

THE COLLEGE OF STATEN ISLAND
Department of Mathematics
Math 230/231 Final Exam
Fall 2008
Form I

1

Name: _____

Part A I – Answer all questions in this part.

a) Evaluate $\int \frac{5 + 2x - x^4}{x^2} dx$

(4%)

a _____

b) Evaluate $\int \frac{\sin \theta}{\sqrt{1 - \cos \theta}} d\theta$

(4%)

b _____

c) Evaluate $\int_0^{\ln 2} e^{7x} dx$

(4%)

c _____

Mth 230/231 Final (Continued)

Part A II - Find the following derivatives:

a) Find $\frac{dy}{dx}$ for $y = \frac{5}{x^3} - \sqrt[4]{x} + e^3$

a _____

(4%)

b) Find $f'(x)$ where $f(x) = \frac{2x}{x^3 + 1} - x^2 \sqrt{2x + 1}$

b _____

(4%)

c) Find the second derivative $f''(x)$ of $f(x) = \ln(3x^2) + \cos(2x)$

(4%)

Mth 230/231 Final (Continued)

Part III Find the following limits: Your answer may be a real number, $+\infty$, $-\infty$, or DNE (does not exist). Justify your answers.

a) $\lim_{x \rightarrow 6} \left(\frac{x^2 - 36}{6 - x} \right)$

a _____

(4%)

b) $\lim_{x \rightarrow 2^-} \left(\frac{5}{2 - x} \right)$

b _____

(4%)

c) $\lim_{x \rightarrow \infty} \left(\frac{x^3 - 4x^4 + \sqrt{x}}{2x^5 - 3\sqrt{x}} \right)$

c _____

(4%)

Mth 230/231 Final (Continued)

Part A IV – Let $f(x) = \frac{3x^2}{x^2 + 2}$

- a) Find the equation of each vertical asymptote in the x-y plane, if any. Justify your answer.

(4%)

a _____

- b) Find the equation of each horizontal asymptote in the x-y plane, if any. Justify your answer.

(4%)

b _____

Part A V – Let $f(x) = x^3 - \frac{3}{2}x^2 + 2$ Answer all questions below and show work.

- a) Determine the intervals on which $f(x)$ is increasing or decreasing

(3%)

$f(x)$ increasing on: _____

$f(x)$ decreasing on: _____

- b) Find the x-values of all relative maxima or relative minima (if any).

(3%)

Rel maxima: _____

Rel minima: _____

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- (3%) c) Determine the interval(s) where $f(x)$ is concave up or concave down.
f conc. up on: _____
f conc. down on: _____

- (3%) d) Find the x-value(s) of the point(s) of inflection (if any)
pt(s) of inflection: _____

- (3%) e) Sketch the graph of $f(x)$.

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Part B – Do any TWO (2) of the following: (10% each)

- B-1 Water is dripping from a ceiling to create a circular puddle on the floor. Assume that the radius of the puddle is increasing at the rate of three inches per minute. Find the rate at which the area is increasing when the radius is 7 inches. (Remember: Area of circle = πr^2).



B1 _____

- B-2 A curve is defined by the equation $x^3 + y^3 = 4xy + 1$

- a) Find a formula for the derivative $\frac{dy}{dx}$ in terms of x and/or y.

a _____

- b) Find the slope of the tangent line to the curve at the point (1,2).

