	x	$f(x)$	linearization.	error	percentage error
good approx	121	11	$\frac{121}{20}+5 = 11.05$	0.05.	$\frac{0.05}{11} \times 100$.
bad approx	400	20	$\frac{400}{20}+5 = 25$	5	$\frac{5}{20} \times 100$.

$$\text{percentage error} = \left| \frac{\text{error}}{\text{actual value}} \right| \times 100$$

<u>observation</u>:

When is a linear approximation a good approximation?

$f''(x)$ small $\qquad f''(x)$ big

§4.2 Extreme values (maxima/minima)

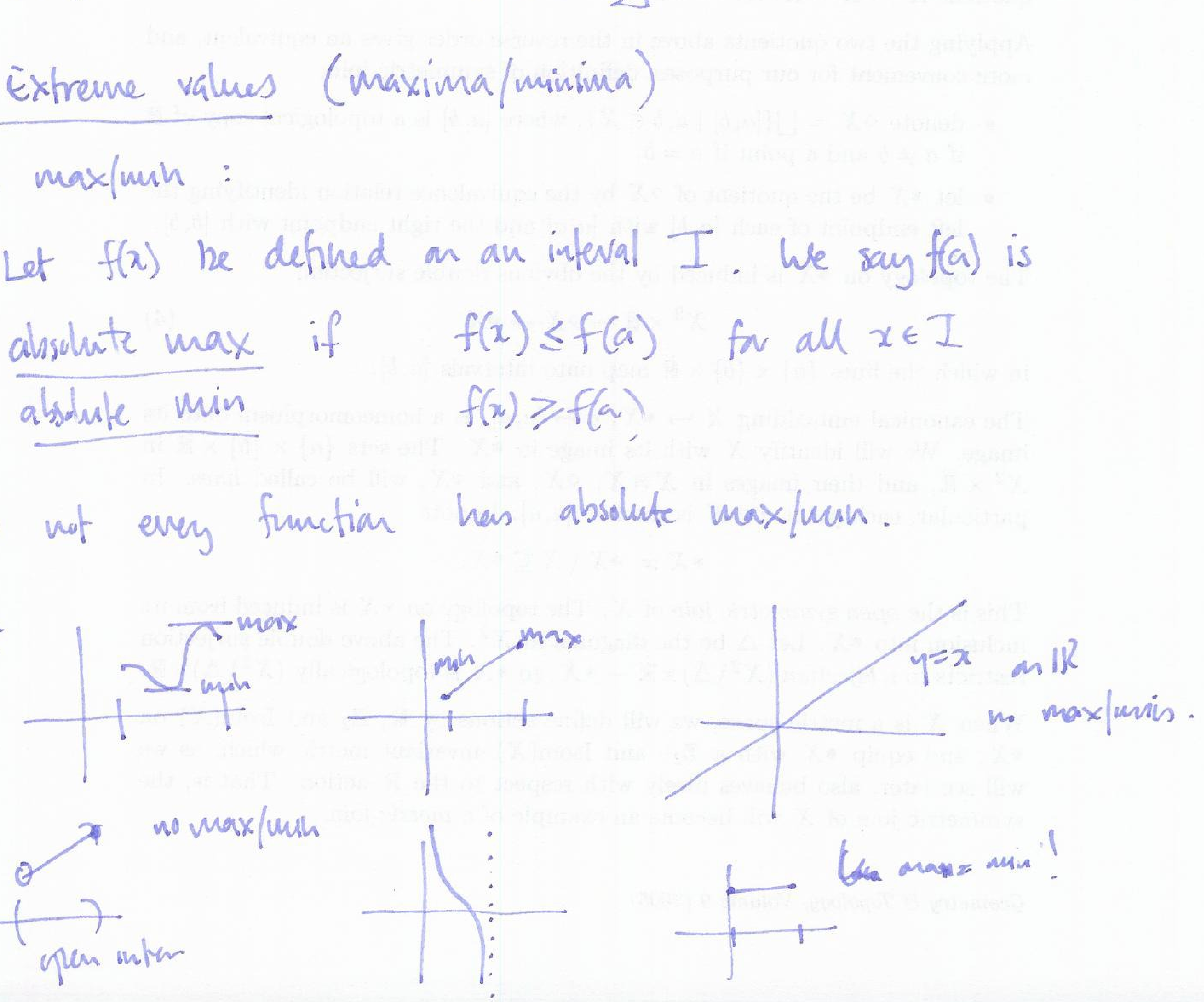
Absolute max/min:

