

Linear Algebra Spring 10 Sample midterm 1

- (1) (8 pts.) State whether each of the following statements is Always true, Sometimes true, or Never true.
- (a) A homogeneous 3×5 linear system has a nontrivial solution.
 - (b) If $\det(A) = 0$, then $\det(A + B) = \det(B)$.
 - (c) If $\det(A) = 0$, then $\det(BA) = 0$.
 - (d) A square matrix which has two identical columns is invertible.

- (2) (16 pts.) Justify three out of the following four statements with a short general argument:
- (a) If A is a non-singular $n \times n$ matrix then:

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

- (b) If A and B are non-singular $n \times n$ matrices, then AB is also non-singular.
 - (c) A non-singular matrix has a unique inverse.
 - (d) If A and B are symmetric matrices, then AB is also symmetric.
- (3) (20 pts.) Write “impossible” or give an example of:
- (a) A 3×3 matrix with no zeros but which is not invertible.
 - (b) A system with two equations and three unknowns that is inconsistent.
 - (c) A system with two equations and three unknowns that has a unique solution.
 - (d) A system with two equations and three unknowns that has infinitely many solutions.

- (4) (10 pts.) Consider the following linear system:

$$\begin{cases} 2x_1 - x_2 + x_4 = 0 \\ -x_1 + 2x_2 - x_3 = 1 \\ -x_2 + 2x_3 = 0 \end{cases}$$

Write its associated augmented matrix. Reduce the matrix to its row-echelon form. Use the procedure to solve the system.

- (5) (25 pts.)
- (a) Let:

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 0 & -1 & 5 \\ -4 & 2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 1 & 3 \\ 5 & 1 & 0 \end{pmatrix}$$

1

Compute $A + B$, AB , B^T , $\det(A)$, $\det(A^T)$ and $\det(3A)$.

- (b) Use elementary operations to find the inverse of:

$$C = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

- (6) (20 pts.)

- (a) Let $L : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation defined by $L(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 5 \\ 3 & 2 & 4 \end{pmatrix}$$

Is the vector $(1, 2, 3)$ in the range of L ?

- (b) Let $L : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by $L(x, y) = (2x + 3y, -2x + 3y, x + y)$. Find the standard matrix representing L .