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- math tutoring: 15-214

- students with disabilities

Text: Calculus, Early Transcendentals by Rogawski§1.1 Functions and graphsFunctions: to define a function is you need 3 things.• domain X • target set Y • for each element $x \in X$ an element $f(x) \in Y$. f is the name of the function.the range of the function is all possible values of $f(x)$.
i.e. $\{y \in Y \mid y = f(x) \text{ for some } x \in X\}$
 ~~$\{f(x) \in Y \mid x \in X\}$~~ $\mathbb{R} = \text{real numbers.}$ domain $\mathbb{R}$.
range $\mathbb{R}$.range $[0, \infty)$ or $x \geq 0$ Example

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$x \mapsto x^2$$

$$f(x) = x^2$$

Example

$$\{\text{people in this room}\} \rightarrow \mathbb{R}$$

person $\mapsto$ height of person.

Real numbers: $\mathbb{R}$.

- can be written as decimals:

$$\frac{1}{4} = 0.25$$

finite decimal.

$$\frac{1}{3} = 0.333\dots = 0.\overline{3}$$

repeating decimal

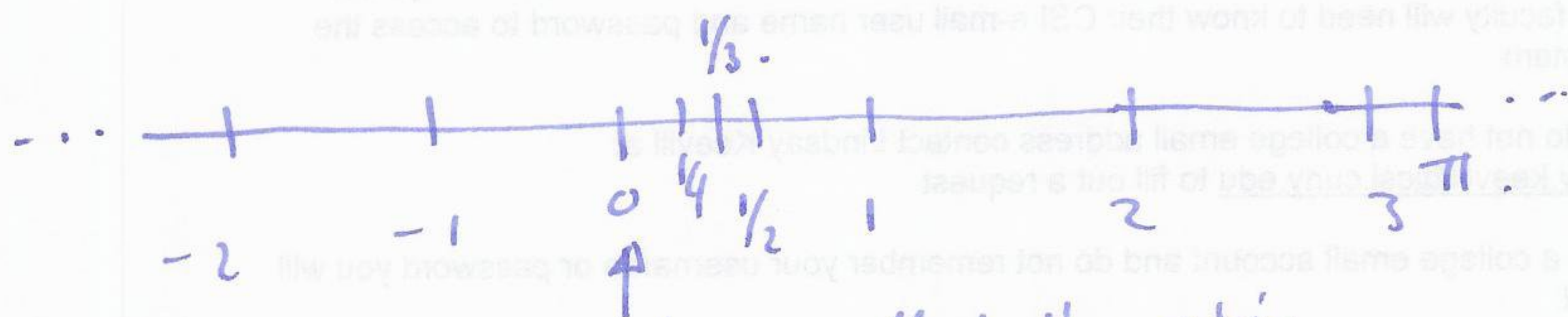
$$\pi = 3.14159\dots$$

non-repeating decimal

fractions/
rational
numbers.

irrational

- can be drawn on the number line:



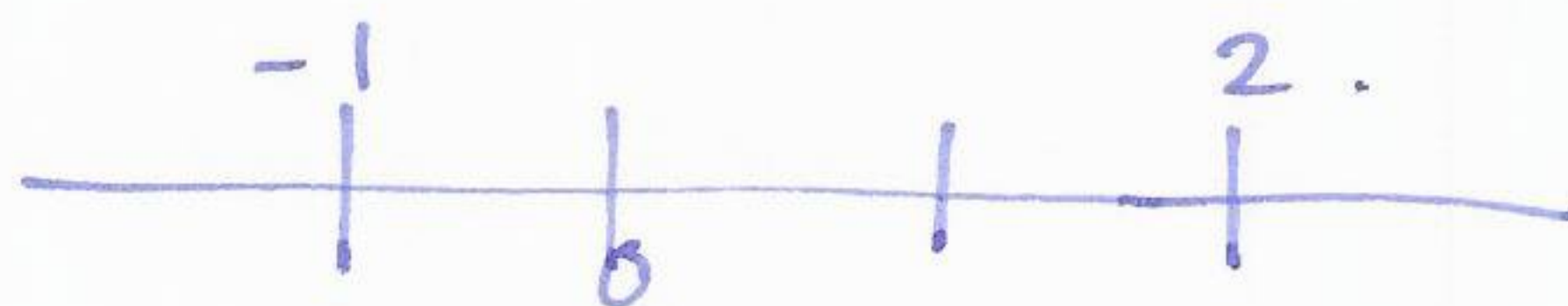
0 sometimes called the origin.