<u>Def</u>: Let $f(x)$ be defined on an interval I. We say $f(a)$ is the <u>absolute max</u> if $f(x) \le f(a)$ for all $x \in I$

<u>absolute min</u> $f(x) \ge f(a)$

<u>warning</u> not every function has absolute max/min.

<u>Example</u>



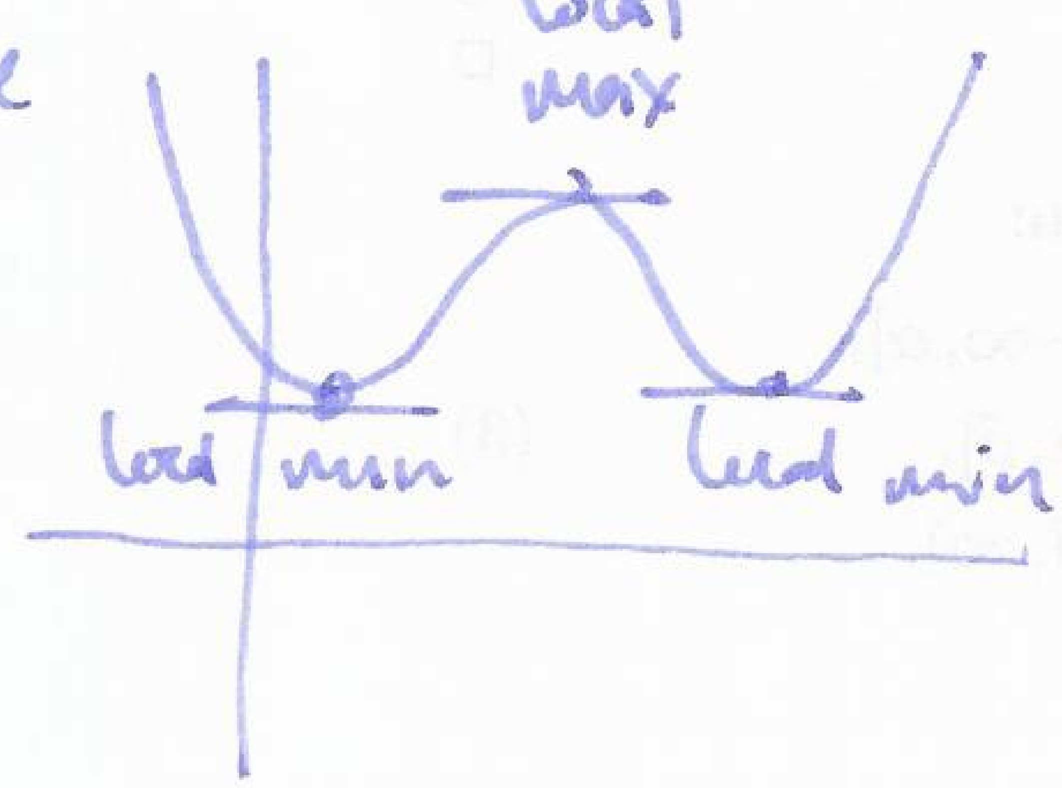
Thm If $f(x)$ is cts on a closed bounded interval I, then $f(x)$ has both an absolute max and absolute min on I.
$I = [a,b]$

Local max/min:

Def^n $f(x)$ has a local min ~~max~~ at $x=c$ if $f(c)$ is the minimum value of $f(x)$ on some open interval containing c.

local max max/minimum.

