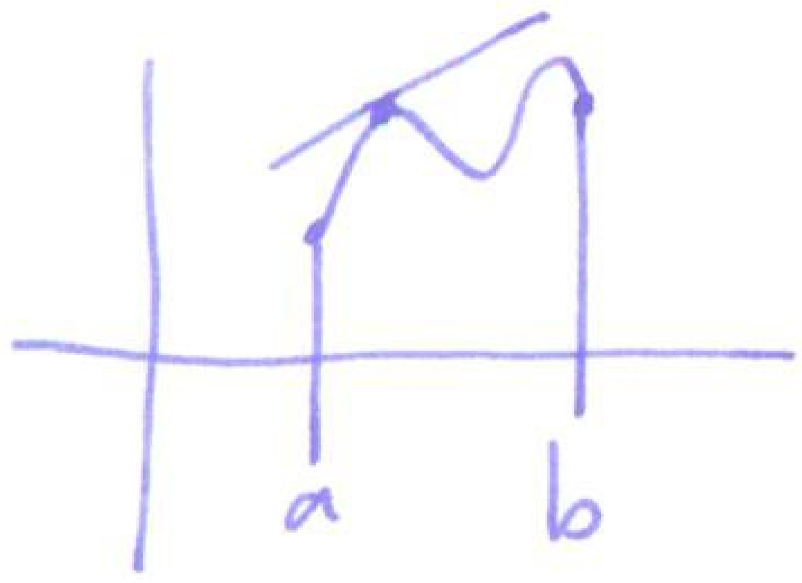
#4a

Mean value theorem $f(x)$ cts on $[a,b]$ and differentiable on (a,b), then there is $c \in (a,b)$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

average rate of change.



Corollary if $f(x)$ is differentiable, and $f'(x)=0$, then $f(x)$ is constant.

Example classify critical points of $f(x) = \cos^2 x + \sin x$ on $[0, \pi]$.

find critical points : $f'(x) = 2\cos x \cdot (-\sin x) + \cos x$

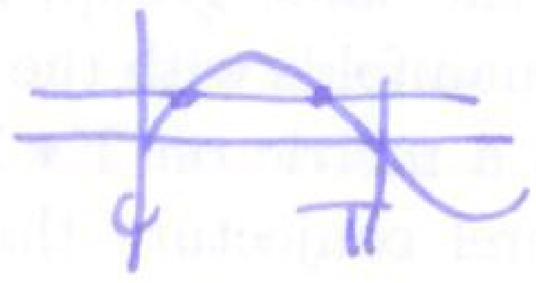
$= \cos x\,(1 - 2\sin x)$

solve $f'(x) = 0$ $\quad \cos x = 0$

$\Rightarrow x = \pi/2$.

