• You may use a graphing calculator.
• You may use a $3 \times 5$ index card of notes.

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<th>Midterm 3</th>
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(1) (10 points) Solve the following triangle: $a = 4\text{cm}$, $b = 6\text{cm}$, $c = 8\text{cm}$.

\[a^2 = b^2 + c^2 - 2bc \cos A\]
\[16^2 = 36 + 64 - 96 \cos A\]
\[\cos A = \frac{84}{96}\]
\[A = \cos^{-1}\left(\frac{84}{96}\right) \approx 0.505\]
\[28.96^\circ\]

\[b^2 = a^2 + c^2 - 2ac \cos B\]
\[36 = 16 + 64 - 64 \cos B\]
\[\cos B = \frac{-44}{-64}\]
\[B = \cos^{-1}\left(\frac{-44}{64}\right) \approx 0.813\]
\[46.6^\circ\]

\[C = \pi - 0.505 - 0.813 \approx 1.824\]
\[104.7^\circ\]
(2) (15 points) Find all solutions that are in \([0, 2\pi]\) of the equation 

\[4\sin^2 x - 1 = 0\]

\[\sin x = \frac{1}{2}\]
\[\sin \theta = \pm \frac{1}{2}\]

\[\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\] other solutions \(\pi - \frac{\pi}{6} = \frac{5\pi}{6}\)

\[\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}\] solution in \([0, 2\pi]\) is \(2\pi - \frac{\pi}{6} = \frac{11\pi}{6}\)

other solution is \(\pi + \left(\frac{5\pi}{6} - \frac{\pi}{6}\right)\)

\[\pi + \frac{\pi}{6} = \frac{7\pi}{6}\]

solutions are: \(\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\)
(3) (10 points) Simplify \( \tan(\sin^{-1}(x/3)) \).

\[
\tan \theta = \frac{x}{\sqrt{9-x^2}}
\]
(4) (15 points) Find all the angles in both triangles with the following properties:
\( a = 12 \text{ cm}, \ b = 10 \text{ cm}, \ B = 46^\circ \).

\[
\frac{a}{\sin A} = \frac{b}{\sin B}
\]

\[
\sin A = \frac{12 \sin 46^\circ}{10} \approx 0.863
\]

\( A \approx 1.042 \)

\( 59.7^\circ \)

\( +\pi & -1.042 \)

\( \text{two solutions in } (9, \pi) \)

\( \text{other solution } -\pi - 1.042 \approx 2.100 \)

\( 120.3^\circ \)
(5) (10 points) Prove the following identity:

\[
\frac{1}{\tan \theta + \cot \theta} = \sin \theta \cos \theta
\]
(6) (15 points) Sketch one period of the graph $y = 20 \cos \left( \frac{1}{2} \left( x - \frac{\pi}{4} \right) \right)$. Label the lowest points, the highest points and the x-intercepts of the graph with their coordinates.
(7) (15 points) Consider the complex number $z = -2 - 2i$.
(a) Write $z$ in trigonometric form as $z = r(\cos(\phi) + i \sin(\phi))$.
(b) Compute $z^2$ using the standard $a + bi$ form.
(c) Compute $z^2$ using the trigonometric form, e.g. by using De Moivre's theorem.

\[ r = \sqrt{(-2)^2 + (-2)^2} = 2\sqrt{2} \]
\[ \theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4} \]
\[ z = 2\sqrt{2} \left( \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right) \]

b) $z^2 = (-2-2i)^2 = (-2)^2 + 2(-2)(-2i) + (-2i)^2 = 4 + 8i - 4 = 8i$

c) $z^2 = (2\sqrt{2})^2 \left( \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right) = 8 \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) = 8i$