Linear Algebra Sample midterm 1 Solutions

Q1 a) Always

b) Sometimes

c) Always

d) Never

Q2 a) A non-singular $\Rightarrow \det(A) \neq 0$ and A^{-1} exists.

$\det(AB) = \det(A)\det(B)$

so $\det(AA^{-1}) = \det(I) = 1$

$\det(AA^{-1}) = \det(A)\det(A^{-1})$

so $\det(A^{-1}) = \dfrac{1}{\det(A)}$

b) A, B non-singular $\Rightarrow \det(A) \neq 0,\ \det(B) \neq 0$

so $\det(AB) = \det(A)\det(B) \neq 0$ so AB is non-singular.

c) spose A has inverses B, C. Then

$AB = I = BA$
$AC = I = CA$

so $AB = I = AC$

$BAB = BAC$

$IB = IC \Rightarrow B = C$.

d) false.

Q3 a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ A not invertible.

b) $0x + 0y + 0z = 1$
$0x + 0y + 0z = 0$

c) impossible

d) $0x + 0y + 0z = 0$
$0x + 0y + 0z = 0$

Q4

$$\left[\begin{array}{cccc|c} 2 & -1 & 0 & 1 & 0 \\ -1 & 2 & -1 & 0 & 1 \\ 0 & -1 & 2 & 0 & 0 \end{array}\right]$$

swap r_1, r_2 multiply by -1

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & -1 \\ 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 \end{array}\right]$$

$r_2 - 2r_1$
$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & -1 \\ 0 & 3 & -2 & 1 & 2 \\ 0 & -1 & 2 & 0 & 0 \end{array}\right]$$

swap r_2, r_3 multiply by -1

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -2 & 1 & 2 \end{array}\right]$$

$$r_3 - 3r_1 \quad \begin{array}{c} \left[\begin{array}{cccc|c} 1 & -2 & 1 & 0 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 4 & 1 & 2 \end{array}\right] \\ \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \end{array}$$

$$x_4 = t \qquad 4x_3 + t = 2 \qquad x_2 - 2x_3 = 0$$

$$x_3 = \frac{1}{2} - \frac{1}{2}t \qquad x_2 = 1 - t$$

$$x_1 - 2x_2 + x_3 = -1$$

$$x_1 = -1 + 2 - 2t - \frac{1}{2} + \frac{1}{2}t = \frac{1}{2} - \frac{3}{2}t$$

$$\underline{x} = \begin{bmatrix} \frac{1}{2} - \frac{3}{2}t \\ 1 - t \\ \frac{1}{2} - \frac{1}{2}t \\ t \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1/2 \\ 0 \end{bmatrix} + t \begin{bmatrix} -3/2 \\ -1 \\ -1/2 \\ 1 \end{bmatrix}$$

Q5 a) $A + B = \begin{bmatrix} 4 & 1 & 3 \\ -1 & 0 & 8 \\ 1 & 3 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 5 \\ -4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 3 \\ 5 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 18 & 4 & 3 \\ 26 & 4 & -3 \\ -5 & 3 & 6 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 2 & -1 & 5 \\ 0 & 1 & 1 \\ 0 & 3 & 0 \end{bmatrix}$$

$$\det(A) = -0\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ -4 & 1 \end{vmatrix} - 5\begin{vmatrix} 2 & 1 \\ -4 & 2 \end{vmatrix}.$$

$$= -14 - 40 = -54$$

$$\det(A^T) = \det(A) = -54$$

$$\det(3A) = 3^3 \det(A) = -27 \cdot 54.$$

b)

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & +1 & -1 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right].$$

check!

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

Q6 a) solve $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 2 & -1 & 5 & 2 \\ 3 & 2 & 4 & 3 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & -5 & 5 & 0 \\ 0 & -4 & 4 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

A: yes.

b) $\begin{bmatrix} 2 & 3 \\ -2 & 3 \\ 1 & \end{bmatrix}$