or $1 - 2\sin x = 0$

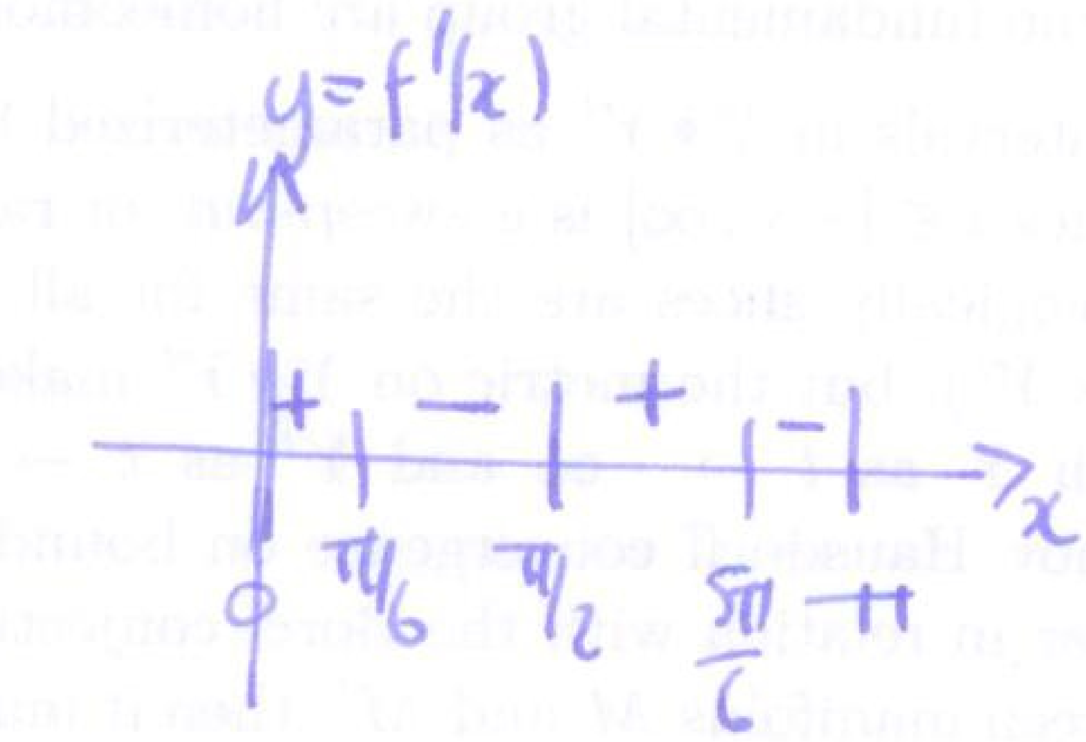
$\sin x = \frac{1}{2}$

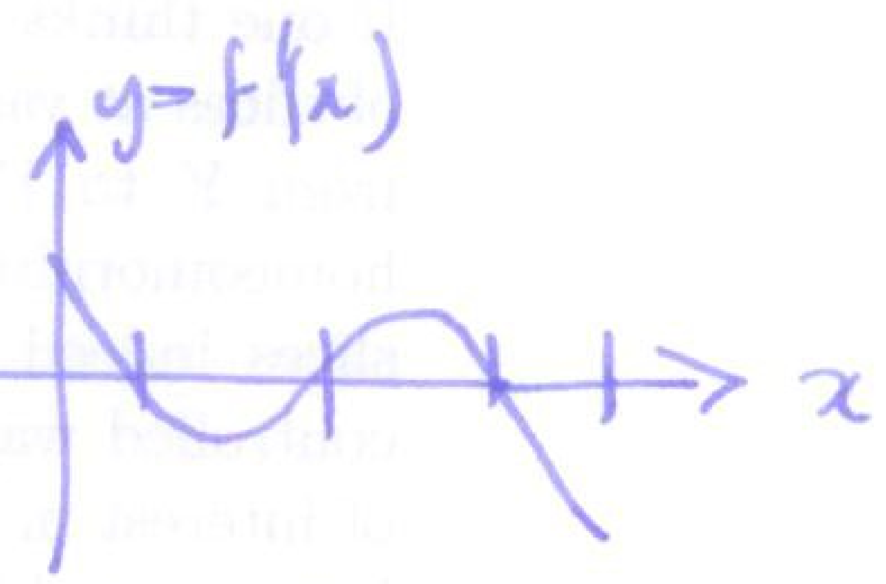


$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

critical points: $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$.

