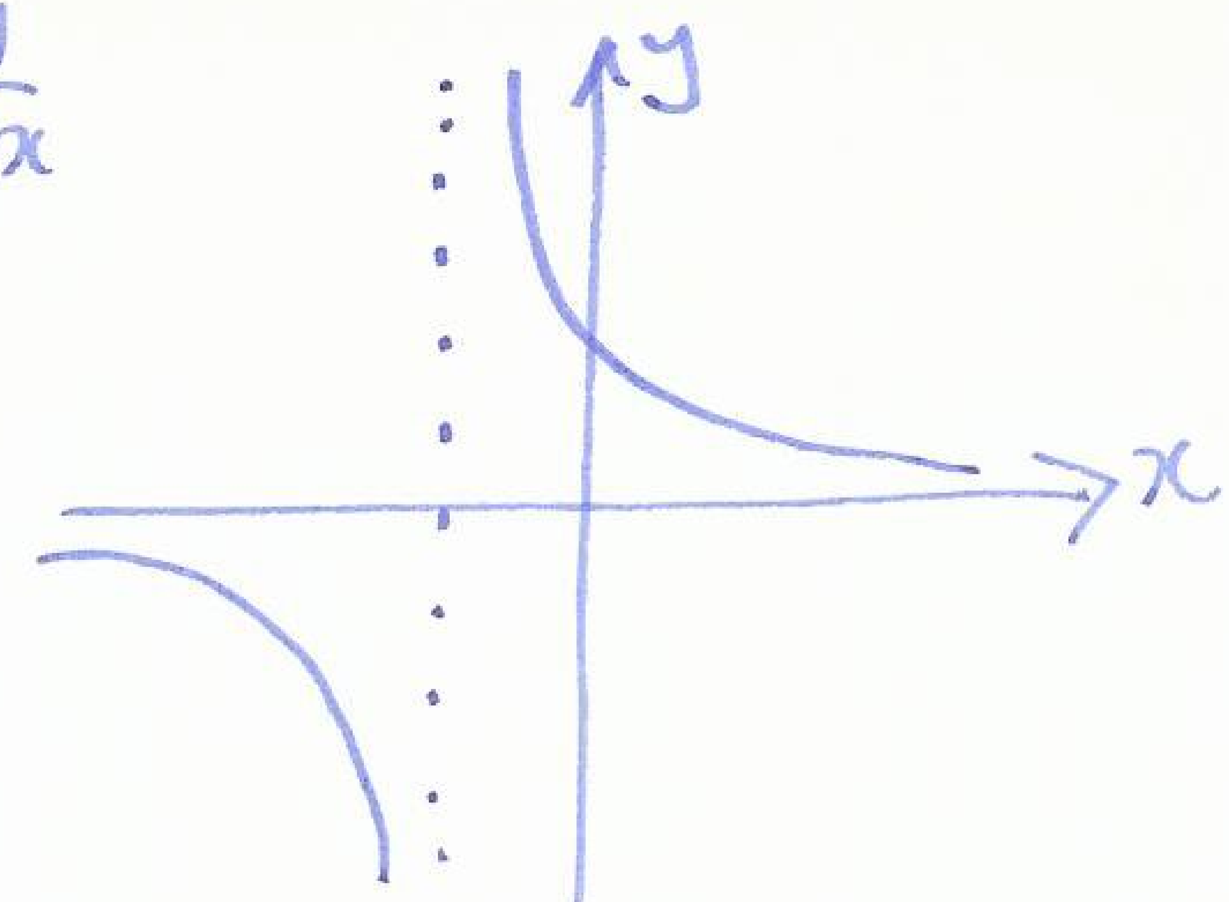
domain $[0,\infty)$.