Example

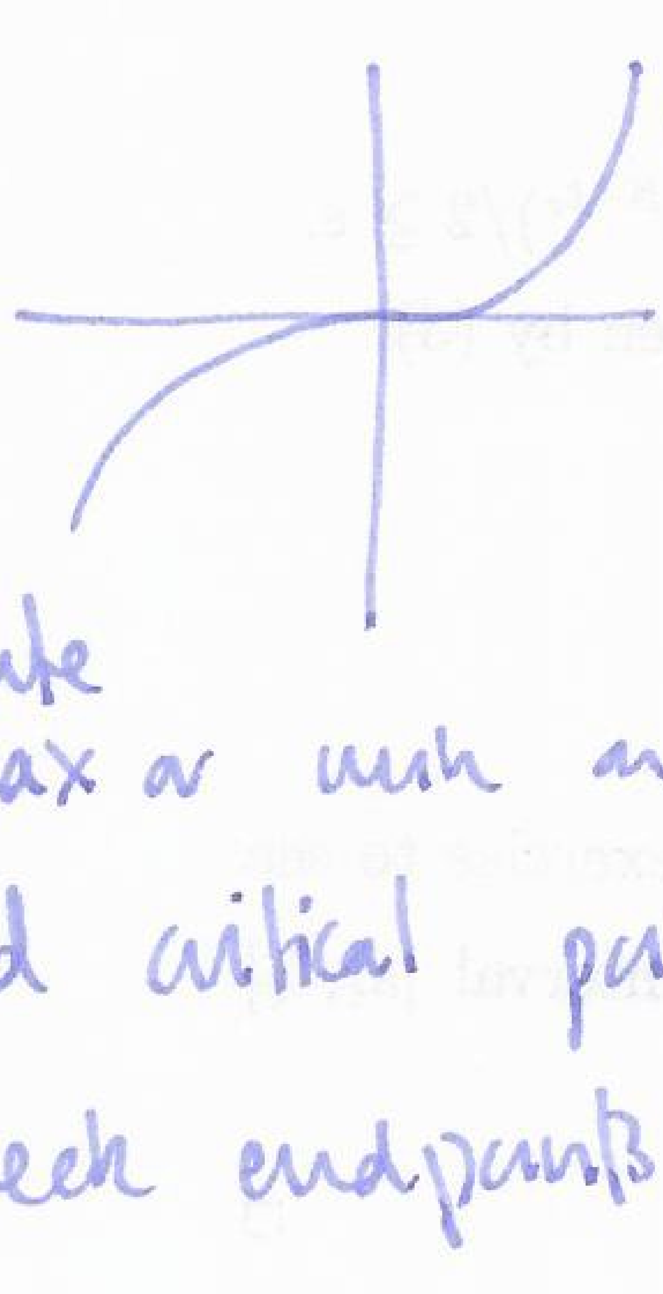


Def^n $x=c$ is a critical point of $f(x)$ if $f'(c)=0$.

Thm If c is a local max/min of a differentiable function, then c is a critical point.

warning c critical point $\not\Rightarrow$ c local max or min.

Example



$y=x^3$ $y'=2x^2$ $x=0$ critical point not local max/min

Finding absolute max or min on closed intervals:

① find critical points & evaluate $f(c_i)$.

② check endpoints

<u>Example</u> find the max/min of $2x^3 - 15x^2 + 24x + 7$ on $[0,6]$

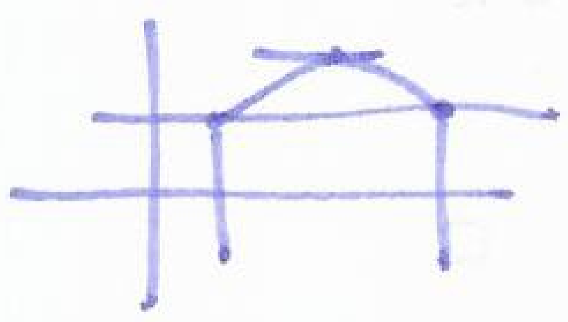
$x^2 - 8\ln x$ on $[1,4]$

$\sin x \cos x$ on $[0,\pi]$.

