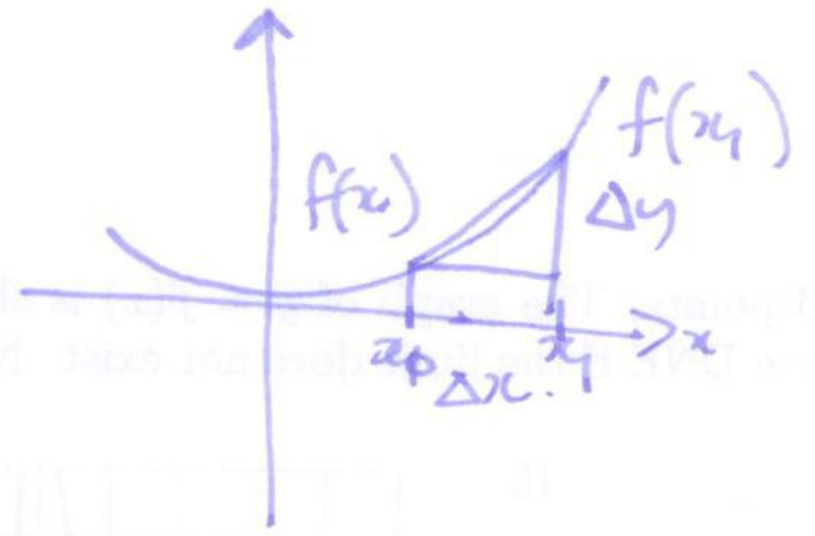
§3.4 Rates of change



Recall: average rate of change

$$= \frac{\Delta x}{\Delta y} = \frac{f(x_1)-f(x_0)}{x_1-x_0}$$

(instantaneous) rate of change.

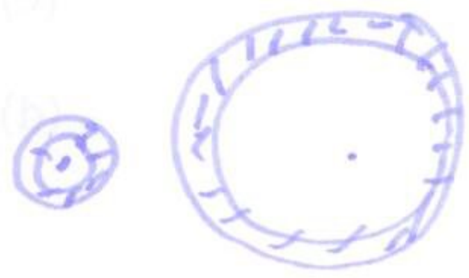
$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_1 \to x_0} \frac{f(x_1)-f(x_0)}{x_1-x_0}$$

if Δx is small, we can use average ROC to approximate ROC (and vice versa)

Example area of circle $= \pi r^2$.

calculate rate of change of area with respect to radius: $\frac{dA}{dr} = 2\pi r$.

$$\left.\frac{dA}{dr}\right|_{r=2} = 4\pi \qquad \left.\frac{dA}{dr}\right|_{r=5} = 10\pi.$$



[rate of change of area = perimeter!].

For small h, $f'(x) \simeq \frac{f(x_0+h)-f(x_0)}{h}$ or. $f(x_0+h) \simeq f(x_0)+hf'(x_0)$

Example stopping distance in feet $f(s) = 1.1s + 0.05s^2$ (s speed in mph $30 \le s \le 75$)

