Math 130 Precalculus Spring 10 Midterm 3a

Name: Solutions

- You may use a graphing calculator.
- You may use a 3×5 index card of notes.

1	10	
2	15	
3	10	
4	15	
5	10	
6	15	
7	15	
	90	

Midterm 3	
Overall	

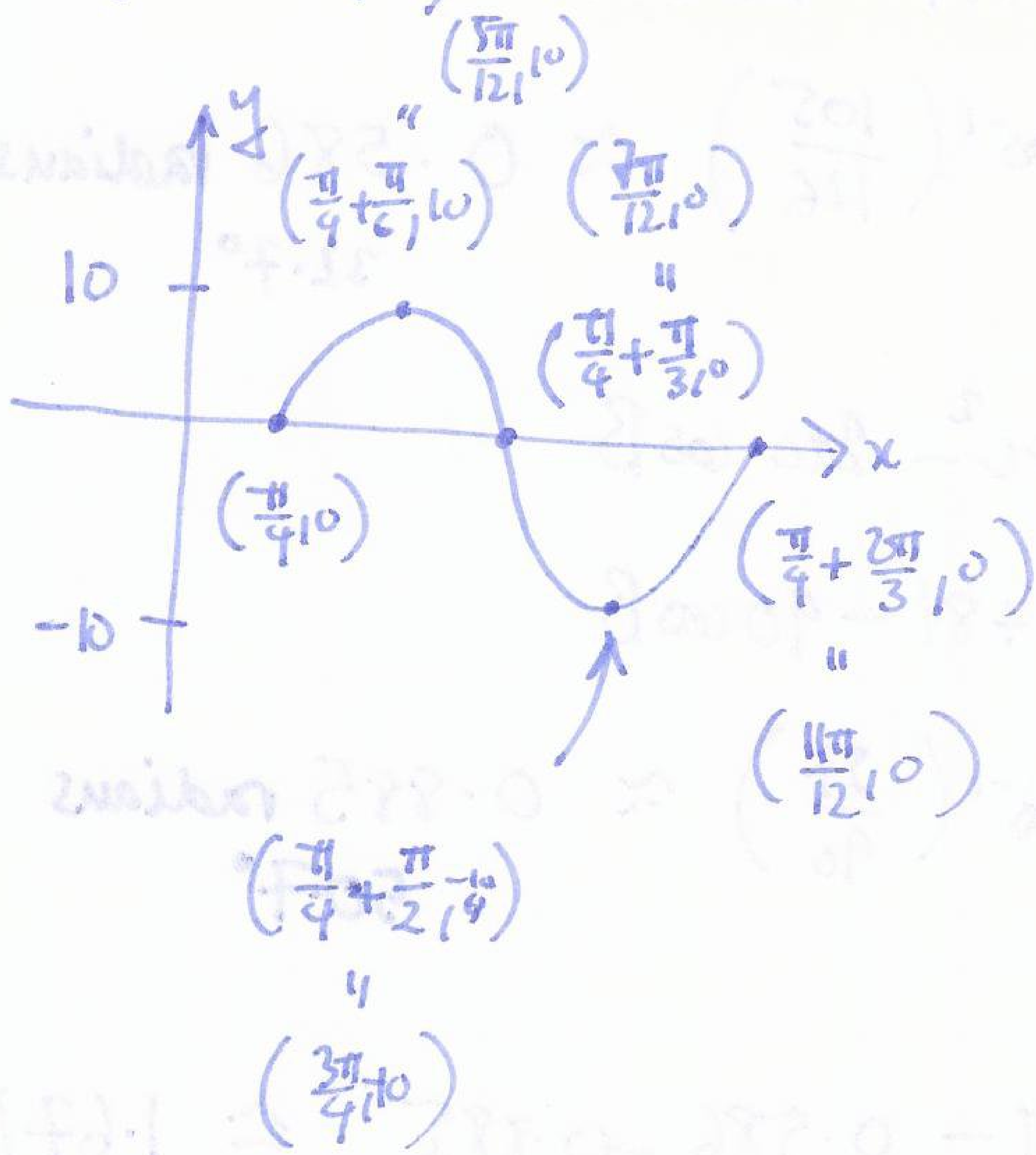
(1) (10 points) Prove the following identity:

