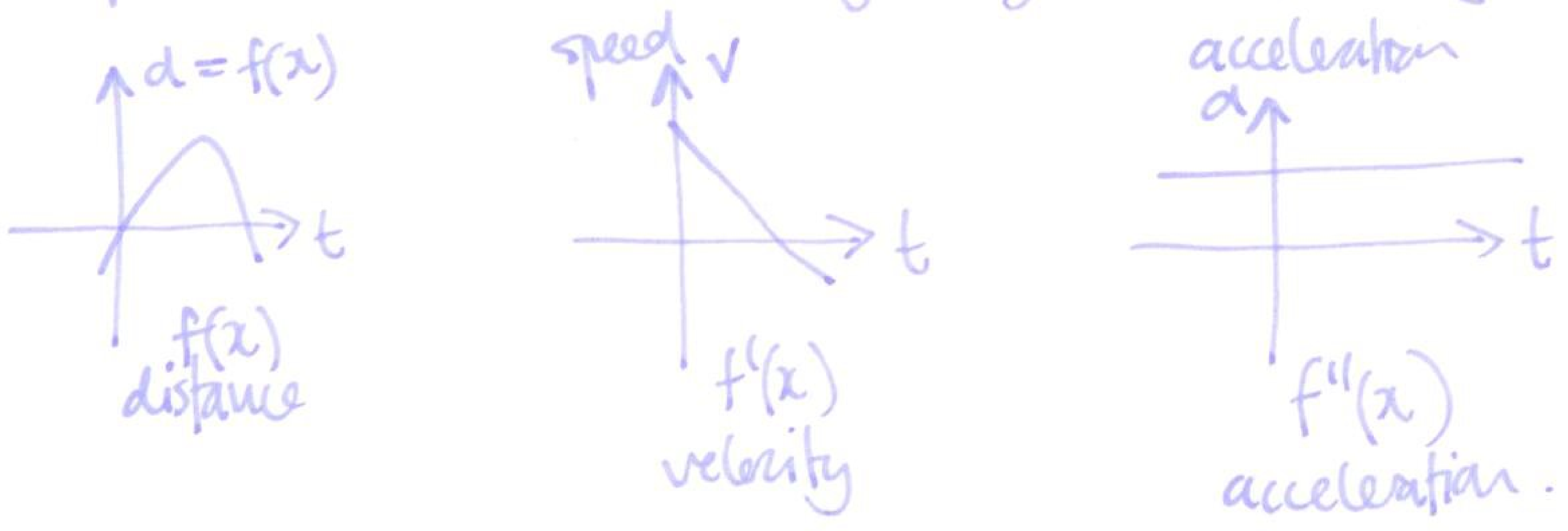
Example acceleration due to gravity: constant $-g$ m/s.



$$f''(x) = -g.$$

$$f'(x) = -gt + v_0$$

$$f'(x) = -\tfrac{1}{2}gt^2 + v_0 t + d_0.$$

§3.6 Trigonometric functions

Thm $\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x.$

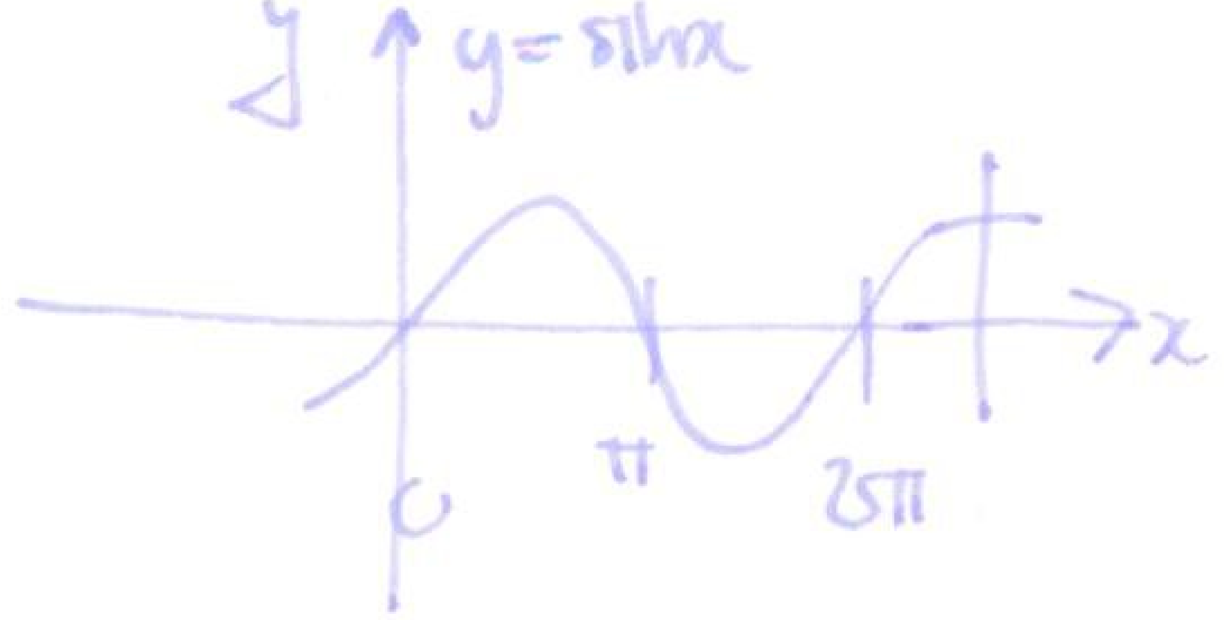
Proof $\frac{d}{dx}(\sin x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$