calculate stopping distance when $s=30$ $f(30) = 1.1.30 + 0.05.30^2 = 78.$

calculate ROC of stopping distance at $s=30$

$f'(s) = 1.1 + 0.1s$ $\quad f'(30) = 4.1$ feet/mph

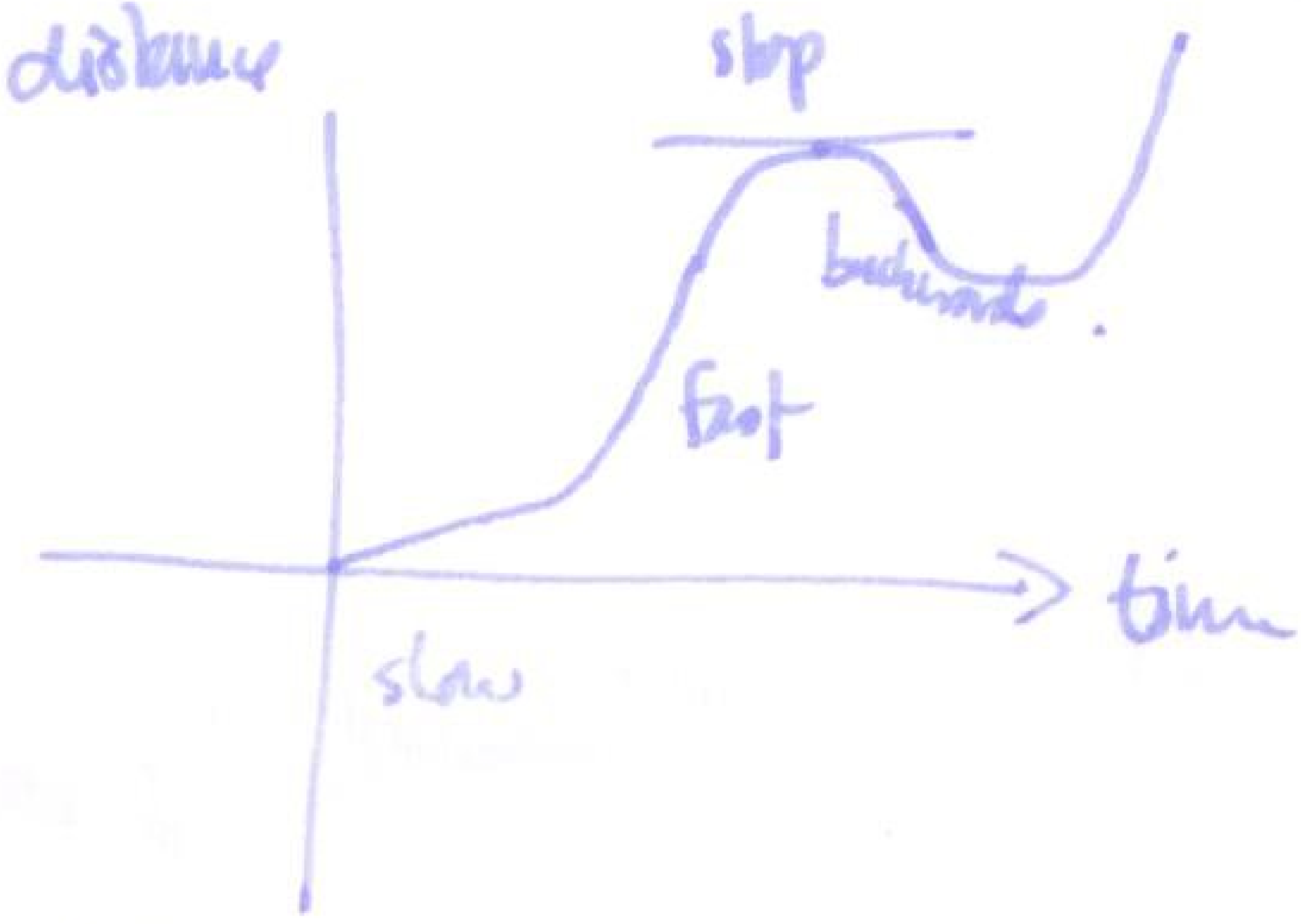
estimate stopping distance at $s = 31$

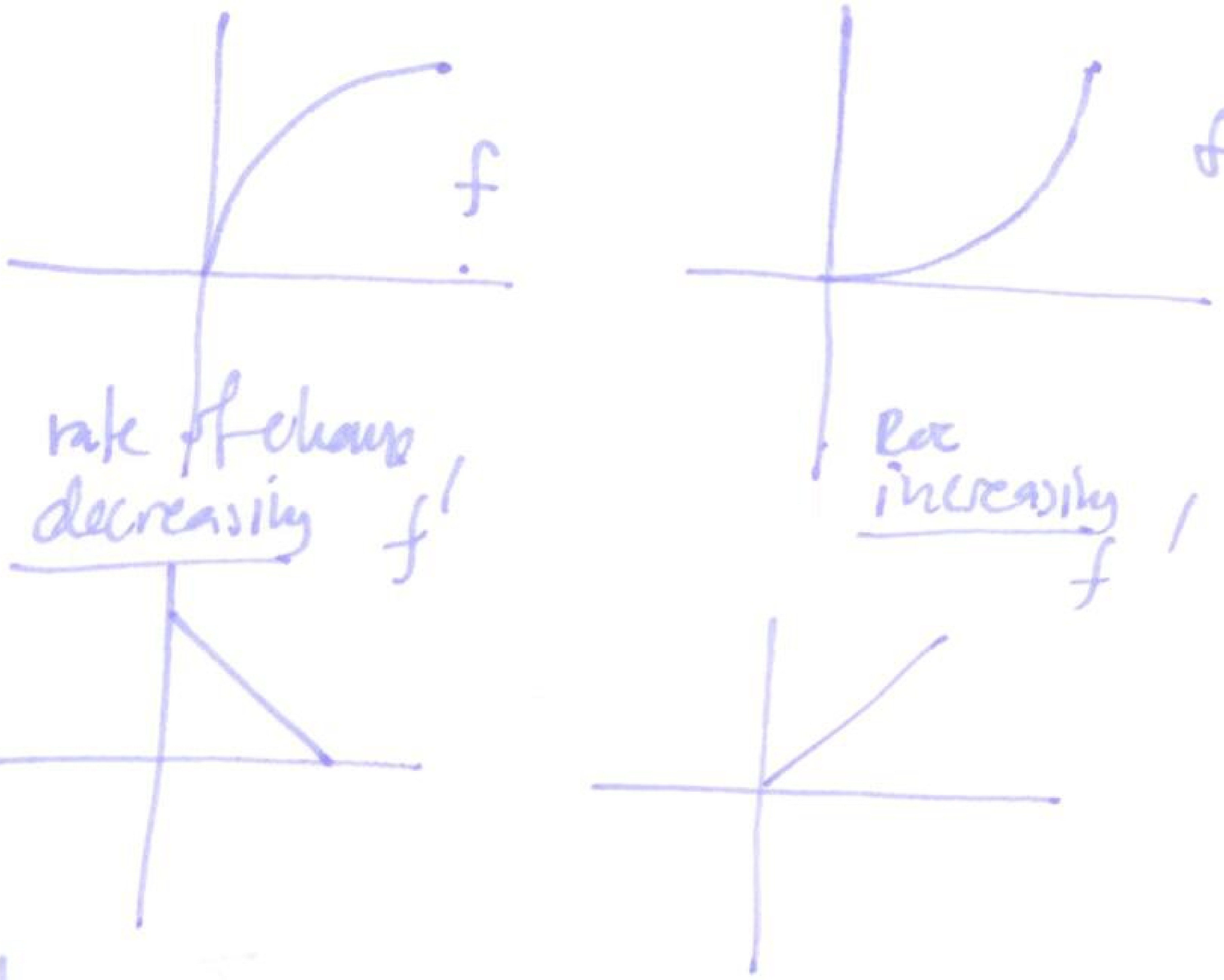
