Math 130 Precalculus Spring 10 Midterm 2a

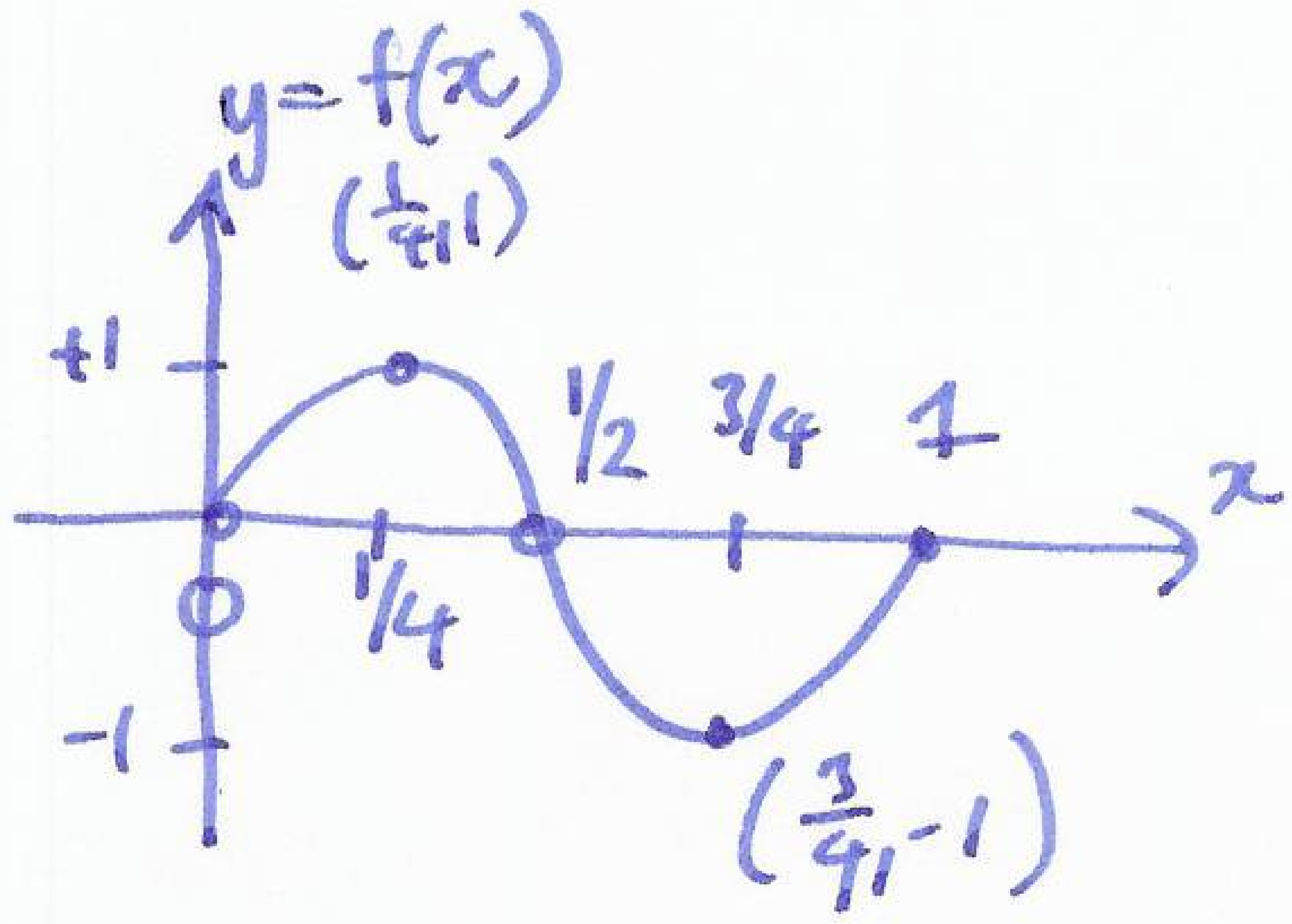
Name: Solutions

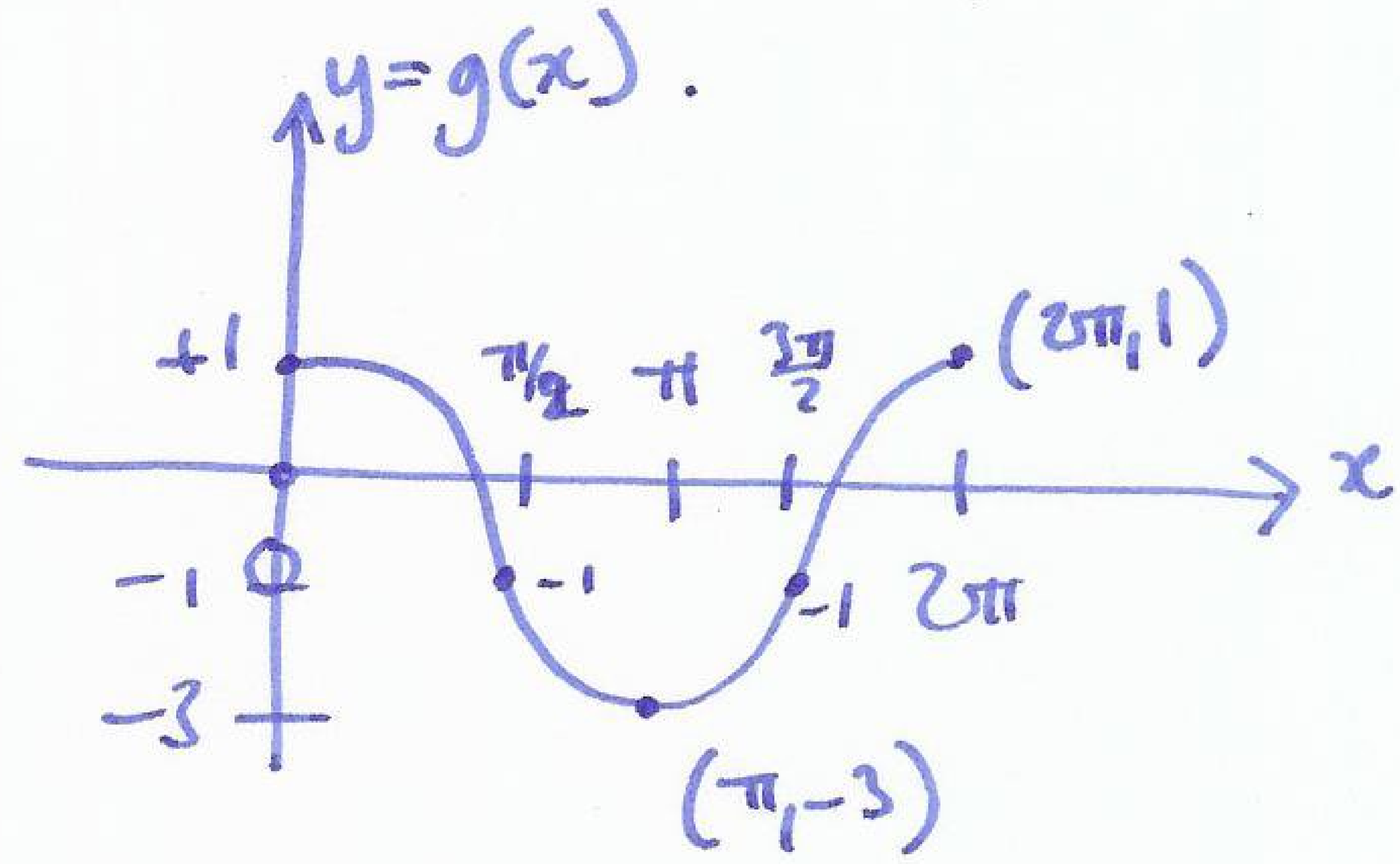
- You may use a graphing calculator.
- You may use a 3 × 5 index card of notes.

1	15	
2	10	
3	15	
4	10	
5	10	
6	15	
7	10	
8	15	
	100	

(1) (10 points) Sketch one period of the graphs of the following functions.