$$\frac{1}{\tan\theta + \cot\theta} = \sin\theta\cos\theta$$

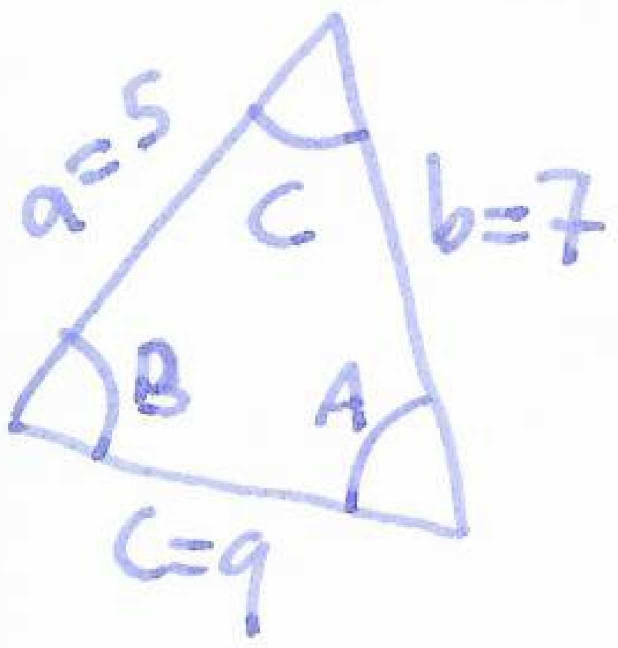
$$\frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} = \frac{1}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}} = \frac{1}{\frac{1}{\sin\theta\cos\theta}} = \sin\theta\cos\theta$$

(2) (15 points) Sketch one period of the graph $y = 10\sin\left(3x - \frac{3\pi}{4}\right)$. Label the lowest points, the highest points and the x-intercepts of the graph with their coordinates.

$10\sin\left(3\left(x - \frac{\pi}{4}\right)\right)$



(3) (10 points) Solve the following triangle: $a = 5$cm, $b = 7$cm, $c = 9$cm.



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$25 = 49 + 81 - 126\cos A$$

$$A = \cos^{-1}\left(\frac{105}{126}\right) \approx 0.586 \text{ radians}$$

32.7°

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$49 = 25 + 81 - 90\cos B$$

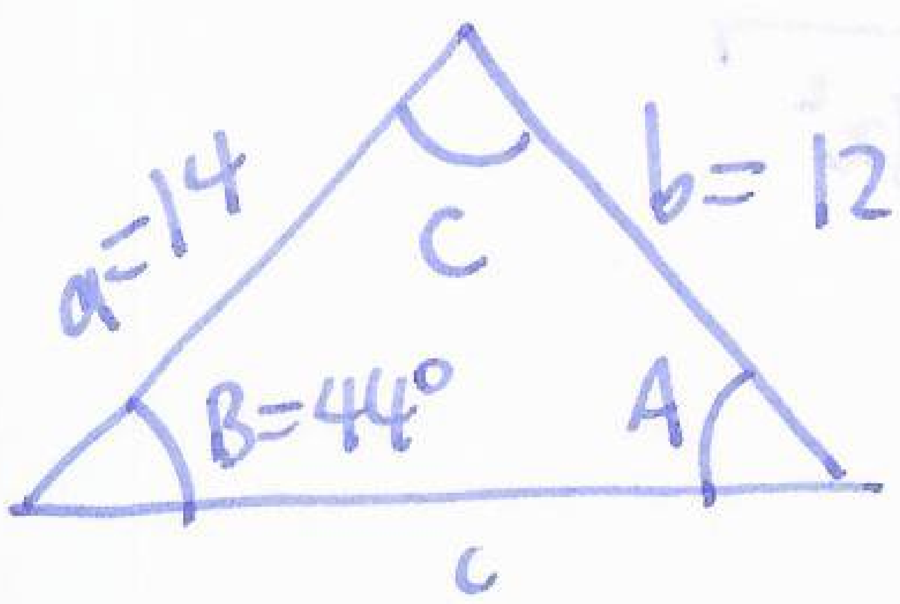
$$B = \cos^{-1}\left(\frac{57}{90}\right) \approx 0.885 \text{ radians}$$

50.7°

$$C = \pi - 0.586 - 0.885 \simeq 1.671$$

96.6°

(4) (15 points) Find all triangles with the following properties: $a = 14$cm, $b = 12$cm, $B = 44°$.