b _____

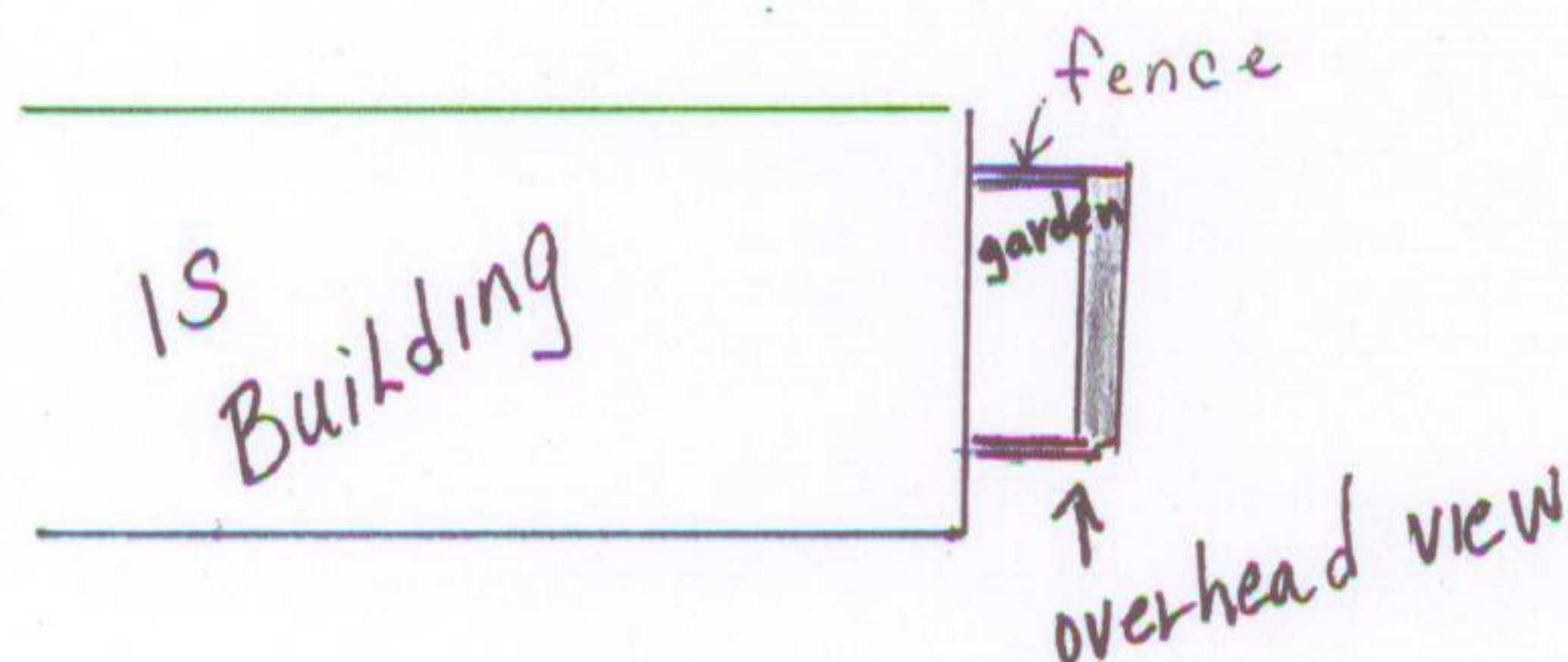
- c) Find the equation of the tangent line at this point.

c _____

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B-3 Assume that the maintenance crew at this college wishes to create a rectangular garden alongside the Math 1S – building. The garden is to be bounded on three sides by a chain link fence. See sketch. No fence is needed along the building.

What dimensions minimize the amount of fencing needed if the garden area is to be 250 ft^2 ?



Length _____

Width _____

Part C – Do any TWO (2) of the following. (10% each)

C -1a) – Using the definition of the derivative as a limit find $f'(x)$ for the function $f(x) = x^2 + 2x$