<u>Rolle's theorem</u> Suppose $f(x)$ is cts on $[a,b]$ and differentiable on (a,b)

if $f(a) = f(b)$ then $\exists\ c \in (a,b)$ s.t. $f'(c) = 0$

<u>Proof</u>

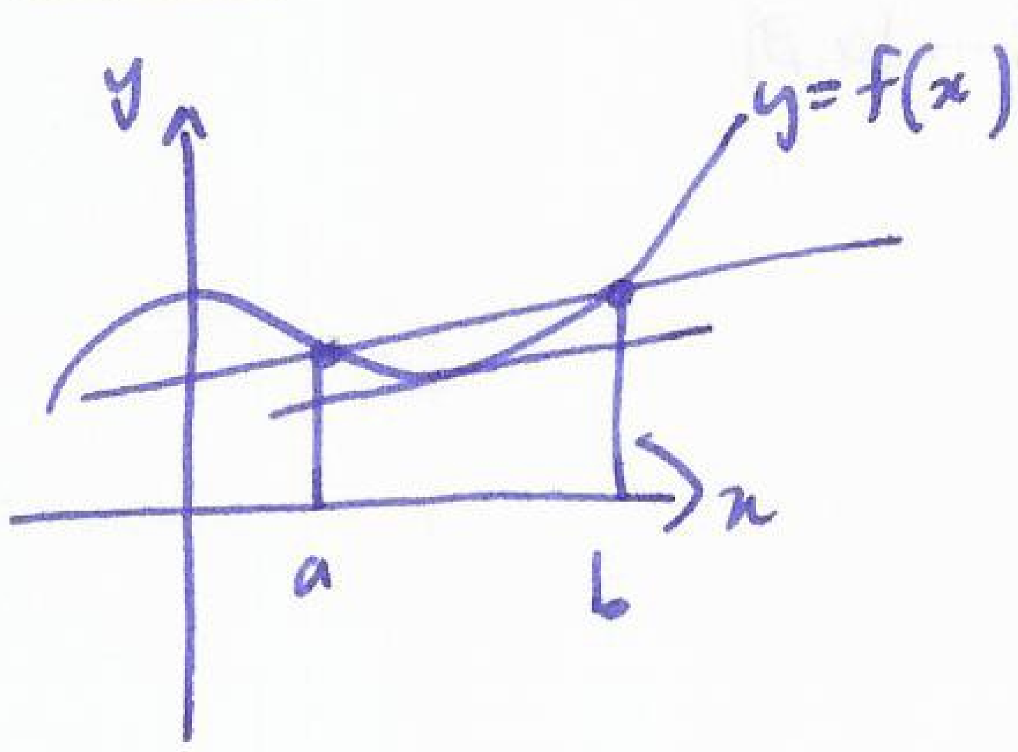


if there is a local max/min in (a,b) then $\exists\ c$ s.t. $f'(c)=0$

if not then $f(a) = f(b) = \max/\min \Rightarrow f(x)$ constant!

$\Rightarrow \exists\ c$ s.t. $f'(c) = 0$.

§4.3 Mean value theorem and monotonicity



<u>Thm</u> (Mean value theorem MVT)

Suppose f is cts on $[a,b]$ and differentiable on (a,b), then there is a $c \in (a,b)$ s.t.

$$f'(c) = \frac{f(b) - f(a)}{b-a} \quad \text{(average rate of change)}$$

<u>Corollary</u> If $f(x)$ is differentiable, and $f'(x) = 0$, then $f(x) = c$ constant.

<u>Proof</u> spse $\exists\ a,b$ s.t. $f(a) \neq f(b)$. Then $\exists\ c \in (a,b)$ s.t. $f'(c) \neq 0$ ↯.

