Math 130 Precalculus Spring 10 Midterm 1b

Name: Solutions

- You may use a graphing calculator.
- You may use a 3 x 5 index card of notes.

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(1) (10 points) Given $u = 3 + 4i$ and $v = 5 - 2i$, compute $u + v$, $uv$ and $u/v$.

$u + v = 8 + 2i$

$uv = (3 + 4i)(5 - 2i) = 15 - 6i + 20i - 8i^2 = 23 + 14i$

$rac{u}{v} = \frac{(3 + 4i)(5 + 2i)}{(5 - 2i)(5 + 2i)} = \frac{15 + 6i + 20i + 8i^2}{25 + 4} = \frac{7}{29} + \frac{26}{29}i$
(2) (10 points)
(a) What is the maximum number of distinct roots a cubic polynomial can have? Write down a cubic polynomial with this number of roots.

(b) What is the smallest number of distinct roots a cubic polynomial can have? Write down a cubic polynomial with this number of roots.

\text{a) } 3 \quad (x-1)(x-2)(x+3) \\
\text{b) } 1 \quad x^3
Find the equation of the graph obtained by taking the graph of $f(x) = x^2$ and shifting it 4 units to the right, then reflecting it across the $y$-axis, and finally shifting it up 6 units.

- Shift 4 units to the right: $(x+4)^2$
- Reflect across $y$-axis: $(-x-4)^2$
- Shift up 6 units: $(-x-4)^2 + 6$
(4) (10 points) Find the domain and the range of the following function and sketch its graph.

\[ f(x) = 3 + \sqrt{3x - 1} \]

domain: \( x \geq \frac{1}{3} \) or \((\frac{1}{3}, \infty)\)

range: \( x \geq 3 \) or \([3, \infty)\)
(5) (10 points) Consider \( f(x) = x - 3x^5 \). Check \( f \) algebraically for symmetries. Graph \( f \) using the calculator and find (using the calculator) all zeros, local maxima and local minima, if any.

\[
\begin{align*}
f(-x) &= (-x) - 3(-x)^5 = -x + 3x^5 \neq f(x) \quad \text{not even} \\
f(-x) &= -f(x) \quad f \text{ is odd}
\end{align*}
\]
(6) (20 points) Let \( p(x) = 3x^3 + 7x^2 - 22x - 8 \).

(a) Give a complete list of all possible rational zeros.

(b) Check, using either long division or synthetic division, that \( x = 2 \) is a rational zero.

(c) Find all remaining zeros.

(d) Write \( p \) as a product of linear factors.

(e) Sketch the graph of \( p \).

\[ \pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3} \]

\[ \begin{array}{c}
\frac{3x^2 + 13x + 4}{x - 2} \\
\frac{3x^3 + 7x^2 - 22x - 8}{3x^3 - 6x^2} \\
13x^2 - 22x - 8 \\
13x^2 - 26x \\
4x - 8 \\
0
\end{array} \]

\[ x = 2 \text{ is a zero.} \]

\[ 3x^2 + 13x + 4 = (3x + 1)(x + 4) \]

Zeros are \( 2, -4, -\frac{1}{3} \).

\[ p(x) = (x - 2)(3x + 1)(x + 4) \]

\[ p(0) = -8 \]
(7) (20 points) Consider the function
\[ f(x) = \frac{x + 2}{3x^2 - 9x + 6} \]
(a) Find the domain of \( f \) and the vertical asymptotes.
(b) Find the horizontal asymptote of \( f \), if it has one.
(c) Find the zeros of \( f \) and the value of \( f(0) \).
(d) Sketch the graph of \( f \).

\[ 3x^2 - 9x + 6 = 3(x^2 - 3x + 2) = 3(x-2)(x-1) \]

\[ \text{domain: } \mathbb{R} \setminus \{1, 2\} \cup (-\infty, 1) \cup (1, 2) \cup (2, \infty) \]

b) \( f(x) = \frac{p}{q} \) with \( \deg p < \deg q \implies y=0 \text{ is horizontal asymptote.} \)

c) \( f(x) = 0 \) at \( x = -2 \) \( f(-2) = \frac{1}{3} \)

d)
(8) (15 points) Consider the polynomials \( p \) and \( q \) given by
\[
p(x) = 2x^4 + 3x^3 + 6x^2 + 12x - 8, \quad q(x) = x^2 + 4
\]
(a) Calculate \( \frac{p}{q} \) using long division.
(b) Find all real and complex zeros of \( p \).

\[
\begin{array}{c|ccccc}
& 2x^3 & +3x^2 & -2x & -8 \\
\hline
x^2 + 4 & 2x^4 & +3x^3 & +6x^2 & +12x & -8 \\
 & 2x^4 & +8x^2 \\
\hline
 & 3x^3 & -2x^2 & +12x & -8 \\
 & 3x^3 & +12x \\
\hline
 & -2x^2 & -8 \\
 & -2x^2 & -8 \\
\hline
 & 0 & & & & 
\end{array}
\]

\[
f(x) = 2x^2 + 3x - 2
\]

b) \( \text{zeros: } +2i, -2i, -2, \frac{1}{2} \)

\[
2x^2 + 3x - 2 = (x + 2)(2x - 1)
\]