find sign of $f'(x)$

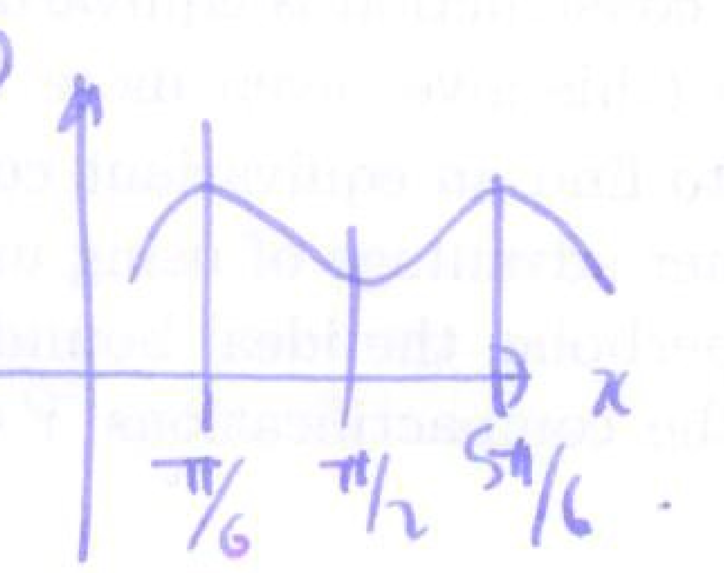




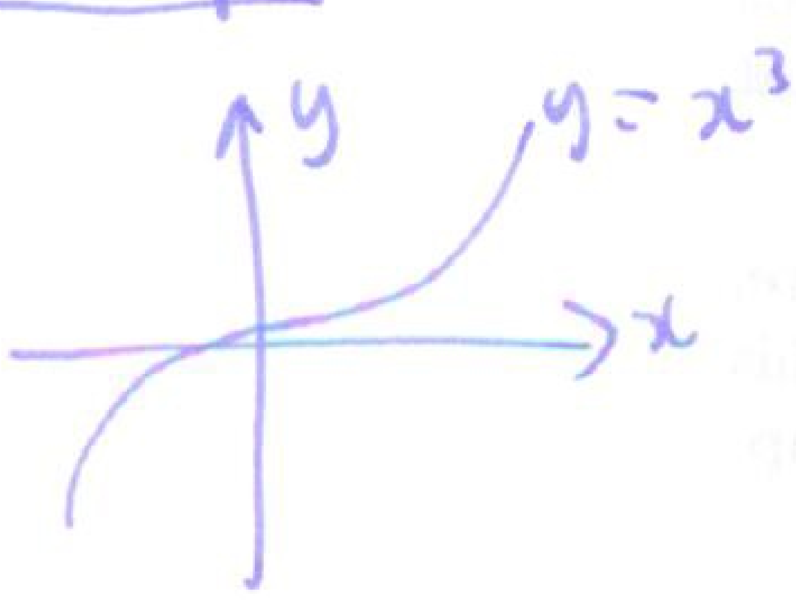
$\frac{\pi}{6}$ local max

$\frac{\pi}{2}$ local min

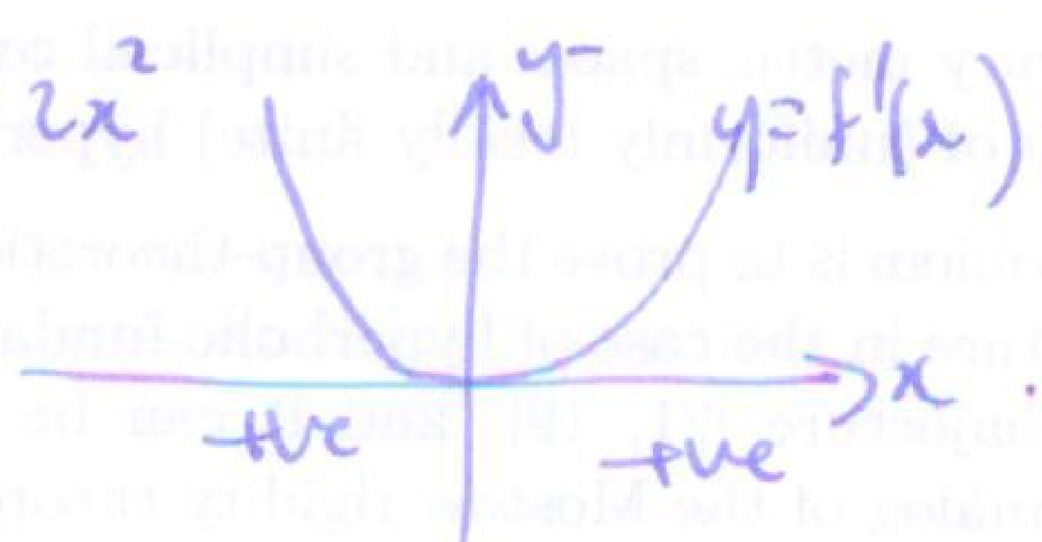
$\frac{5\pi}{6}$ local max.