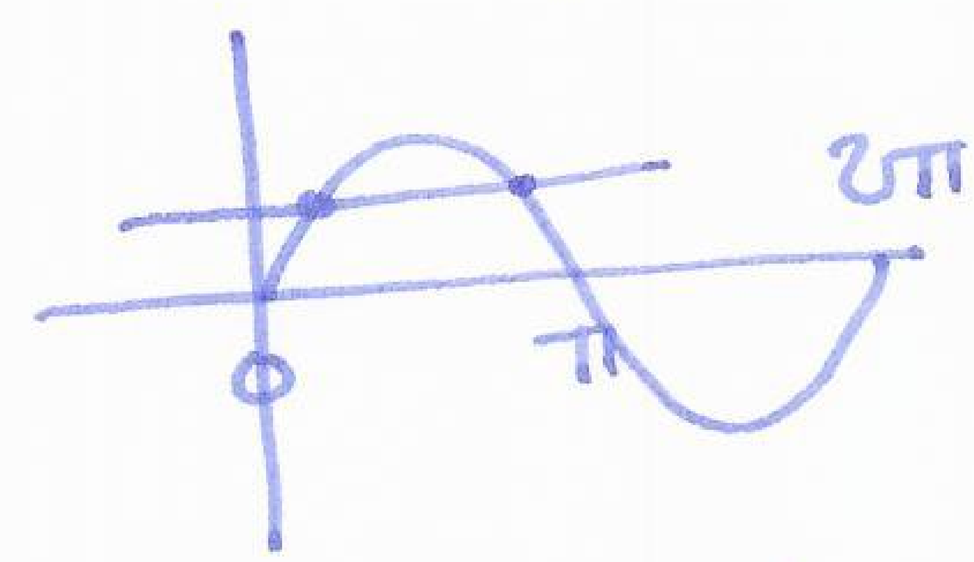


$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin A = \frac{14 \sin 44°}{12}$$