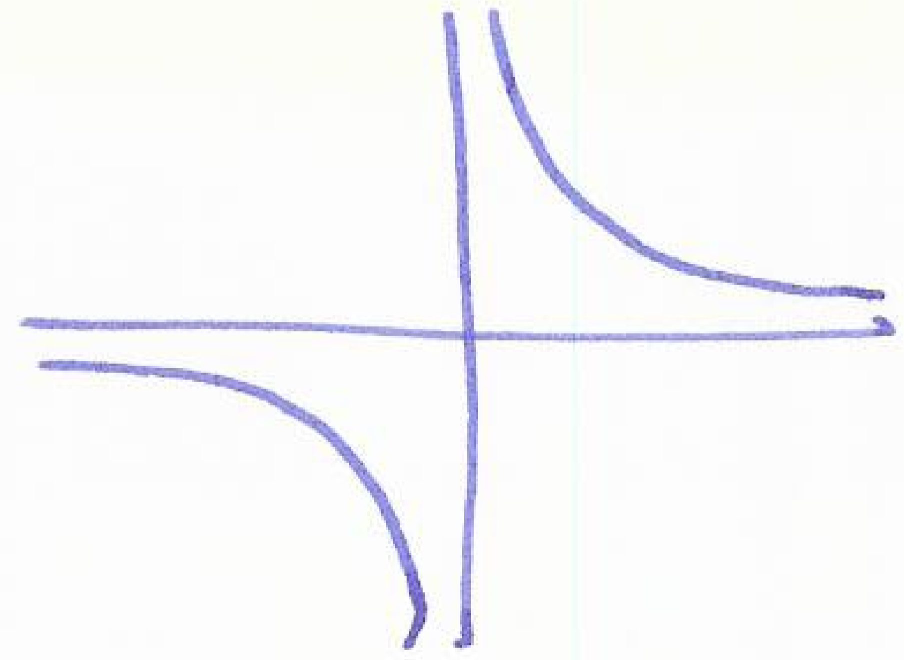
~~First derivative test~~ <u>Monotonicity</u>

Suppose f is differentiable on (a,b).

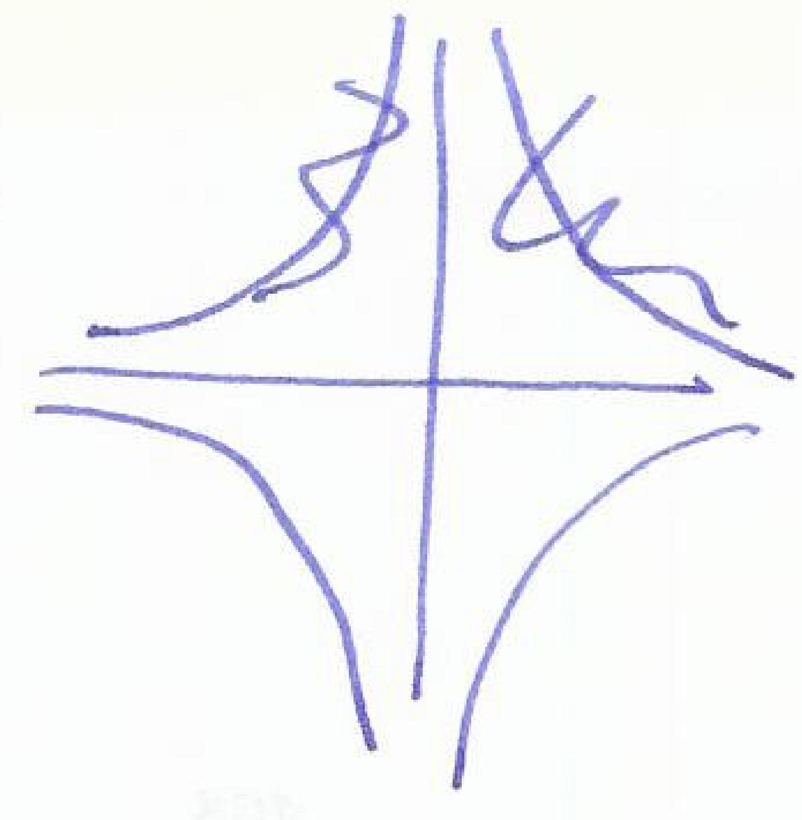
If $f'(x) > 0$ for <u>all</u> $x \in (a,b)$ then f is <u>increasing</u> on (a,b).

$f'(x) < 0$ <u>decreasing</u> on (a,b).

