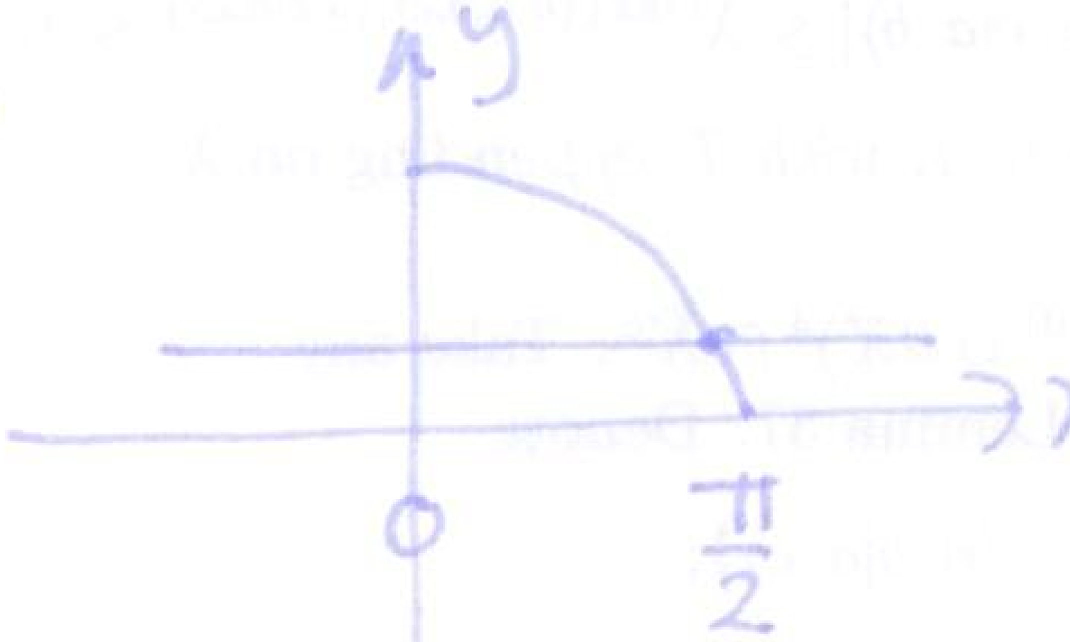
Thm (Intermediate value theorem IVT)

If $f(x)$ is a continuous function on a closed interval $[a,b]$ and $f(a) \neq f(b)$. Then for every M between $f(a)$ and $f(b)$ there is at least one $c \in [a,b]$ such that $f(c) = M$.

Application: showing equations have solutions.

Example show the equation ~~sin~~ $\cos(x) = 1/4$ has at least one solution.

cos cts.