- decimal expansion of a number is unique, with the following exception: every finite decimal is equal to an infinite decimal ending in repeating 9's.

i.e. $1 = 0.999\dots$ $\frac{1}{4} = 0.25 = 0.24999\dots$

- absolute value: if $a \in \mathbb{R}$ then $|a| = \text{distance to } 0$.

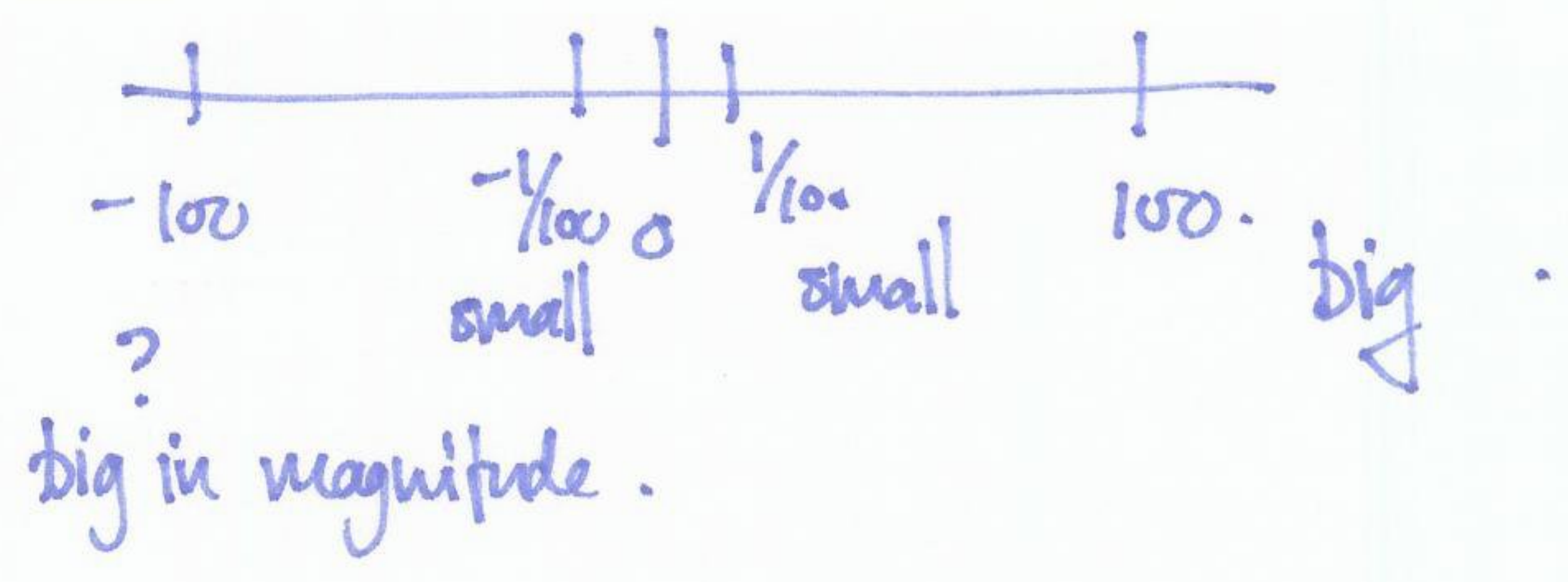
$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$



$$|2| = 2$$

$$|-1| = 1$$

Language problem:



useful facts:

$$|-a| = |a|$$

$$|ab| = |a||b|$$

$$|a+b| \leq |a| + |b|$$

distance from a to b is $|b-a| = |a-b|$

interval notation

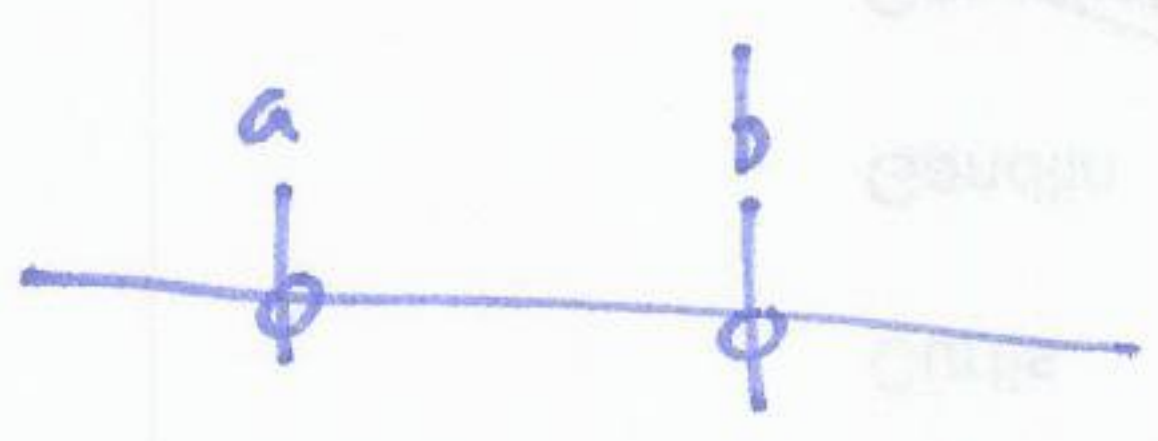


closed interval



$$\{x \mid a \leq x \leq b\}$$

open interval
no endpoints.



$$\{x \mid a < x < b\}$$

includes endpoints

half open intervals:



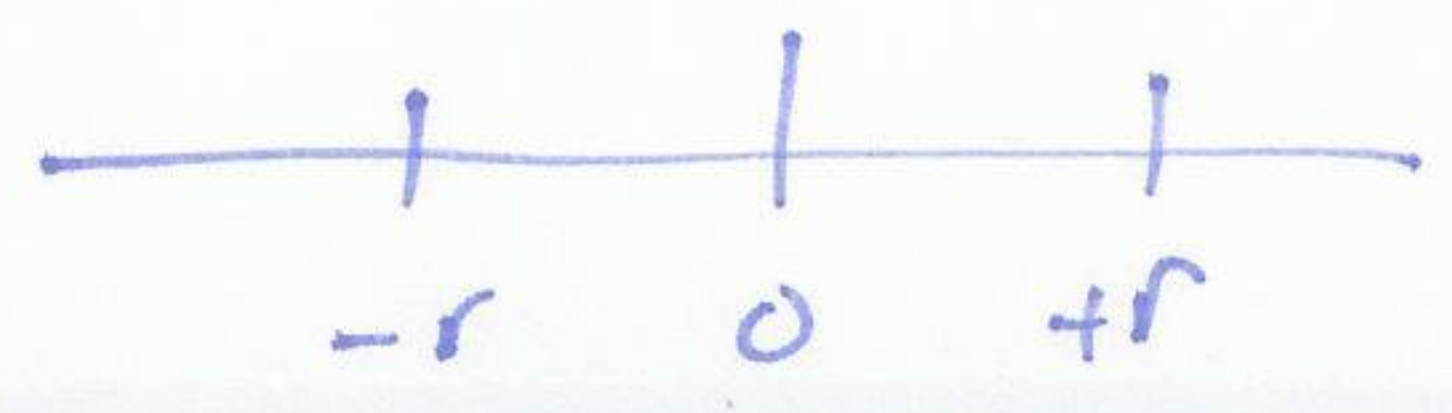
$$[a, b) \quad \{x \mid a \leq x < b\}$$