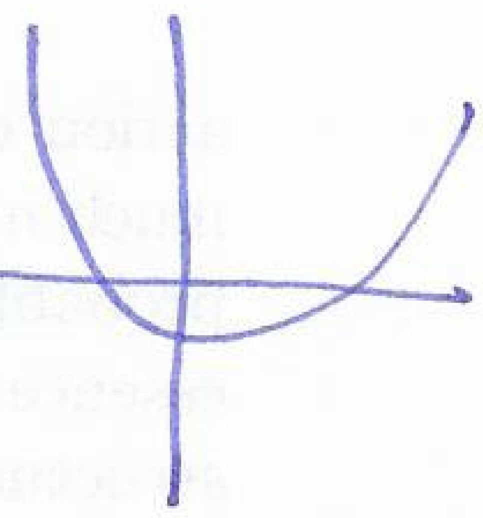
<u>Example</u> $f(x) = \frac{1}{x}$



$f'(x) = -\frac{1}{x^2}$



<u>Example</u> where is $f(x) = x^2 - 2x - 3$ increasing?



$f'(x) = 2x - 2$

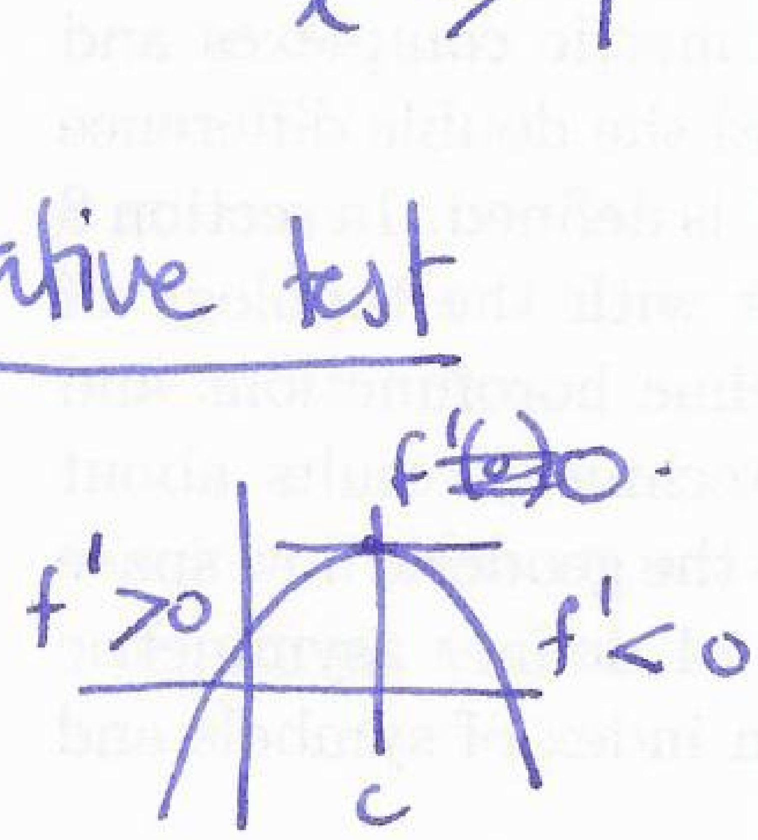
Q: where is $2x - 2 > 0$

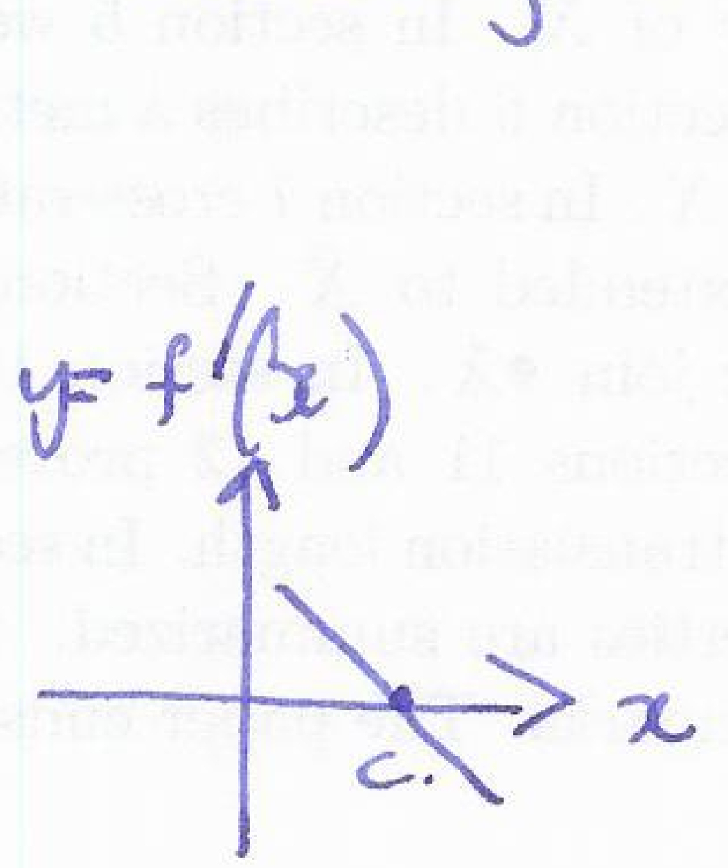
$2x > 2$

$x > 1$ so increasing on $(1, \infty)$.

<u>~~Second~~ First derivative test</u>