$$\approx 0.810$$

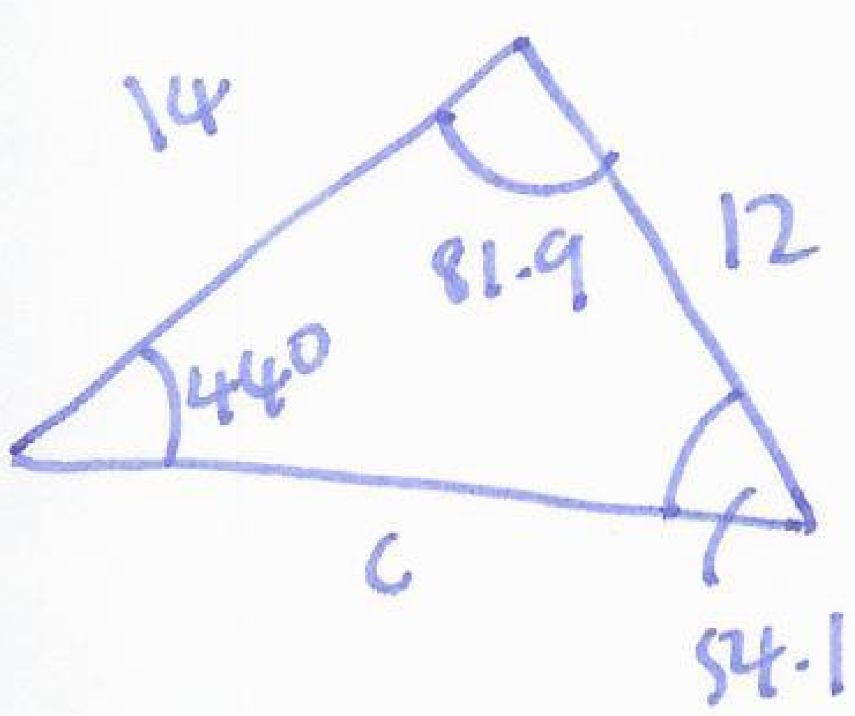
$$A \simeq 0.945 \simeq 54.1°$$

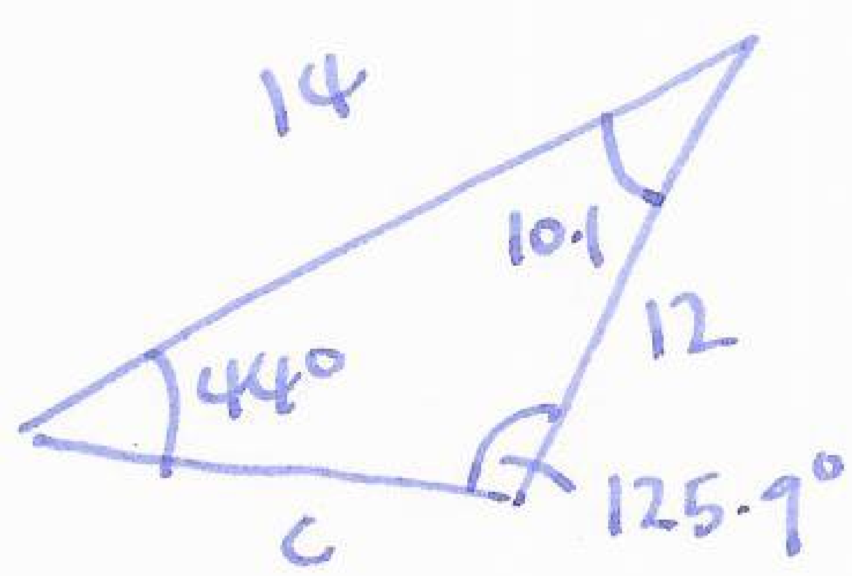


two solutions $A \simeq 0.945 \simeq 54.1°$

$A \simeq \pi - 0.945$

$\simeq 180 - 54.1° = 125.9°$





$$\frac{c}{\sin 81.9°} = \frac{12}{\sin 44°}$$

$$c = 12 \cdot \frac{\sin 81.9°}{\sin 44°}$$

$$c \simeq 17.1$$

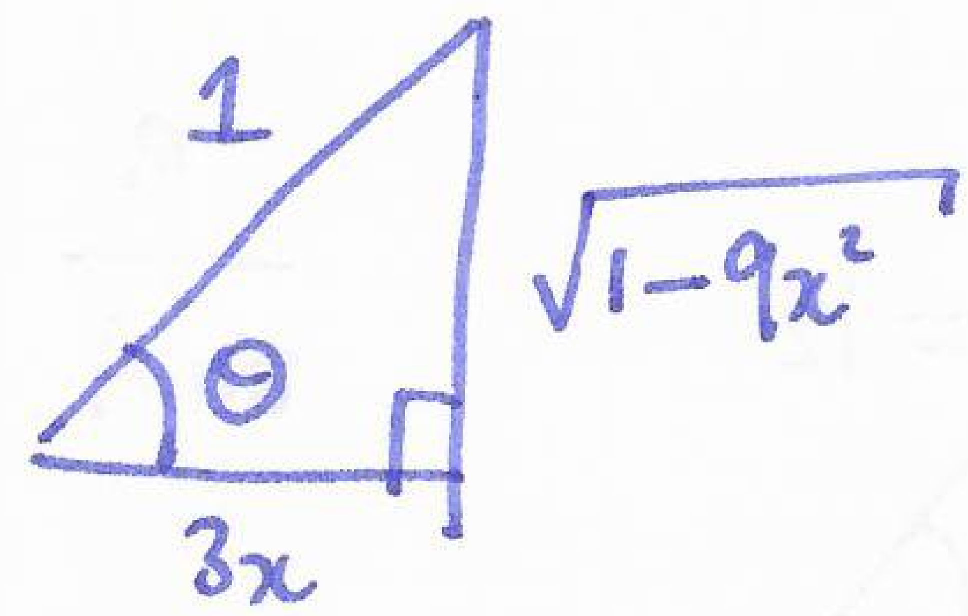
$$\frac{c}{\sin 10.1} = \frac{12}{\sin 44°}$$

