Example $f(x,y) = 2x^3 + 6xy + 3y^2 + 1$

$\frac{\partial f}{\partial x} = 6x^2 + 6y = 0$ $\qquad 6x^2 - 6x = 0 \qquad 6x(x-1) \qquad x=0, \quad x=+1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad y=0 \quad y=-1$

$\frac{\partial f}{\partial y} = 6x + 6y = 0 \iff x = -y$

$(0,0) \quad (+1,-1)$.

TUE 10	MatLab 3	
FRI 13	ww 19, 20	
MON 16	SMTS	
WED 18	MTS.	

$f_{xx} = 12x$

$f_{xy} = 6 \qquad D = f_{xx} f_{yy} - (f_{xy})^2 = 72x - 36$

$f_{yy} = 6 \qquad D(0,0) = -36 \Rightarrow$ saddle.

$D(1,-1) = 36 > 0 \quad f_{xx} > 0 \Rightarrow$ local min.

Example $f(x,y) = x^2y^2$.

$\left.\begin{array}{l} f_x = 2xy^2 \\ f_y = 2x^2y \end{array}\right\} \Rightarrow$ x,y axis are critical points.

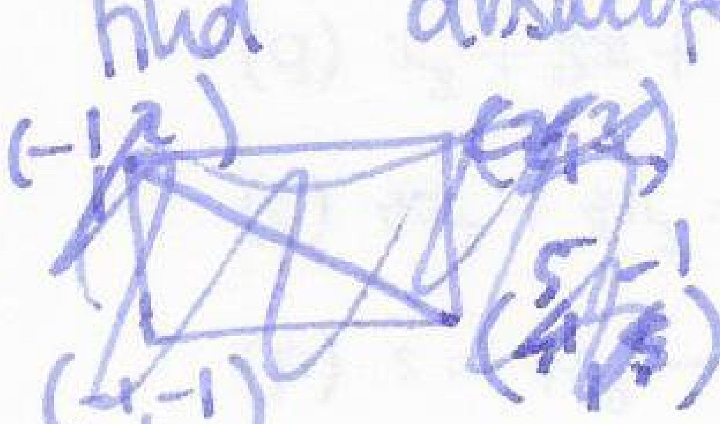
$f_{xx} = 2y^2$

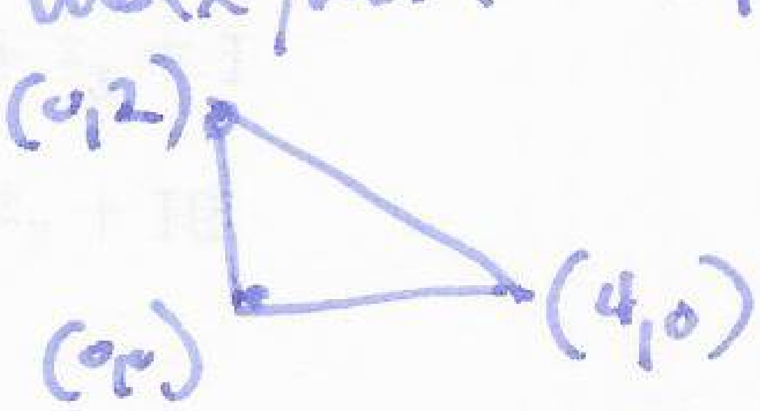
$f_{xy} = 4xy \qquad D(x,y) = 2x^2 \cdot 2y^2 - (4xy)^2 = -12x^2y^2$

$f_{yy} = 2x^2 \qquad D = 0$ on x,y axis : <u>no information</u>!

<u>Finding absolute max/min in simple regions.</u>

<u>Example</u> find absolute max/min of $f(x,y) = x^2 - x + y^2 - y$ in the ~~rectangle~~ triangle





• if max/min value occurs in <u>interior</u> then it is a local max/min.

• otherwise max/min occurs in boundary : 1 variable max min problem.

so ① find critical points in region – evaluate f on each point

② solve 1d problem on boundary.

① $f_x = 2x-1$, $f_y = 2y-1$ $\Big\}$ $x = 1/2$, $y = 1/2$. $(1/2, 1/2)$ in triangle

$f(1/2, 1/2) = -1/2$.

② boundary: $(0,y)$ $0 \le y \le 2$ $\quad f(0,y) = y^2 - y \quad f'(y) = 2y-1, y = \frac{1}{2}$.

$(x,0)$ $0 \le x \le 4$ $\quad f(x,0) = x^2 - x \quad f'(x) = 2x-1, x = \frac{1}{2}$.

$y = 2 - \frac{1}{2}x$. $\quad (x, 2-\frac{1}{2}x)$ $0 \le x \le 4$. $\quad f(x, 2-\frac{1}{2}x) = x^2 - x + (2-\frac{1}{2}x)^2 - 2 + \frac{1}{2}x$

$= \frac{5}{4}x^2 - \frac{3}{2}x + 4$.