consider $[0, \frac{\pi}{2}]$

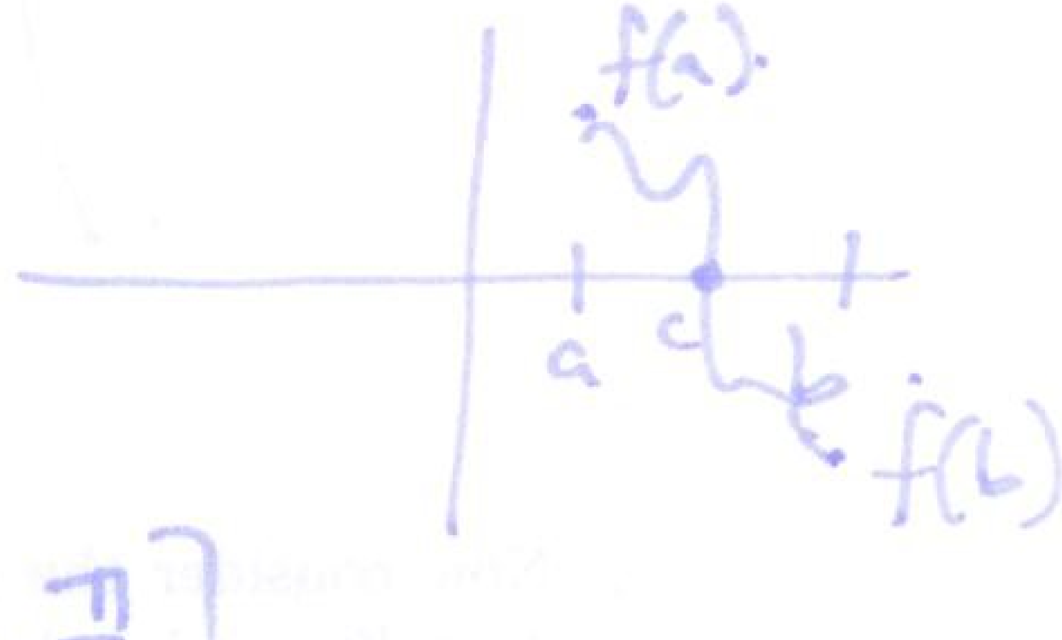
~~sin~~ $\cos(0) = 1$

$\cos(\frac{\pi}{2}) = 0$.

$0 < 1/4 < 1$ so $\cos(x) = 1/4$ has at least one solution.

Special case: finding zeros.

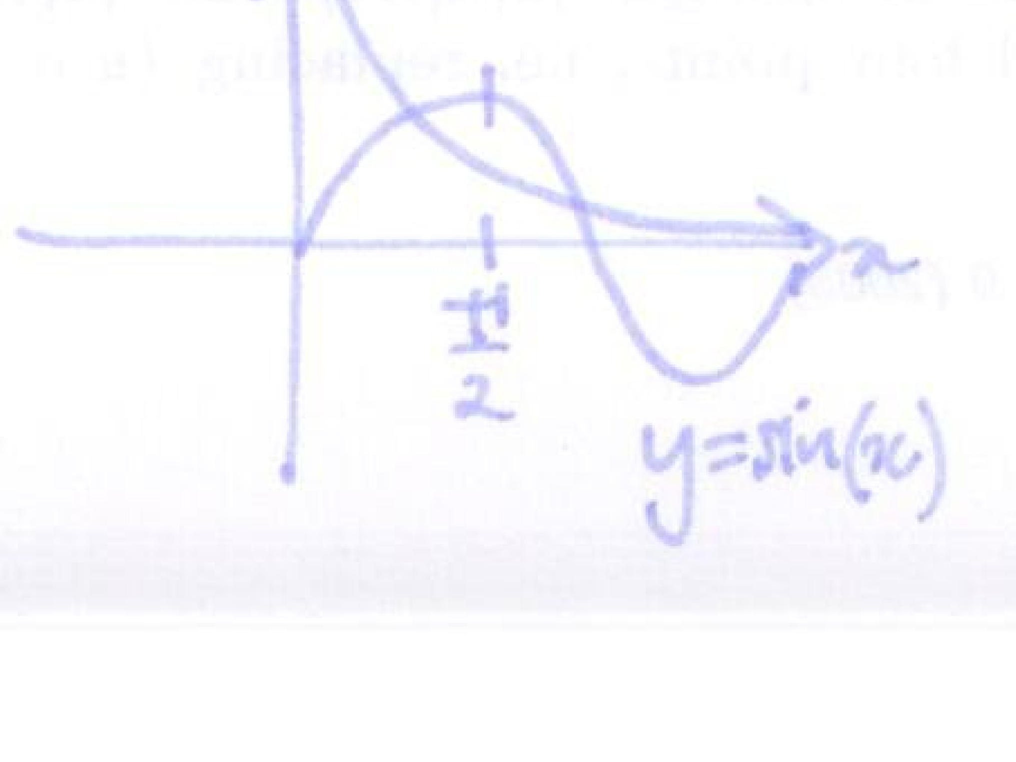
Corollary If the function $f(x)$ is cts on $[a,b]$ and $f(a)$ and $f(b)$ have different signs, then there is at least one $c \in [a,b]$ s.t. $f(c) = 0$