$$F(s+h) \approx F(s) + hF'(s)$$

$$F(31) \approx F(30) + 1\,F'(30)$$

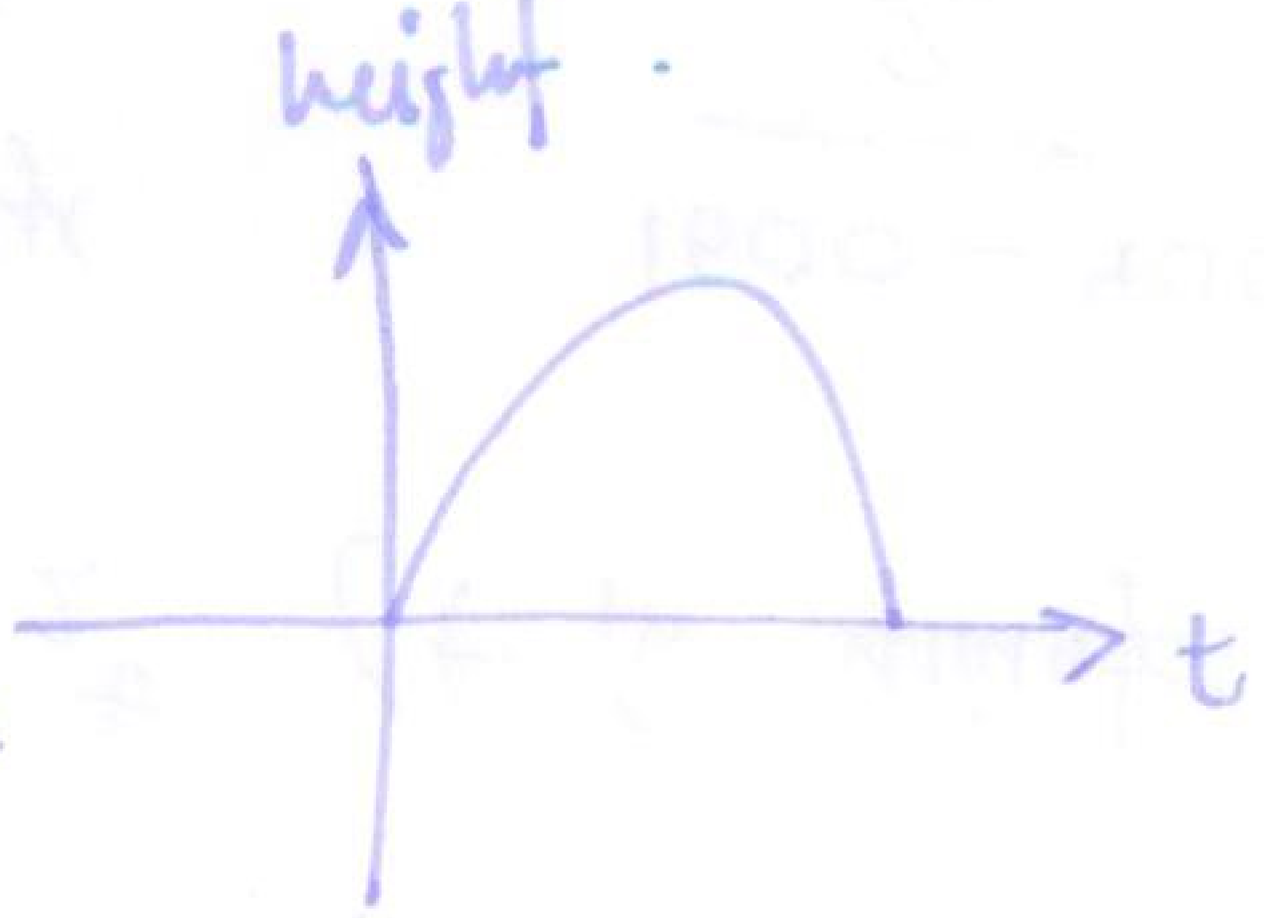
$$F(31) \approx 78 + 4.1 = 82.1$$

On the interpretation of graphs





Motion under gravity



$g = 9.8 m/s$
$= 32 ft/s$.

$$s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$$

$$v(t) = v_0 - gt$$
$= s'(t)$

$s(0) = s_0$ = height at time zero.

$v(0) = v_0$ = velocity at time zero.

maximum height: when $v(t) = 0$ i.e. $v_0 - gt = 0$ $t = v_0/g$.

Example throw a stone vertically upwards at 10m/s from a height of 2m. what's the maximum height?

