Example $f(x) = \log_b(x) = \log \dfrac{\ln(x)}{\ln(b)}$

$$f'(x) = \frac{1}{\ln(b)\, x}.$$

Example $\ln(x^2)$ $(\ln(x))^2$.

Example (logarithmic differentiation)

$$f(x) = \frac{(x+1)^2(2x^2-3)}{\sqrt{x^2+1}}$$

$$\ln(f(x)) = 2\ln(x+1) + \ln(2x^2-3) - \tfrac{1}{2}\ln(x^2+1)$$

implicit differentiation / chain rule.

$$\frac{1}{f(x)} f'(x) = \frac{2}{x+1} + \frac{1}{2x^2-3}\cdot 4x - \frac{1}{2(x^2+1)}\cdot 2x.$$

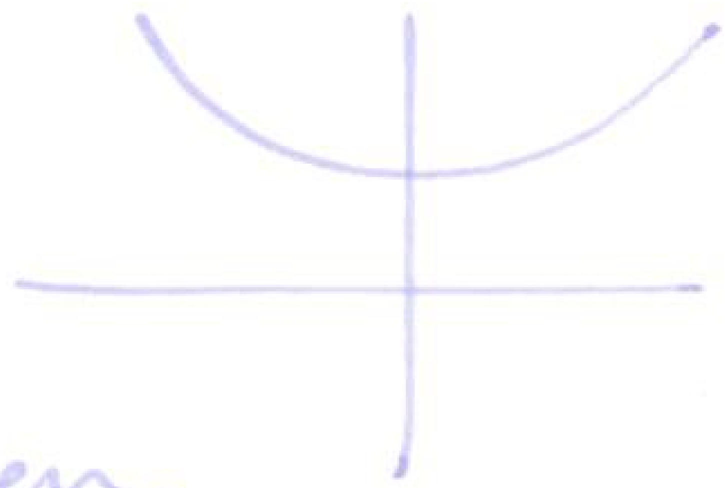
$$\frac{f'(x)}{f(x)} = \frac{2}{x+1} + \frac{4x}{2x^2-3} - \frac{x}{x^2+1}.$$

$$f'(x) = f(x)\left(\text{---}''\text{---}\right) = \frac{(x+1)^2(2x^2-3)}{\sqrt{x^2+1}}\left(\frac{2}{x+1} + \frac{4x}{2x^2-3} - \frac{x}{x^2+1}\right)$$

Hyperbolic trig functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

odd



even.

$$\frac{d}{dx}(\sinh(x)) = \cosh x \qquad \frac{d}{dx}(\cosh(x)) = \sinh(x).$$

Check: $\frac{d}{dx}(\sinh(x)) = \frac{d}{dx}\left(\frac{1}{2}(e^x - e^{-x})\right) = \frac{1}{2}e^x + \frac{1}{2}e^{-x} = \cosh(x).$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} \quad \text{etc...}$$

Example $\frac{d}{dx}(\tanh(x)) = \frac{d}{dx}\left(\frac{\sinh(x)}{\cosh(x)}\right) = \frac{\cosh x . \cosh x - \sinh x \sinh x}{\cosh^2 x}$

$$= \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x.$$

Example $\frac{d}{dx}(\cosh(3x^2+1)) \qquad \frac{d}{dx}(\sinh x \tanh x)$

Inverse functions

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$$

<u>check</u> $\frac{d}{dx}\left(\sinh^{-1}x\right)$ $\quad f(x) = \sinh x \quad f'(x) = \cosh x$

$$g(x) = \sinh^{-1}(x).$$

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{\cosh\left(\sinh^{-1}(x)\right)}$$

$\cosh t$, $\quad t = \sinh^{-1}(x)$

$\sinh t = x.$ $\quad$ use $\quad \cosh^2 t - \sinh^2 t = 1.$

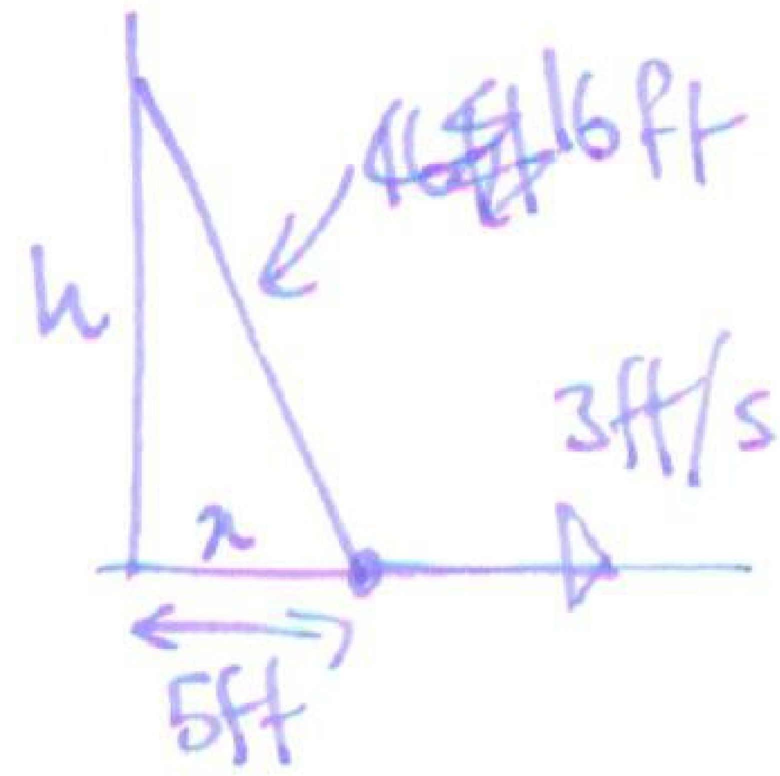
$$\cosh^2 t = 1 + \sinh^2 t = 1 + x^2.$$

$$\cosh t = \sqrt{1+x^2}.$$

so $\quad \frac{d}{dx}\left(\sinh^{-1}(x)\right) = \frac{1}{\sqrt{1+x^2}} \quad \square.$

§3.11 Related rates

<u>Example</u> falling ladders.



$x(t)$ distance of foot of ladder from wall.
$h(t)$ height of top of ladder.

$\frac{dx}{dt} = 3\text{ft/s}.$ $\quad$ <u>Q</u>: what is $\frac{dh}{dt}$?

$$x^2 + h^2 = 16^2$$

differentiate wrt t: $\quad 2x\underbrace{\frac{dx}{dt}}_{=3} + 2h\frac{dh}{dt} = 0.$

$$\frac{dh}{dt} = -3\frac{x}{h}\ \text{ft/s}.$$

$t=0$: $\quad x = 5$, $h \approx 15.2$ $\quad \frac{dh}{dt} \simeq -1$ $\qquad t = 1$ $\quad x = 8$, $h \simeq 13.9$ $\quad \frac{dh}{dt} \simeq -1.7$ ft/s.