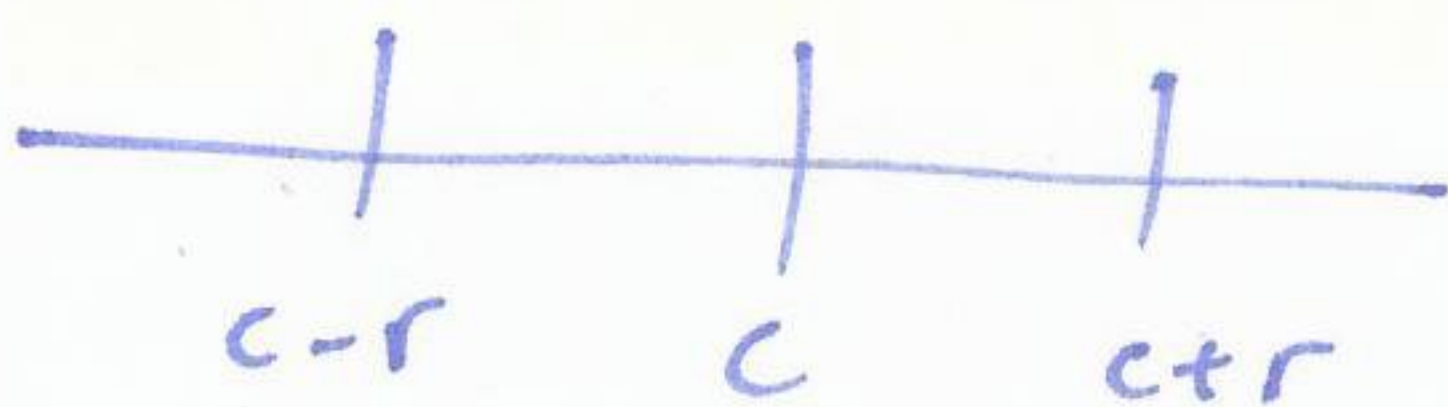
$$(a, b] \quad \{x \mid a < x \leq b\}$$

describing intervals with inequalities:

$$\{x \mid |x| \leq r\}$$



$$\{x \mid |x-c| \leq r\}$$



④

Exercise convert $[a,b]$ to inequality description.

Graphing / Pictures of functions

$$\{\text{people}\} \rightarrow \mathbb{R}$$

person $\mapsto$ height of person

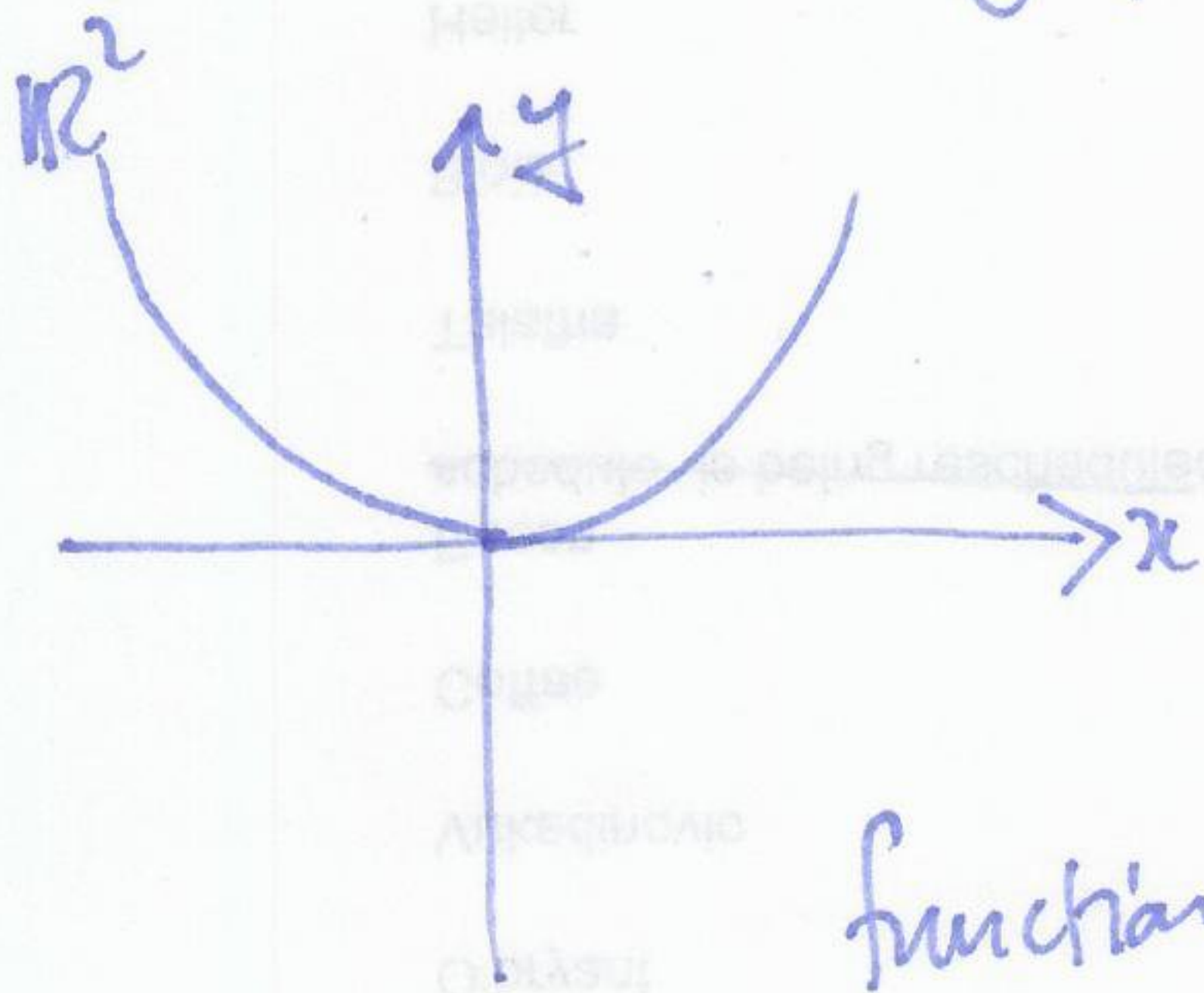
} Q. how to draw a picture?