a _____

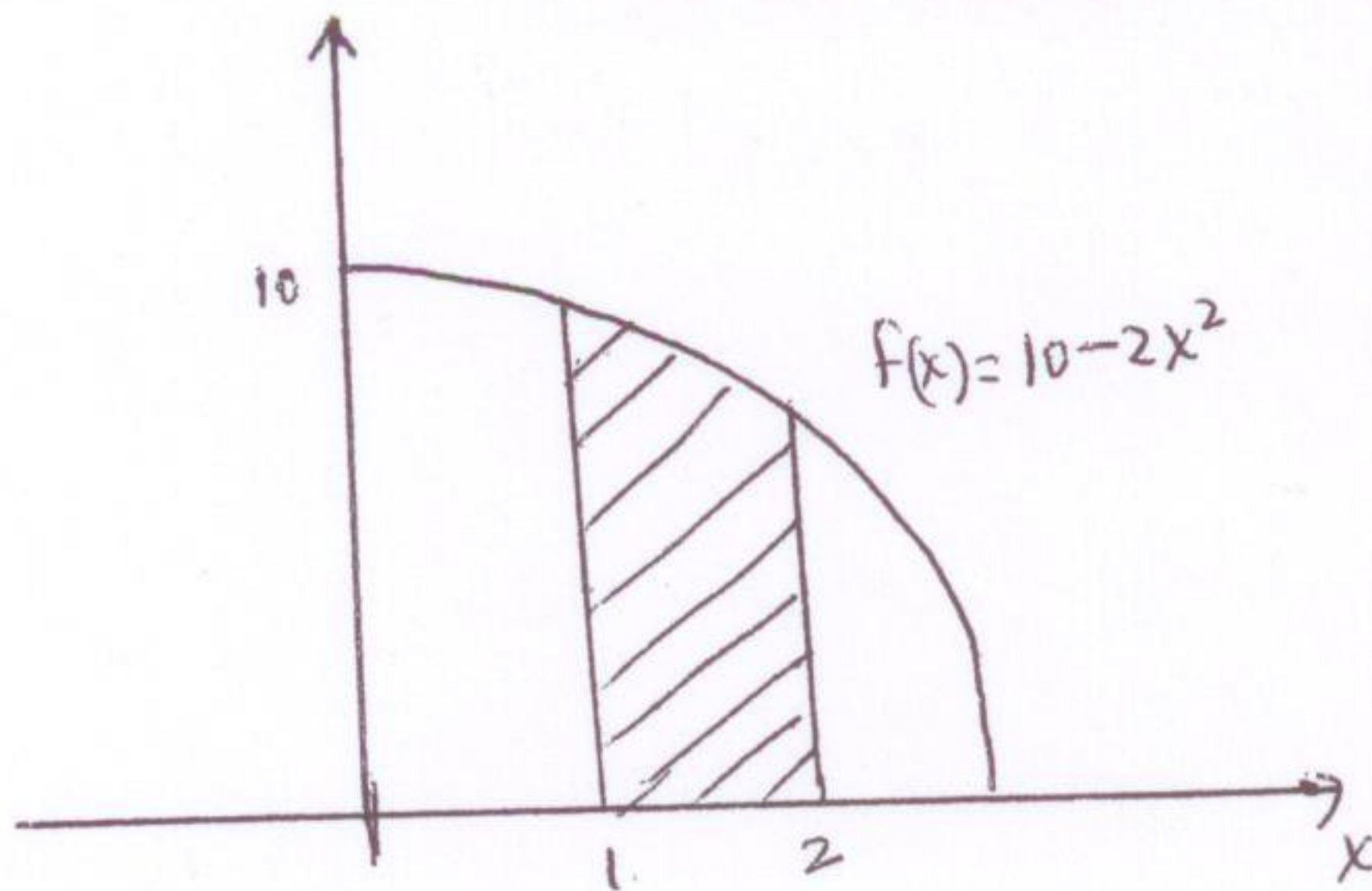
Mth 230/231 Final (continued)

b) Check your answer by using the appropriate derivative rules.

b _____

C-2

a) Find an estimate at the area of the region shown by using a Riemann sum with four rectangles of equal width and by using the right end points of each rectangle to find the height. The region is bounded by $f(x) = 10 - 2x^2$, the y -axis, $x = 1$, and $x = 2$.



Estimate _____

b) Should this estimate be less than or greater than the actual area?

Explain your answer.

b _____

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- C- 2 c) Find the exact area by setting up and evaluating an appropriate integral.

c _____

- C- 3 Determine whether each of the following is true or false. If it is true explain why. If it is false, you must explain why or give an example that shows it is false. Otherwise, you will not receive any credit.**

- a) Rolle's Theorem can be applied to the function $f(x) = x^2 - 2x$ on $[0,2]$

a _____

- b) The graph of a rational function has at least one vertical asymptote.

b _____

- c) The graph of $f(x) = \frac{x^2 - 1}{x - 1}$ has a removable discontinuity at $x=1$.

c _____

Mth 230/231 Final (continued)

d) If $f(x) = |x|$ then $f'(0)$ does not exist.

d _____

e) The function $f(x) = (x+1)^{2/3}$ is differentiable for all values of x .

e _____

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