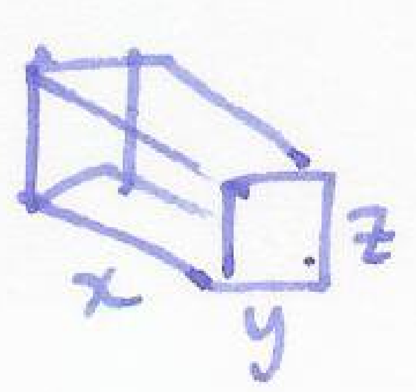
$f'(x) = \frac{5}{2}x - \frac{3}{2} \quad x = 3/5$.

evaluate f at: $(1/2, 1/2)$ $(0, \frac{1}{2})$, $(\frac{1}{2}, 0)$, $(3/5, 1.7)$. and corners $(0,0)$ $(0,2)$ $(4,0)$.

biggest value: absolute max.
smallest value: absolute min.

§13.9 Applications of extrema.

Example in order to send a package by USPS, cheap rate. the sum of the length and girth (perimeter of smallest perpendicular cross-section) may not exceed 108 inches. What's the largest volume box you can send?



$x + 2y + 2z = 108$. $\quad V = xyz$.

$x = 108 - 2y - 2z$.

so max: $V = yz(108 - 2y - 2z) = 108yz - 2y^2z - 2yz^2$.

$V_y = 108z - 4yz - 2z^2 = 0 \quad 2z(54 - 2y - z) = 0$

$V_z = 108y - 2y^2 - 4yz = 0 \quad 2y(54 - y - 2z) = 0$

assume $z \ne 0, y \ne 0$ $\quad 54 - 2y - z = 0$ ① $\quad$ ① − 2②: $-54 + 3z = 0$.

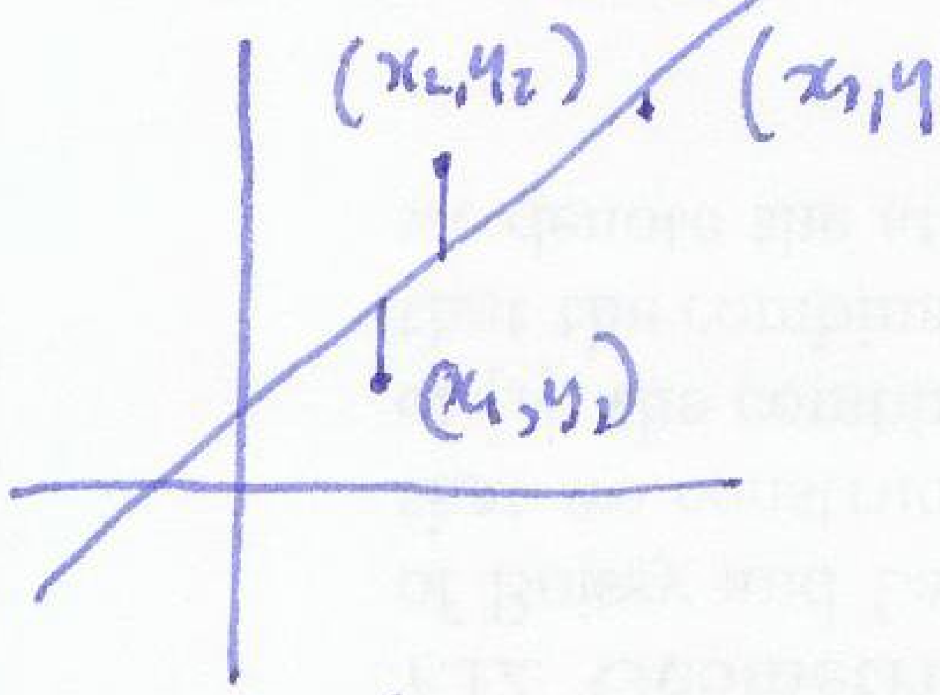
$54 - y - 2z = 0$ ② $\quad z = 18$.

$y = 18$

$x = 36$.

Linear regression

$y = ax + b$



"best fit line"

one method is to choose a line which minimise the sums of squares of vertical distances

$$E = \text{"vertical error"} = \sum_{i=1}^{n} (ax_i + b - y_i)^2$$ a function of a and b!

look for critical point

$$\frac{\partial E}{\partial a} = \sum_i 2(ax_i + b - y_i)\cdot x_i = 0$$

$$\frac{\partial E}{\partial b} = \sum_i 2(ax_i + b - y_i) = 0$$

① $a\sum_i x_i^2 + b\sum_i x_i - \sum_i x_i y_i = 0$

② $a\sum_i x_i + nb - \sum y_i = 0$

a pair of linear equations for a and b.

$n① - \sum_i x_i ②$: $a\left[n\sum x_i - (\sum x_i)^2\right] + \left[-n\sum_i x_i y_i + \sum x_i \sum y_i\right] = 0$

so $$a = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i - (\sum x_i)^2}$$

$\sum_i x_i ① - \sum x_i^2 ②$: $b\left[(\sum x_i)^2 - n\sum x_i^2\right] + \left[-\sum_i x_i y_i \sum x_i + \sum x_i^2 \sum y_i\right] = 0$

$$b = \left(\sum x_i y_i \sum x_i - \sum x_i^2 \sum y_i\right) \Big/ \left((\sum x_i)^2 - n\sum x_i^2\right).$$