Example critical point that is <u>not</u> max or min.



$f'(x) = 2x^2$



§4.4 Second derivative test

how do the slopes change?

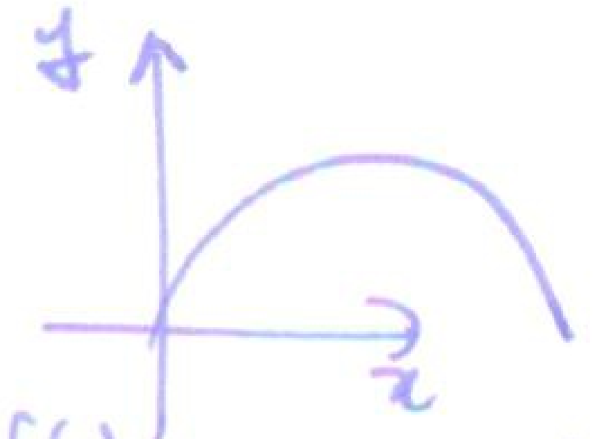
concave down ⟺ slopes <u>decreasing</u>.

concave up ⟺ slopes <u>increasing</u>

<u>Defn</u> Let f be an ~~ops~~ differentiable function on an open interval (a,b). Then

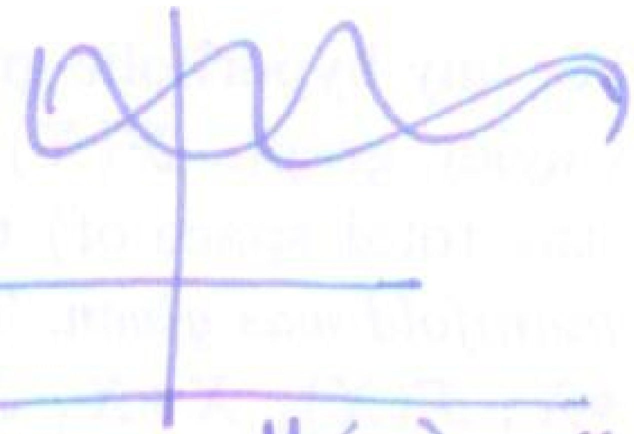
f is <u>concave up</u> ⟺ $f'(x)$ is increasing

f <u>concave down</u> ⟺ $f'(x)$ is decreasing.