$$f(x) = \sin(2\pi x), \qquad g(x) = 2\cos(x) - 1$$



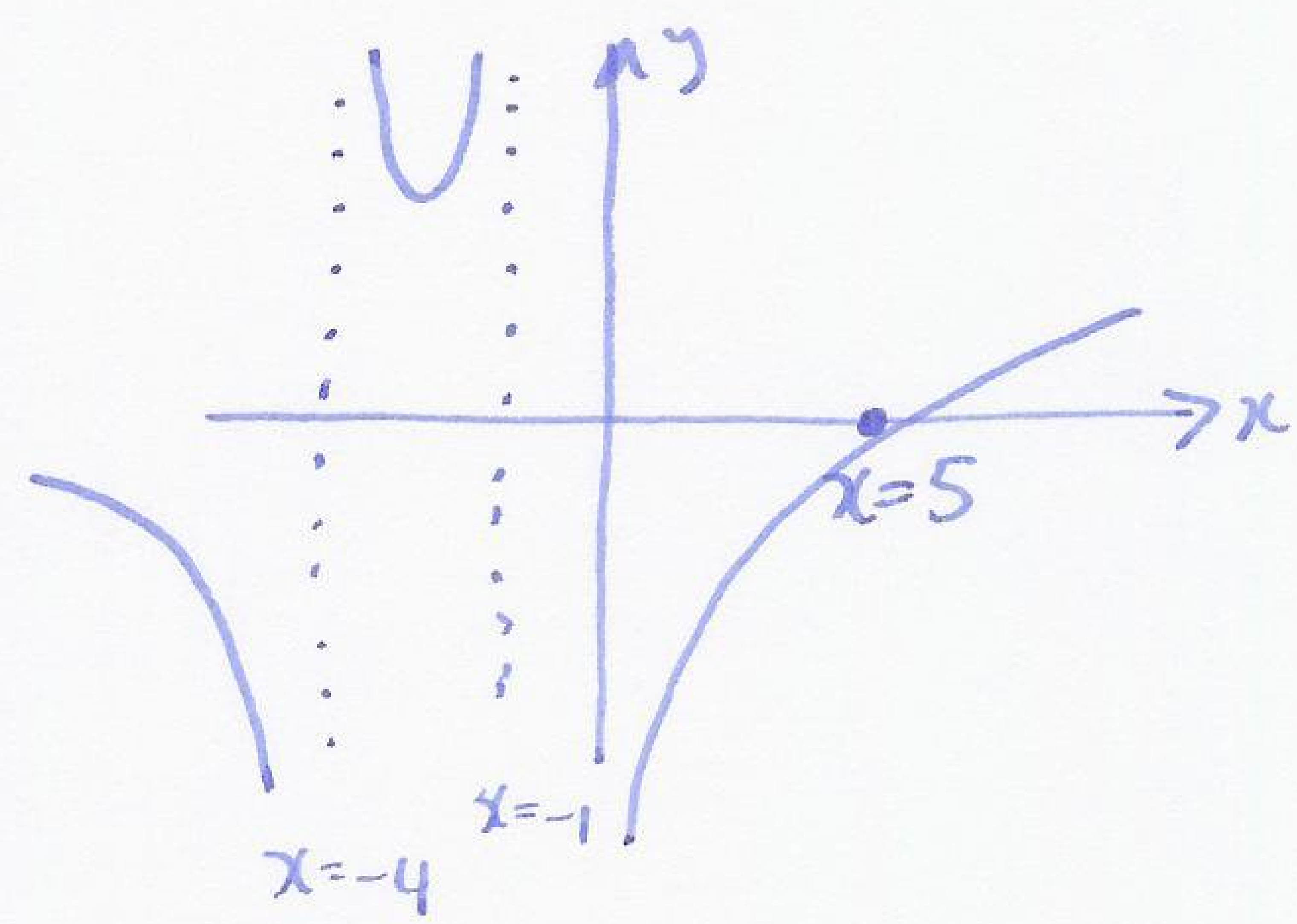


(2) (15 points) Solve the inequality $\frac{x-2}{x+4} \leqslant \frac{x-3}{x+1}$. Write your answer in interval notation.

$$\frac{x-3}{x+1} - \frac{x-2}{x+4} \geqslant 0 \qquad \frac{(x-3)(x+4)-(x-2)(x+1)}{(x+1)(x+4)} \geqslant 0$$

$$\frac{x^2+x-12-x^2+x+2}{(x+1)(x+4)} \geqslant 0 \qquad \frac{2x-10}{(x+1)(x+4)} \geqslant 0$$

$$\frac{2(x-5)}{(x+1)(x+4)} \geqslant 0$$



$x \geqslant 5$	$+/++$	$+$
$-1 < x \leqslant 5$	$-/++$	$-$
$-4 < x < -1$	$-/-+$	$+$
$-4 < x$	$-/--$	$-$

$(-4,-1) \cup [5,\infty)$

(3) (10 points) Let $f(x) = 2x^2 - 3x - 2$ and $g(x) = -3x - 1$. Compute and simplify $(f \circ g)(x)$.

$$f(g(x)) = 2(-3x-1)^2 - 3(-3x-1) - 2$$

$$= 2(9x^2+6x+1) + 9x + 3 - 2$$

$$= 18x^2 + 21x + 3$$

(4) (10 points) If $f(x) = \frac{3x}{5-x}$, find and simplify $f^{-1}(x)$=?

$y = \frac{3x}{5-x}$ swap x, y: $x = \frac{3y}{5-y}$

$x(5-y) = 3y$

$5x - xy = 3y$

$5x = 3y + xy$

$5x = y(3+x)$

$y = \frac{5x}{3+x}$ so $f^{-1}(x) = \frac{5x}{3+x}$

(5) (10 points) Show the following identities:

(a) $(\cos x + \sin x)(\cos x - \sin x) = \cos 2x$

(b)

$$\frac{2\sin x}{\sin 2x} = \sec x$$

a) $(\cos x + \sin x)(\cos x - \sin x) = \cos^2 x - \sin^2 x = \cos 2x$

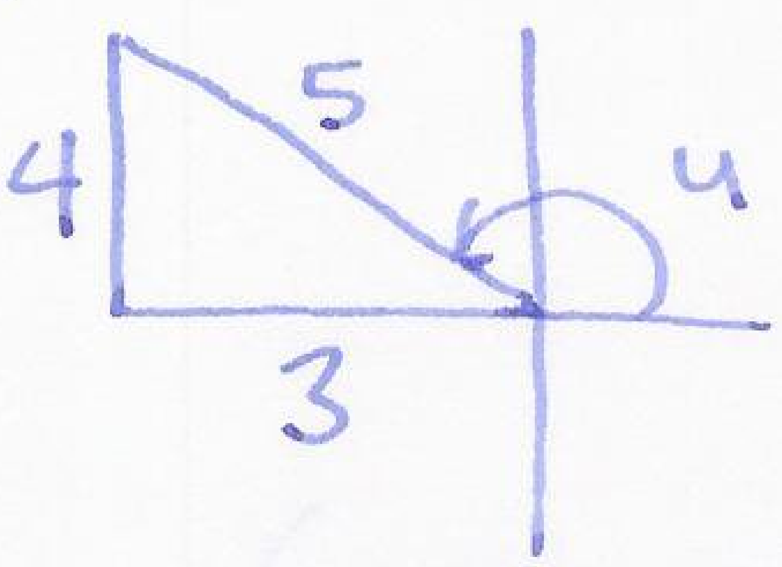
(double angle formula)

b) $\dfrac{2\sin x}{\sin 2x} = \dfrac{2\sin x}{2\sin x\cos x} = \dfrac{1}{\cos x} = \sec x$

(double angle formula)

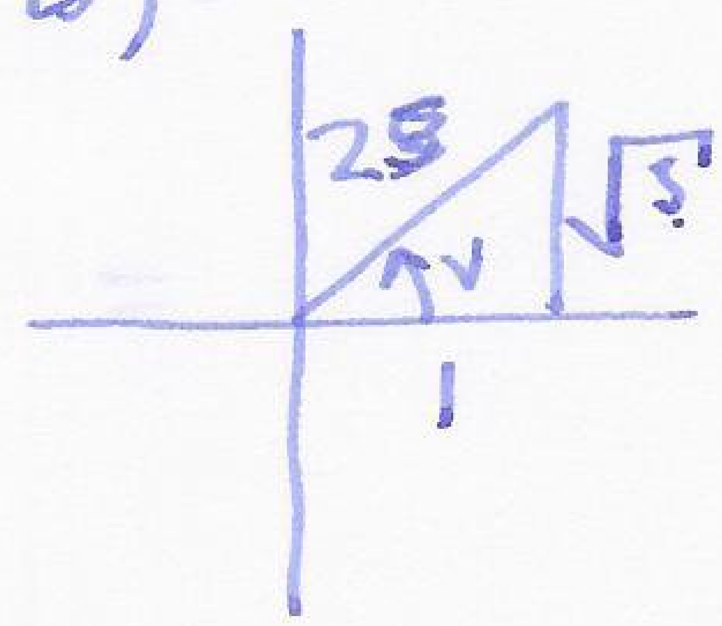
(6) (15 points) If $\cos(u) = -\frac{3}{5}$, (u in quadrant II), and $\sin(v) = \frac{1}{2}$, (v in quadrant I), use properties of right angled triangles and suitable identities to find the values of the following: (a) $\sin(u)$, (b) $\cos(v)$, and (c) $\sin(u+v)$. Write all answers as fractions.

a)



$\sin(u) = \frac{4}{5}$

b)



$\cos(v) = \frac{\sqrt{3}}{2}$

c) $\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cos(u)$

$= \frac{4}{5} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \left(-\frac{3}{5}\right) = \frac{4\sqrt{3}}{10} - \frac{3}{10}$

$= \frac{4\sqrt{3} - 3}{10}$

(7) (10 points) Show the following identity:

$$\sin^3(x) = \sin(x) - \tfrac{1}{2}\sin(2x)\cos(x)$$

$\sin(x) - \frac{1}{2}\sin(2x)\cos(x) = \sin x - \sin x \cos^2 x$

(double angle)

$= \sin x \,(1 - \cos^2 x)$

$= \sin x \sin^2 x$ (Pythagorean identity).

$= \sin^3 x$.

(8) (25 points) Consider the function $f(x) = 10\sin(\frac{1}{2}x - \frac{\pi}{2})$.

(a) What is the amplitude?

(b) What is the period?

(c) What is the phase shift?

(d) Draw a picture of the graph for one period, labelling the x-intercepts and the maximum and minimum.

$f(x) = 10\sin\left(\frac{1}{2}(x-\pi)\right)$

a) 10 b) $\frac{2\pi}{1/2} = 4\pi$ c) π

d)