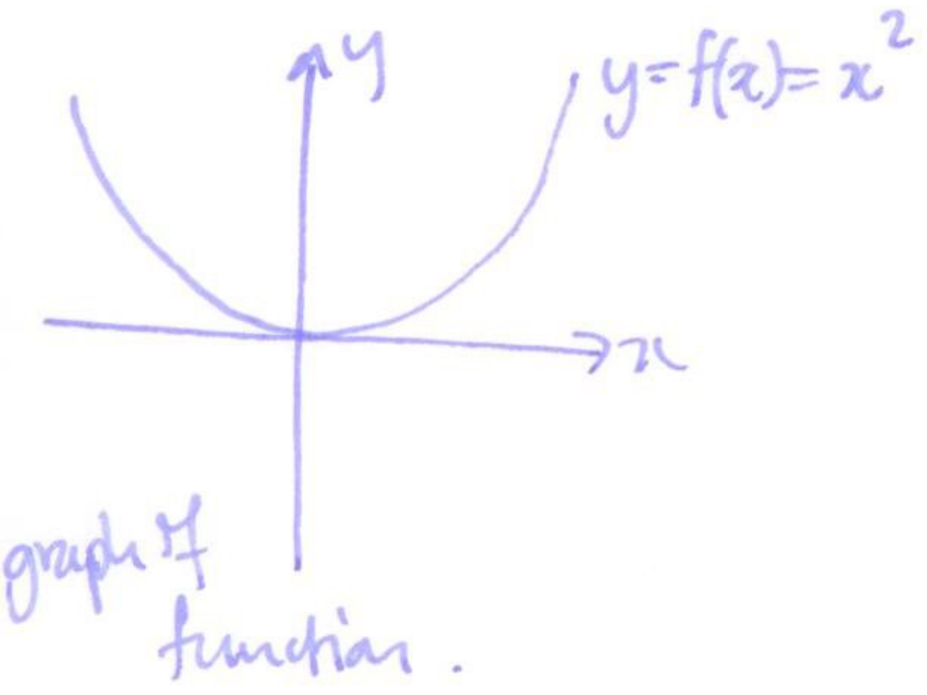
$s(t) = 2 + 10t - \frac{1}{2}gt^2$

$v(t) = 10 - \frac{3}{2}gt$

$v(t) = 0 \qquad t = \frac{10}{g} \approx 1 \qquad s(1) = 2 + 10 - 5 = 7m.$

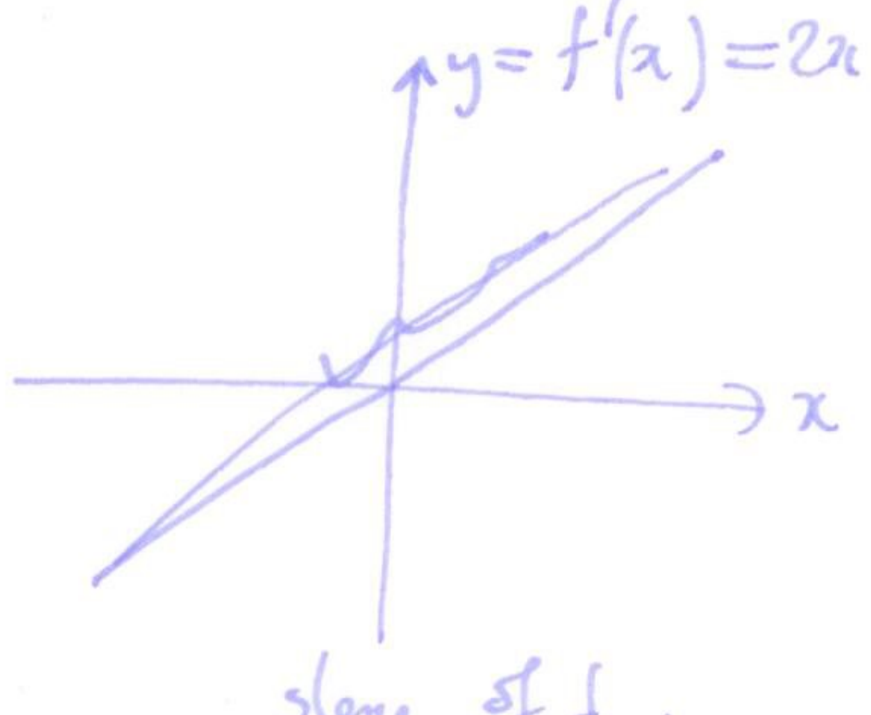
Q: if I can throw a stone 10m high, how fast can I throw it?

3.5 Higher derivatives



graph of function.

$f(x)$

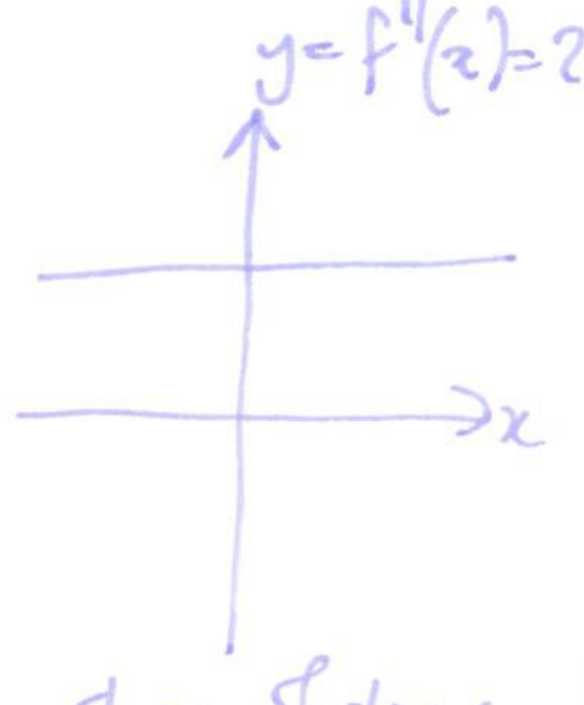


slope of f.

$f'(x)$

first derivative

$\frac{df}{dx}$



slope of slope of f.

$f''(x)$

2nd derivative.

$\frac{d^2f}{dx^2}$.

Example $f(x) = xe^x$.

$f'(x) = xe^x + e^x$

$f''(x) = xe^x + e^x + e^x = xe^x + 2e^x$.

$f^{(3)}(x) = f'''(x) = xe^x + e^x + 2e^x = xe^x + 3e^x$

and so on ...