$$c = \frac{12 \sin 10.1°}{\sin 44°}$$

$$c \simeq 3.03$$

(5) (10 points) Simplify $\tan(\cos^{-1}(3x))$.

$\tan\theta \qquad \cos\theta = 3x$

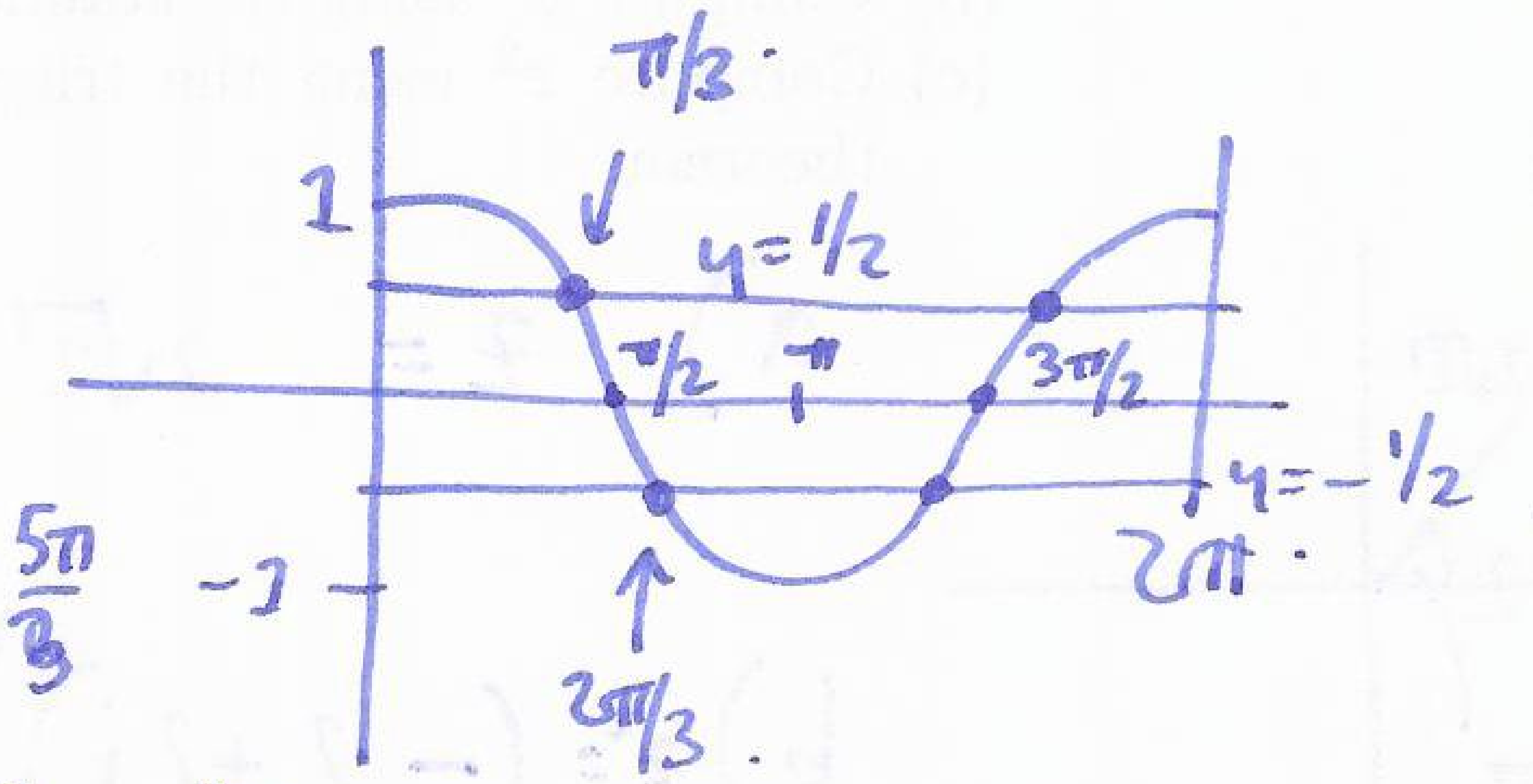


$$\tan\theta = \frac{\sqrt{1-9x^2}}{3x}$$

(6) (15 points) Find all solutions that are in $[0, 2\pi)$ of the equation

$$4\cos^2 x - 1 = 0$$

$\cos^2 x = \frac{1}{4}$ $\quad \cos x = \pm\frac{1}{2}$



$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ other solution $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ other solution $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$

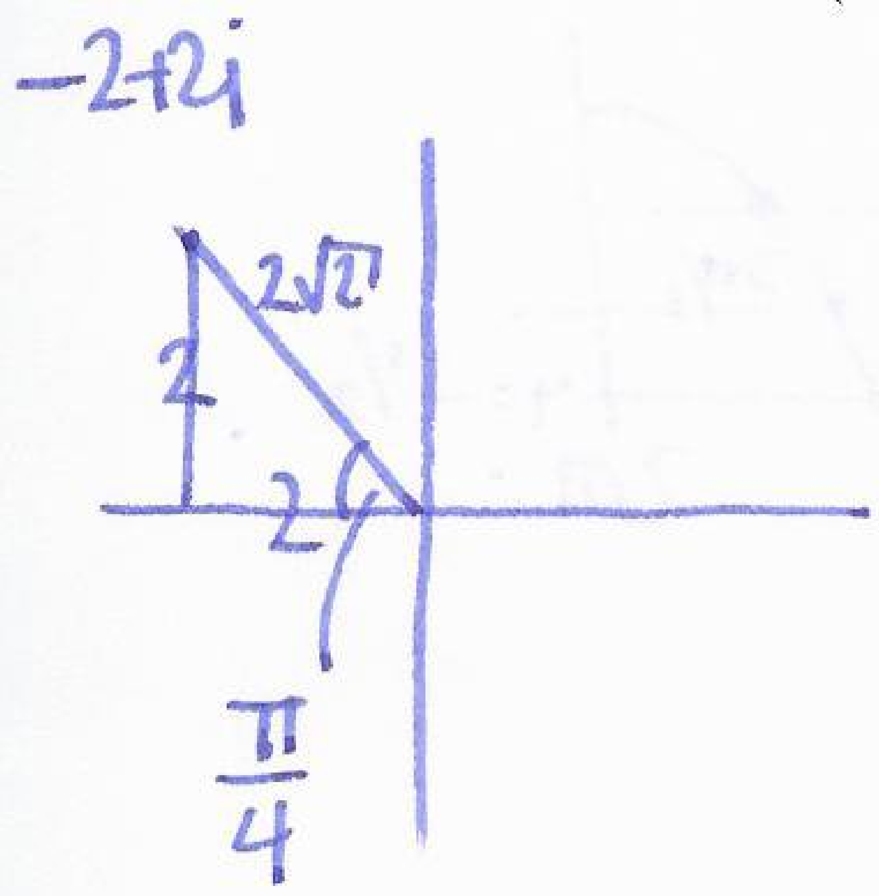
solutions : $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(7) (15 points) Consider the complex number $z = -2 + 2i$.

(a) Write z in trigonometric form as $z = r(\cos(\phi) + i\sin(\phi))$.

(b) Compute z^2 using the standard $a + bi$ form.

(c) Compute z^2 using the trigonometric form, i.e. by using De Moivre's theorem.



a) $z = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

b) $z^2 = (-2+2i)^2 = 4 - 8i + 4i^2$

$= 4 - 4 - 8i = -8i$

c) $z^2 = (2\sqrt{2})^2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

$= 8\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = -8i$