functions between numbers

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$x \mapsto x^2$$

can draw the graph of the function



graph of f :

$$\{(x, f(x)) \in \mathbb{R}^2 \mid x \in \mathbb{R}\} \text{ or domain of } f.$$

function

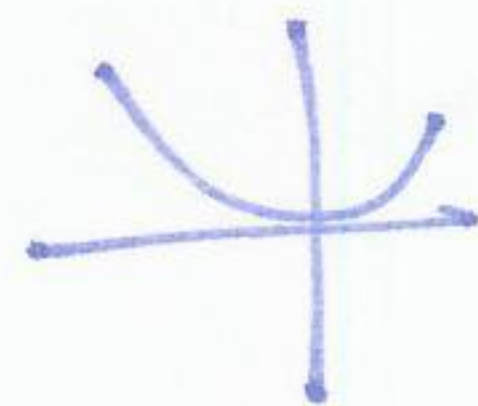
$$x \mapsto x^2$$

domain

$$\mathbb{R}.$$

range.

$$y \geq 0.$$



$$\cos x$$

$$\mathbb{R}$$

$$-1 \leq y \leq 1$$



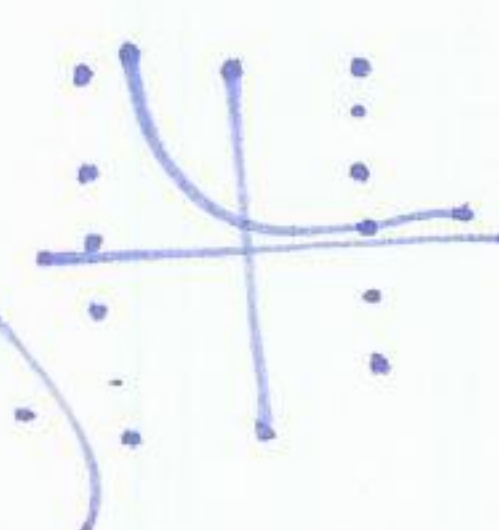
$$\frac{1}{1+x}$$

$$\mathbb{R} \setminus -1.$$

$$\{x \in \mathbb{R} \mid x \neq -1\}.$$

$$\mathbb{R} \setminus 0$$

$$\{y \in \mathbb{R} \mid y \neq 0\}.$$



Examples