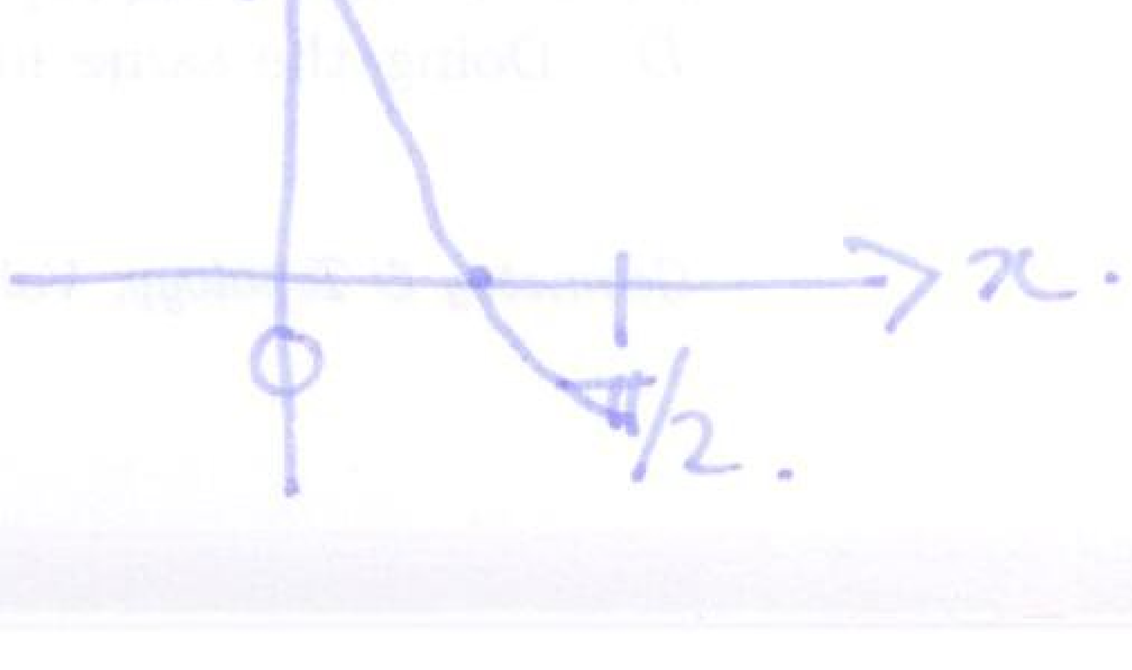


Application: bisection method.

look for a solution to $\sin x = \frac{1}{x}$ in $[0, \frac{\pi}{2}]$.

consider $f(x) = \frac{1}{x} - \sin(x)$

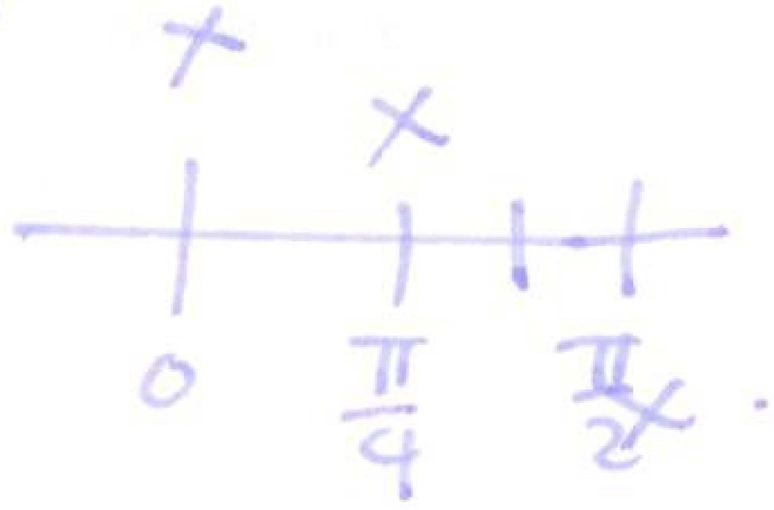




$f(0) = +\infty.$

$f(\frac{\pi}{2}) = \frac{2}{\pi} - \sin(\frac{\pi}{2}) = \frac{2}{\pi} - 1 \simeq -0.3634$

now try halfway point: $\frac{\pi}{4}$



$f(\frac{\pi}{4}) = \frac{4}{\pi} - \sin(\frac{\pi}{4}) \simeq 0.566$ +ve.