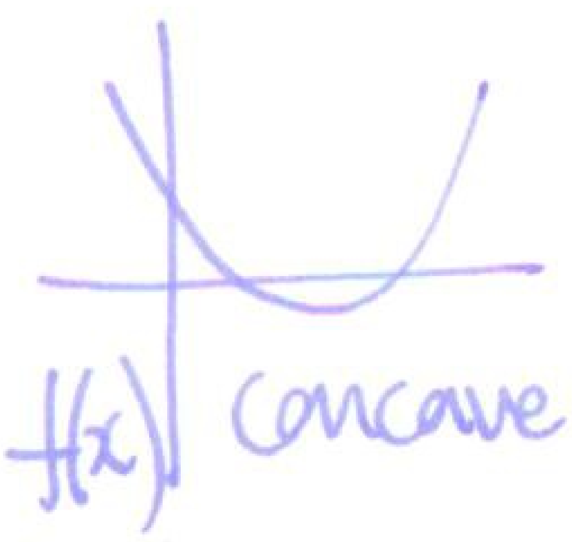


$f(x)$ concave down

$f'(x)$ ~~increasing~~ decreasing

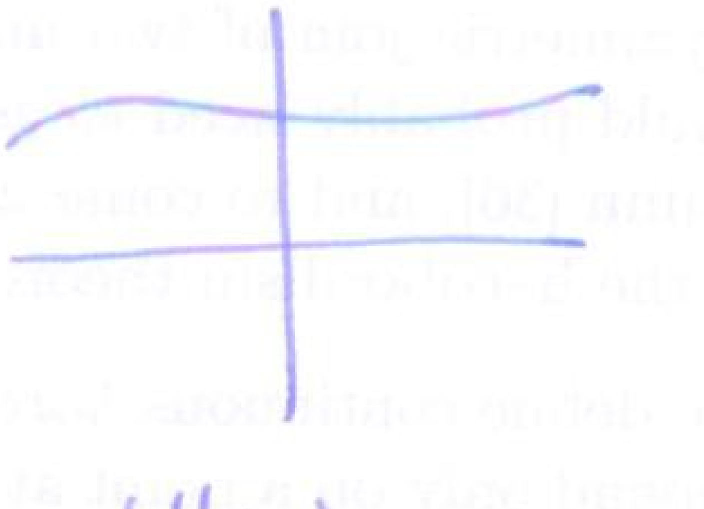


$f''(x)$ ~~>~~ < 0



$f(x)$ concave up

$f'(x)$ increasing



$f''(x) > 0$

<u>Thm</u> <u>Concavity test</u> Suppose $f''(x)$ exists for all $x \in (a,b)$

If $f''(x) < 0$ for <u>all</u> $x \in (a,b)$ then $f(x)$ is <u>concave down</u>.

$f''(x) > 0$ $f(x)$ <u>concave up</u>.

Defn A point of inflection is where the graph changes from concave up to concave down (or vice versa)