$f(x) = \frac{1}{x}$



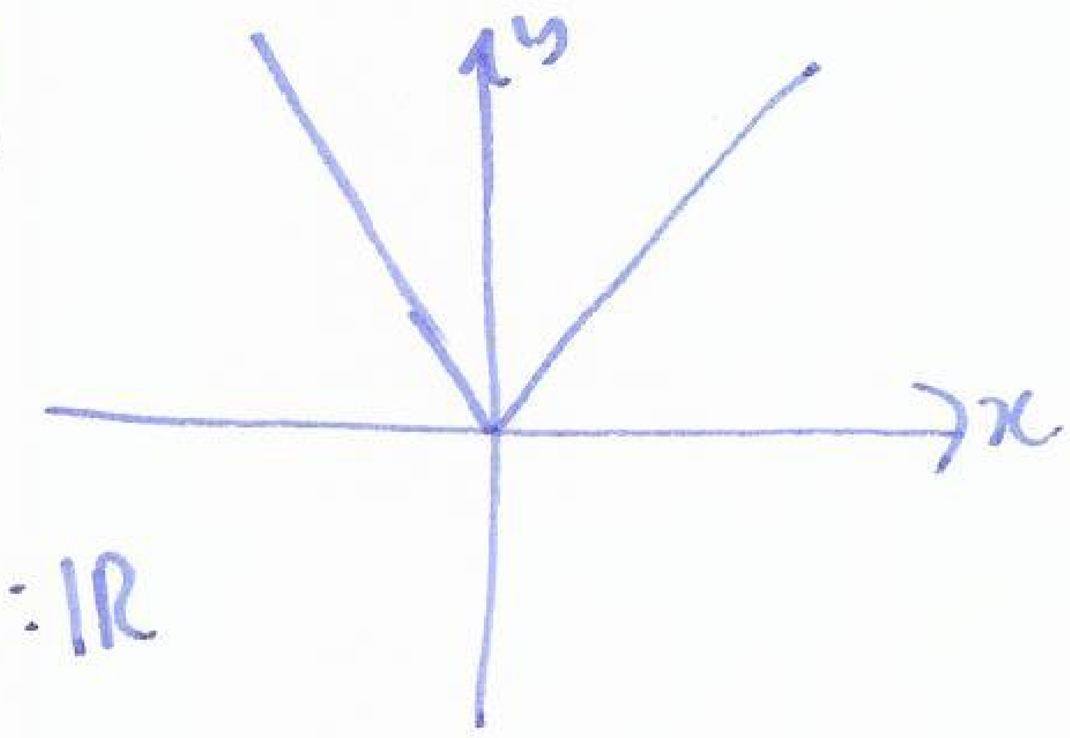
domain: $\mathbb{R}\setminus 0$.
$(-\infty,0)\cup(0,\infty)$