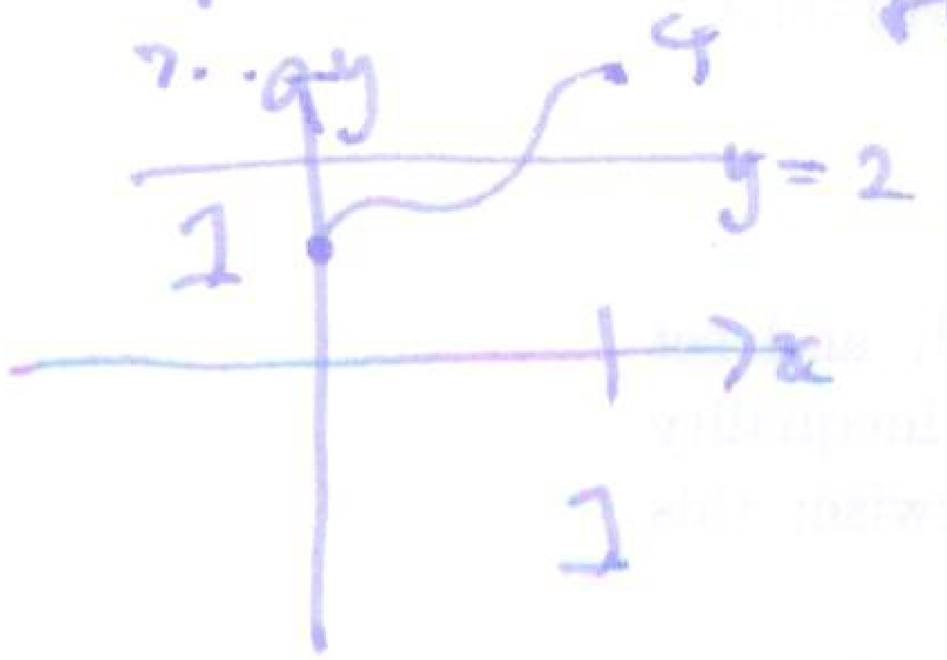
there is a

so 1 solution in $[\frac{\pi}{4}, \frac{\pi}{2}]$ now try $\frac{3\pi}{8}$ ...

and so on.

<u>Example</u> show that $\sqrt{c} + \sqrt{c+1} = 2$ for some number c.

$f(c) =$



c	$f(c)$
0	1
1	$1 + \sqrt{2} \simeq 2.4$

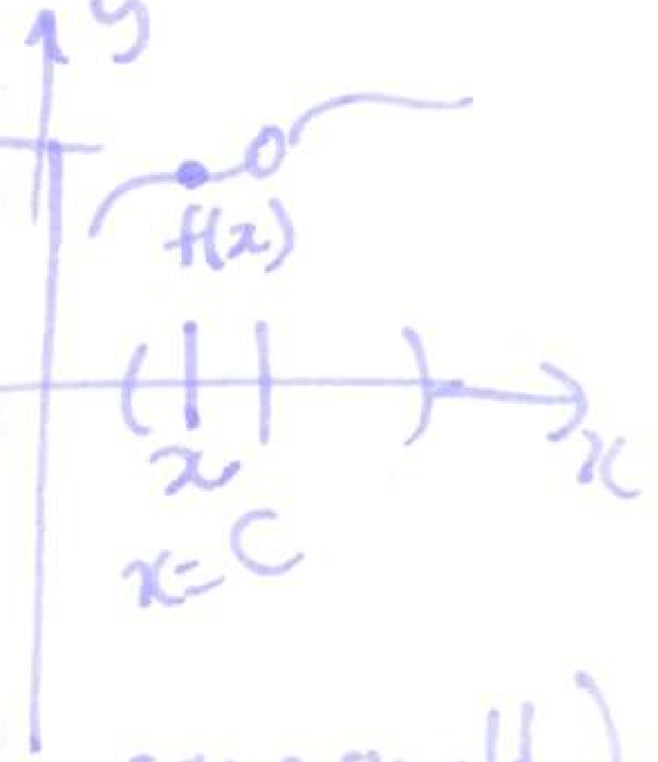
2

3

$\sqrt{2} + \sqrt{3} \simeq$

§2.8 Formal definition of a limit

<u>recall</u>: $\lim_{x \to c} f(x) = L$ if $|f(x) - L|$ becomes arbitrarily small as x gets close to c.



make this precise: give names to things!

$|x - c|$ horizontal gap "small" $< \delta$ ← (think of as some small number)

$|f(x) - L|$ vertical gap "small" $< \epsilon$