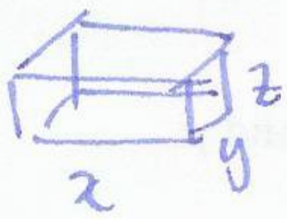
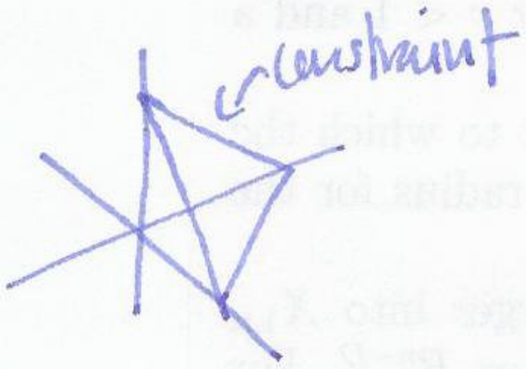
postoffice problem revisited (Lagrange multipliers)..

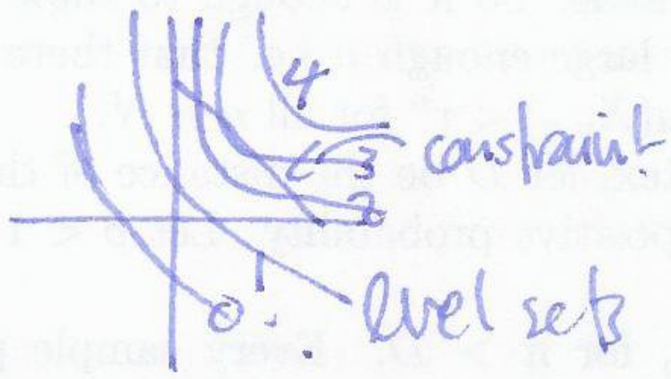


length + girth = 108 max volume $V = xyz$.

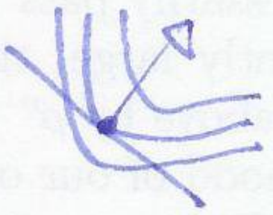
$x + 2y + 2z = 108$

alternate method:





max occurs where level sets are tangent to constraint



i.e. same normal vector directions.

$V = xyz = c$ level set normal vector is $\nabla V = \langle V_x, V_y, V_z \rangle$.

$= \langle yz, xz, xy \rangle$.

constraint $\underbrace{x + 2y + 2z = 108}_{F(x,y,z)}$ $\nabla F = \langle 1, 2, 2 \rangle$.

Q: when is ∇F parallel to ∇V?

solve: $\nabla F = \lambda \nabla V$ $\langle 1, 2, 2 \rangle = \lambda \langle yz, xz, xy \rangle$.

and $x + 2y + 2z = 108$.

$\lambda yz = 1$, $\lambda xz = 2$ $\Big\}$ $z \neq 0$, $\lambda \neq 0$ $\Rightarrow$ $\frac{y}{x} = \frac{1}{2}$. $x = 2y$

$\lambda xy = 2$ $\Big\}$ $y \neq 0$, $\lambda \neq 0$ $\Rightarrow$ $\frac{z}{x} = \frac{1}{2}$ $x = 2z$ so

$x + x + x = 108$.

$x = 36$.

General method: Think level sets $\perp$ to constraint

max $F(x,y,z)$ subject to $G(x,y,z) = c$

solve: $\nabla F = \lambda \nabla G$

$G = c$