$f(x) = \frac{1}{1+x}$



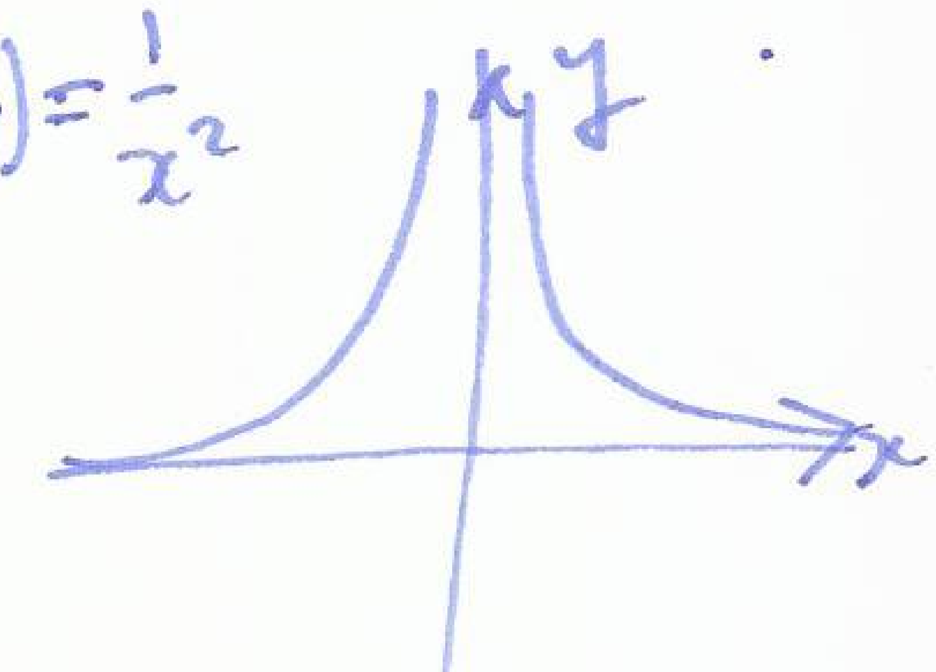
domain $(-\infty,-1)\cup(-1,\infty)$

$f(x) = |x|$

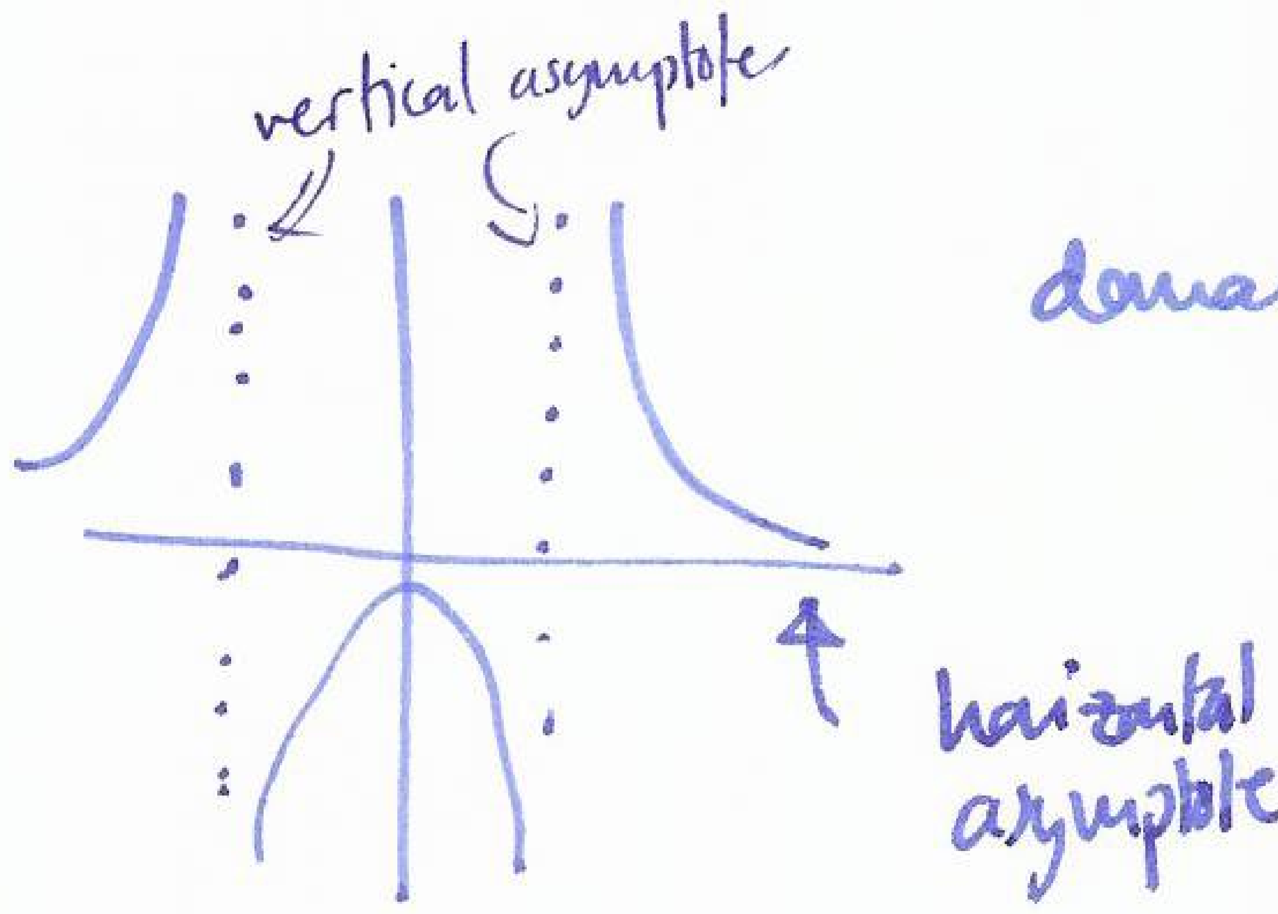


domain: $\mathbb{R}$

$\cancel{f(x)= \frac{1}{1-x^2}}$ $f(x) = \frac{1}{x^2}$

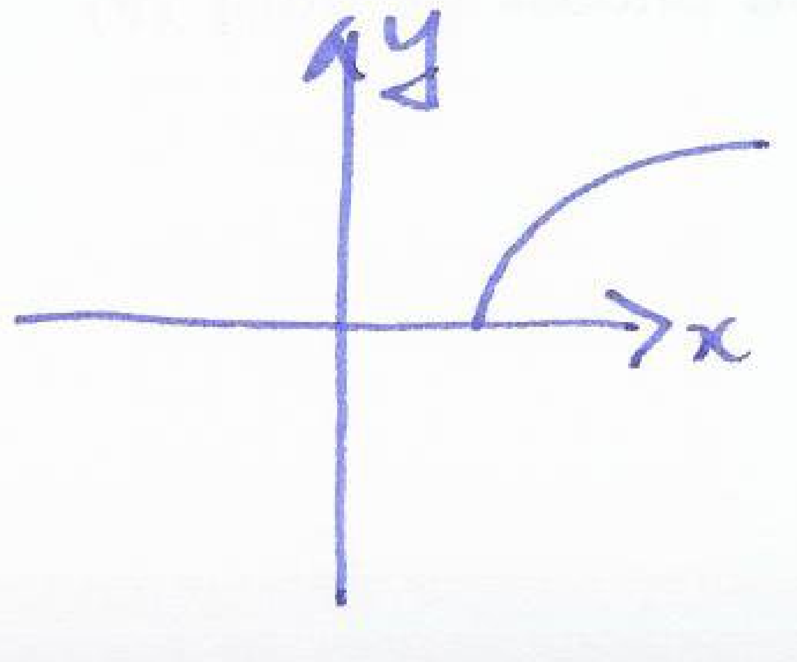


$f(x) = \frac{1}{x^2-1}$



domain: not defined at $(x-1)(x+1)$
$x = \pm 1$.

$f(x) = \sqrt{x-1}$



$f(x) = \sqrt{x} - 1$