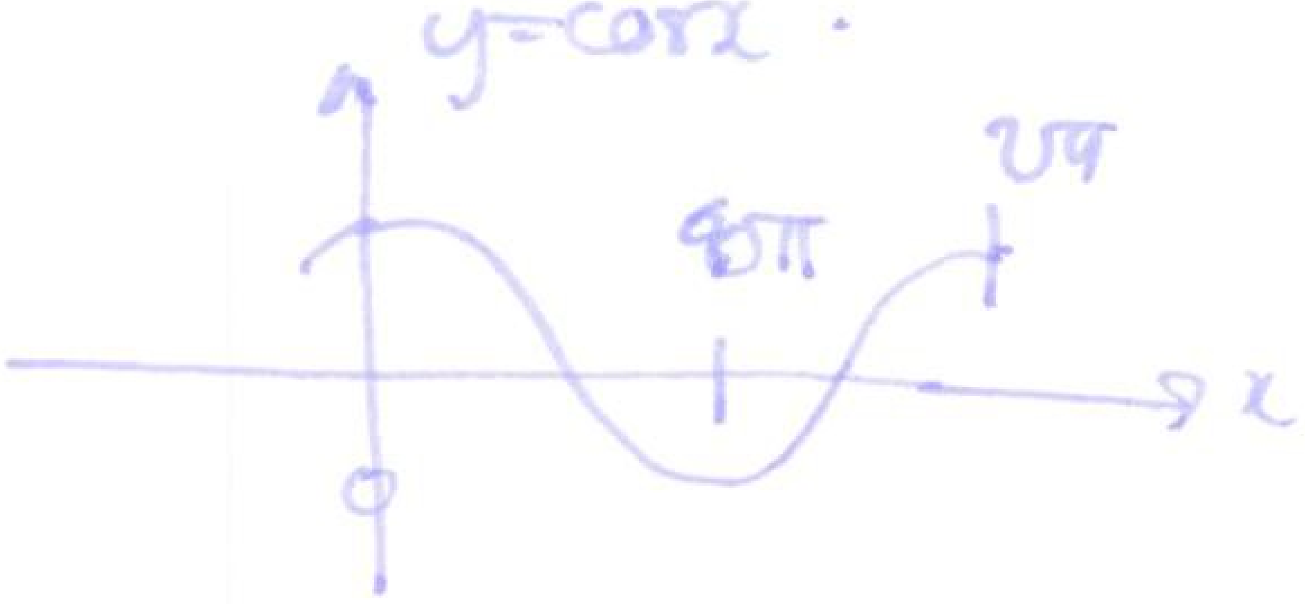
$$\sin(x+h) = \sin x \cos h + \cos x \sin h$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} = \lim_{h\to 0} \sin x \frac{(\cos h - 1)}{h} + \cos x \frac{\sin h}{h}$$

$$= \sin x \underbrace{\lim_{h\to 0} \frac{\cos h - 1}{h}}_{=0} + \cos x \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{=1} = \cos x \quad \checkmark.$$

can this be right?





yes.

<u>Example</u> $f(x) = x \sin x$

$f'(x) = x\cos x + \sin x$.