Example

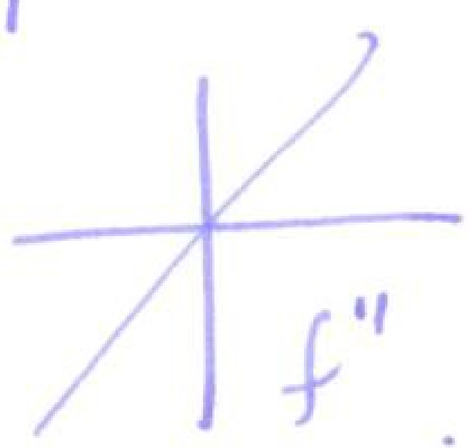
$x(x-1)(x+1) = x(x^2-1) \quad x^3-x.$

$f'(x) = 3x^2 - 1$

$f''(x) = 6x$

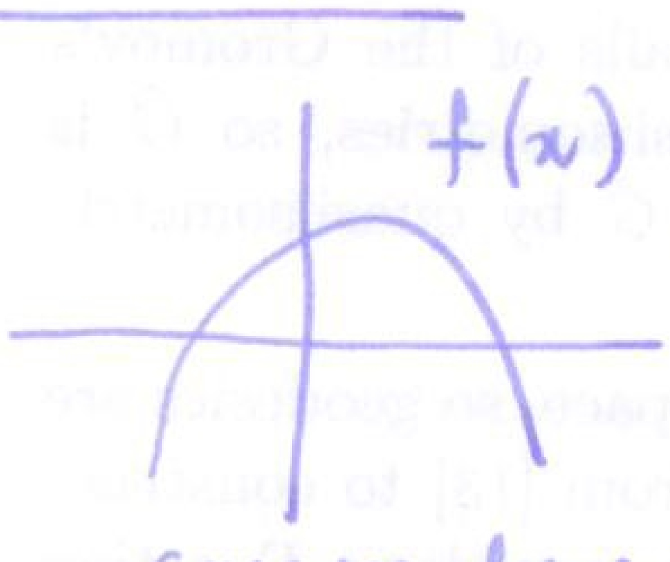
$f''(x) > 0$ for $x > 0$

$< 0 \quad x < 0.$

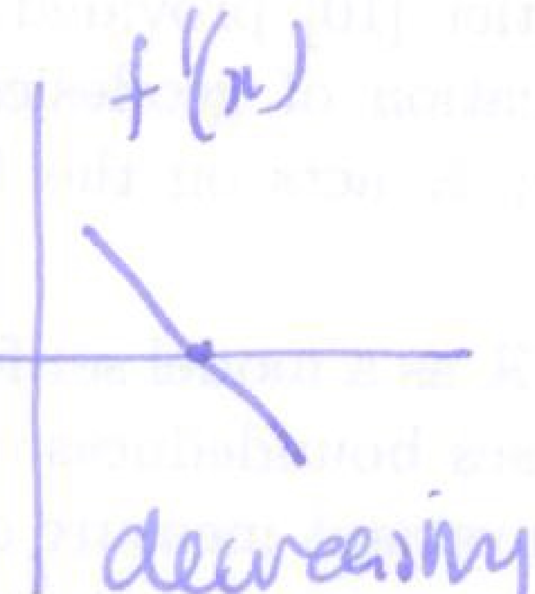


Second derivative test

local max $f(x)$



concave down

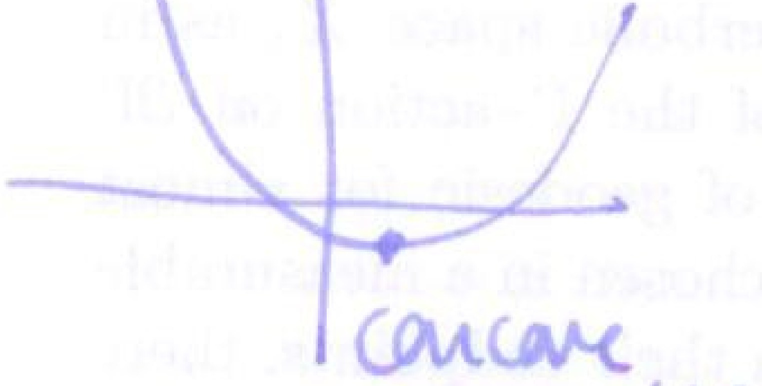


decreasing

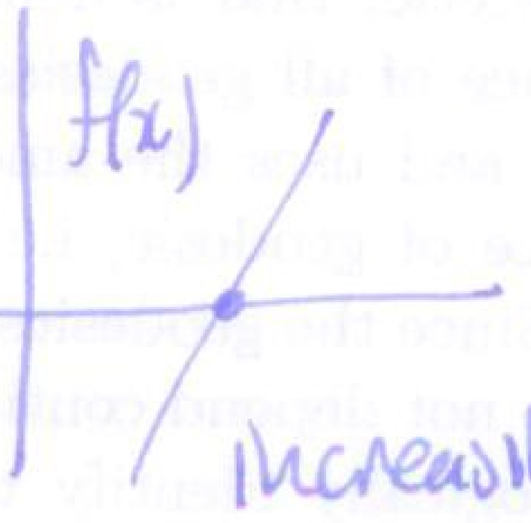
$f''(x) < 0$.

negative

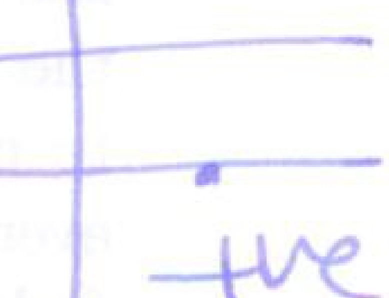
local min



concave ~~down~~ up



increasing



$f(x)$ +ve.

Thm Suppose $f(x)$ differentiable and c is a critical point, and $f''(c)$ exists. Then

if $f''(c) > 0 \Rightarrow f(c)$ is a local max.

$f'(c) < 0 \Rightarrow f(0)$ is a local min

$f''(c) = 0$ NO INFORMATION! may be either local min/max or neither.