Math 130 Precalculus Spring 10 Midterm 1a

Name: Solutions

- You may use a graphing calculator.
- You may use a $3 \times 5$ index card of notes.

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(1) (10 points)
(a) What is the maximum number of distinct roots a cubic polynomial can have? Write down a cubic polynomial with this number of roots.
(b) What is the smallest number of distinct roots a cubic polynomial can have? Write down a cubic polynomial with this number of roots.

a) 3 \((x-1)(x-2)(x-3)\)
b) 1 \(x^3\)
(2) (10 points) Given \( u = 2 - 5i \) and \( v = 1 + 4i \), compute \( u + v \), \( uv \) and \( u/v \).

\[
u + v = 3 - i
\]

\[
u v = (2-5i)(1+4i) = 2 + 8i - 5i - 20i^2 = 22 + 3i
\]

\[
u \frac{v}{u} = \frac{(2-5i)(1-4i)}{(1+4i)(1-4i)} = \frac{2-8i-5i+20i^2}{1+16} = \frac{-18}{17} - \frac{13}{17}i
\]
(3) (10 points) Find the domain and the range of the following function and sketch its graph.

\[ f(x) = 4 - \sqrt{4 - x} \]

- Domain: \( x \leq 4 \) or \((-\infty, 4]\)
- Range: \( x \leq 4 \) or \((-\infty, 4]\)
(4) (10 points) Find the equation of the graph obtained by taking the graph of \( f(x) = x^2 \) and shifting it 7 units to the left, then reflecting it across the y-axis, and finally shifting it down 3 units.

\[
\text{shift to left : } (x+7)^2 \\
\text{reflect across y-axis : } (-x+7)^2 \\
\text{shift down three units : } (-x+7)^2 - 3
\]
(5) (10 points) Consider \( f(x) = 2x^4 - 4x^2 \). Check \( f \) algebraically for symmetries. Graph \( f \) using the calculator and find (using the calculator) all zeros, local maxima and local minima, if any.

\[
f(-x) = 2(-x)^4 - 4(-x)^2 = 2x^4 - 4x^2 = f(x) \quad f \text{ is even.} \]

\[
f(-x) = f(x) \neq -f(x) \quad f \text{ is not odd.}
\]

\[
\text{Local max: } (0, 0)
\]
\[
\text{Local min: } (1, -2), (-1, -2)
\]
(6) (20 points) Let \( p(x) = 2x^3 - x^2 - 7x + 6 \).
(a) Give a complete list of all possible rational zeros.
(b) Check, using either long division or synthetic division, that \( x = 1 \) is a rational zero.
(c) Find all remaining zeros.
(d) Write \( p(x) \) as a product of linear factors.
(e) Sketch the graph of \( p(x) \).

\[
\begin{align*}
a) & \quad x - 1 \quad 2x^2 + x - 6 \\
& \quad 2x^3 - x^2 - 7x + 6 \\
& \quad 2x^3 - 2x^2 \\
& \quad x^2 - 7x + 6 \\
& \quad x^2 - x \\
& \quad -6x + 6 \\
& \quad -2x + 6 \\
& \quad 0 \\

\text{b) } & \quad 2x^2 + x - 6 = (2x - 3)(x + 2) \quad x = -2, 1 \frac{3}{2} \\
\text{c) } & \quad p(x) = (x - 1)(x + 2)(2x - 3) \\
\text{d) } & \quad (x, y) (0, 6), (-2, 0), (1.5, 0) \\
\text{e) } & \quad \text{Graph Sketch}
\end{align*}
\]
(7) (20 points) Consider the function

\[ f(x) = \frac{x + 3}{2x^2 - 2x - 12} \]

(a) Find the domain of \( f \) and the vertical asymptotes.

(b) Find the horizontal asymptote of \( f \), if it has one.

(c) Find the zeros of \( f \) and the value of \( f(0) \).

(d) Sketch the graph of \( f \).

a) \[ 2x^2 - 2x - 12 = 2(2x - 3)(x + 2) \]
\[ = 2(x^2 - x - 6) \]
\[ \text{domain: } \mathbb{R} \setminus \{-2, 3\} \lor (-\infty, -2) \cup (-2, 3) \cup (3, \infty) \]

b) \[ f(x) = \frac{p(x)}{q(x)} \]
where \( \deg p(x) \leq \deg q(x) \) so \( y = 0 \) is horizontal asymptote.

c) \[ f(x) = 0 \text{ when } x+3=0 \text{ i.e. } x=-3. \]
\[ f(-3) = \frac{3}{-12} = -\frac{1}{4} \]

d)\[\begin{array}{c|cccc|c}
\text{Signs} & x>3 & -2<x<3 & -3<x<-2 & x<-3 & f(x) \\
\hline
+ & + & + & + & + & + \\
- & + & - & - & - & - \\
+ & + & - & - & - & + \\
- & - & - & - & - & - \\
\end{array}\]
(8) (15 points) Consider the polynomials $p$ and $q$ given by

$$
p(x) = x^4 + x^3 + 3x^2 + 9x - 54, \quad q(x) = x^2 + 9
$$

(a) Calculate $\frac{p}{q}$ using long division.

(b) Find all real and complex zeros of $p$.

\[\begin{array}{c}
x^2 + q \\
\hline
x^4 + x^3 + 3x^2 + 9x - 54 \\
\hline
x^4 + 9x^2 \\
\hline
x^3 - 6x^2 + 9x - 54 \\
\hline
x^3 + 9x \\
\hline
-6x^2 - 54 \\
\hline
-6x^2 - 54 \\
\hline
0
\end{array}\]

\[\frac{p}{q} = x^2 + x - 6\]

\[b) \quad x^2 + x - 6 = (x + 3)(x - 2)\]

\[\text{roots are: } 3i, -3i, -3, 2 .\]