<u>Thm</u> $\frac{d}{dx}(\tan x) = \sec^2 x$ $\qquad \frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ $\qquad \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$.

<u>Proof</u> $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x(\overset{\cos x}{\cancel{-\sin x}}) - \sin x(\overset{-\sin x}{\cancel{\cos x}})}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$. $\square$.

<u>Example</u> $\frac{d}{dx}(e^x \cos x) = e^x(-\sin x) + e^x \cos x$

$= -e^x \sin x + e^x \cos x$.

<u>§3.7 The chain rule</u>

new functions from old: $f(x) + g(x)$ $\quad f(x)g(x)$ $\quad f(x)/g(x)$.

what about composition: $f(g(x)) = f \circ g(x)$.

<u>Example</u> $\sin(x^2)$, $e^{\sin x}$, $e^{\sqrt{x}}$, etc.....

Thm Chain rule. If f and g are differentiable, then $f\circ g(x) = f(g(x))$ is differentiable, and

$$[f(g(x))]' = f'(g(x))\, g'(x).$$

mnemonic: $[f(g(x))]' =$ outside$'$(inside) . inside$'$.

Example $h(x) = \sqrt{x^3+1} = f(g(x))$ where $f(x) = \sqrt{x} = x^{1/2}$.
$g(x) = x^3+1$.

$$h'(x) = f'(g(x))\, g'(x)$$

$f'(x) = \frac{1}{2}x^{-1/2}$.

$$= \frac{1}{2}(g(x))^{-1/2} \cdot 3x^2$$

$g'(x) = 3x^2$

$$= \frac{1}{2}(x^3+1)^{-1/2} \cdot 3x^2 = \frac{3x^2}{2\sqrt{x^3+1}}.$$

Alternate notation $f(g(x))$ set $u = g(x)$.

$f(u)$, $u = g(x)$.

$$\frac{df}{dx} = f'(u)\frac{du}{dx} = \frac{df}{du}\frac{du}{dx} \qquad \boxed{\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}}.$$

mnemonic "match fractions".

Examples $\cos(x^2)$ $e^{\sqrt{x}}$. sin in degrees.

$\sin x$ $\sin\left(\frac{\pi x}{180}\right)$.

$\sqrt{x+\sqrt{x^2+1}}$

Proof (of chain rule)

$$[f(g(x))]' = \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

answer should be $f'(g(x))g'(x)$.

so write this as:

$$\lim_{h \to 0} \underbrace{\frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)}} \underbrace{\frac{g(x+h) - g(x)}{h}}_{\text{this limit} = g'(x)}$$

set $k = g(x+h) - g(x)$ then as $\underline{g \text{ cts}}$ $h \to 0 \Rightarrow k \to 0$.

then $\displaystyle \lim_{k \to 0} \frac{f(g(x)+k) - f(g(x))}{k} = \lim f'(g(x))$ $\square$.

More examples

$$\frac{d}{dx}\left(g(x)^n\right) = n(g(x))^{n-1} \cdot g'(x)$$

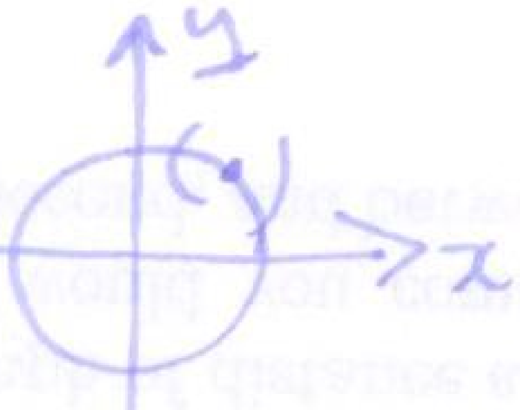
$$\frac{d}{dx}\left(e^{g(x)}\right) = e^{g(x)} \cdot g'(x)$$

$$\frac{d}{dx}\left(f(kx+b)\right) = kf'(kx+b)$$

§3.8 Implicit differentiation

explicit function: $f(x) = x^2 + x + 1$

implicit "function": $x^2 + y^2 = 1$



"local" function. near a point (x, y) on the curve we can think of y as a function of x $(x, y(x))$.

differentiate: $\displaystyle \frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(1)$

$$2x + \frac{d}{dx}\left(y(x)^2\right) = 0$$