Math 130 Precalculus Spring 10 Sample midterm 1

(1) (5 points) Let f be the function defined by $f(x) = x^3 - x$ and let g be the function defined by $g(u) = u^3 - u$. Which of the following statements is correct?
   (a) f and g are exactly the same functions.
   (b) if x and u are different numbers, then f and g are different functions.
   (c) not enough information is given to determine if f and g are the same.
   Write a sentence or two justifying your answer.

(2) (10 points) Find the domain and the range of the following function and sketch its graph.
$$f(x) = \sqrt{x - 30} + 20$$

(3) (10 points) Make a hand-drawn graph of
$$f(x) = \begin{cases} x + 1 & \text{for} & x \leq -1 \\ -2x & \text{for} & -1 < x < 1 \\ x - 1 & \text{for} & x \geq 1 \end{cases}$$

(4) (10 points) Find the equation of the graph obtained by reflecting the graph of $f(x) = x^2$ across the x-axis and shifting it 10 units to the right and two units up.

(5) (10 points) Given $u = 3 - 2i$ and $v = 2 + 3i$, compute $u + v$, $u - v$, uv, u/v and v/u.

(6) (10 points) Consider $f(x) = x^5 - 3x$. Check f algebraically for symmetries. Graph f using the calculator and find (using the calculator) all zeros, local maxima and local minima.

(7) (15 points) Let $p(x) = 3x^3 - 7x^2 - 22x + 8$.
   (a) Give a complete list of all possible rational zeros.
   (b) Check, using long division, whether $x = 2$ and $x = \frac{1}{3}$ are actual rational zeros.
   (c) Find all remaining zeros.
   (d) Write p as a product of linear factors.
   (e) Sketch the graph of p.

(8) (15 points) Consider the function
$$f(x) = \frac{2x^2 - 2}{3x^2 - 3x - 18}$$
   (a) Find the domain of f and the vertical asymptotes.
   (b) Find the horizontal asymptote of f.

(c) Find the zeros of f and $f(0)$.

(d) Does the graph of f intersect with its horizontal asymptote? Check algebraically!

(e) Sketch the graph of f.

(9) (10 points) Solve the following rational inequality:
$$\frac{x+1}{x+2} \leq \frac{x+3}{x-4}$$

(10) (10 points) Consider the polynomials p and q given by
$$p(x) = x^4 + 3x^3 - 6x^2 + 12x - 40, \qquad q(x) = x^2 + 4$$

(a) Calculate $\frac{p}{q}$ using long polynomial division.

(b) Find all real and complex zeros of p.