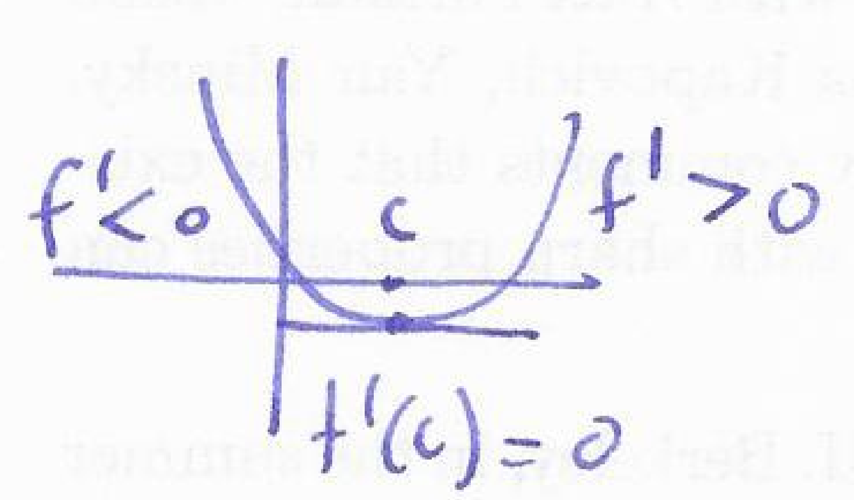
local max:

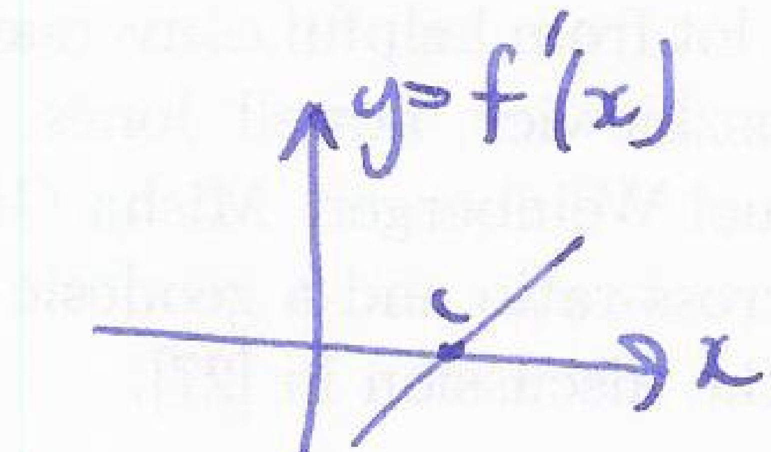




$\left[f''(c) < 0 \right]$.

local min:





$\left[f''(c) > 0 \right]$.

<u>Thm</u>: ~~Second~~ First derivative test

Suppose $f(x)$ is differentiable and c is a critical point (ie. $f'(c) = 0$)

if $f'(x)$ changes from +ve to −ve at $c \Rightarrow$ local max.

if $f'(x)$ −ve +ve $\Rightarrow$ local min.