13.3 Partial derivatives

$f(x,y)$ $f_x = \dfrac{\partial f}{\partial x}(x,y)$ "differentiate wrt x keeping y constant"

$f_y = \dfrac{\partial f}{\partial y}(x,y)$ "differentiate wrt y keeping x constant".

Example $f(x,y) = x^2+y^2$ $\quad f_x = 2x \quad f_y = 2y$

$f(x,y) = xy \quad f_x = y \quad f_y = x.$

$f(x,y) = xye^x + x^2y \quad f_x = ye^x + xye^x + 2xy$

$f_y = xe^x + x^2.$

note $f_x(x,y)$, $f_y(x,y)$ are still functions of two variables

so we can partially differentiate again.

$f_{xx} = \dfrac{\partial^2 f}{\partial x^2}$ $\quad f_{yy} = \dfrac{\partial^2 f}{\partial y^2}$ $\quad f_{xy} = \dfrac{\partial^2 f}{\partial y \partial x}$ $\quad f_{yx} = \dfrac{\partial^2 f}{\partial x \partial y}$

Example $f_{xx} = ye^x + ye^x + xye^x + 2y$

$f_{xy} = e^x + xe^x + 2x$

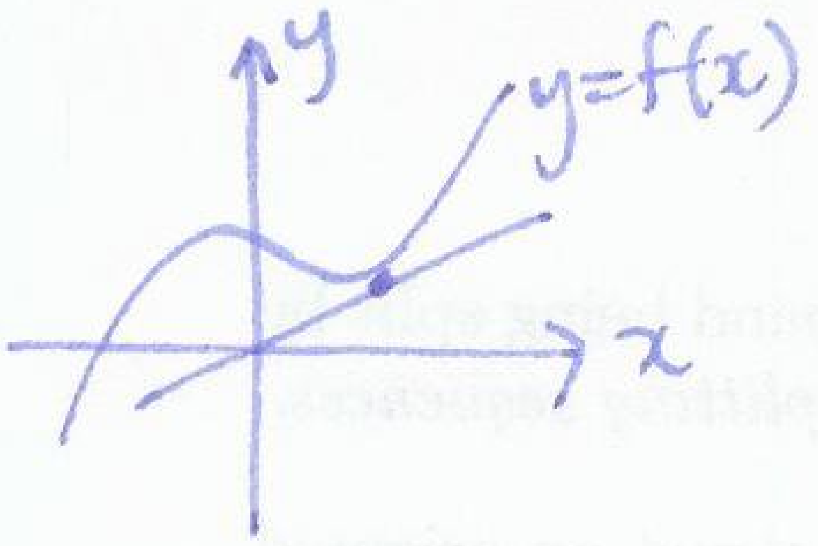
$f_{yx} = e^x + xe^x + 2x$

$f_{yy} = 0$

Thm If f_{xy} and f_{yx} are both cts, then $f_{xy} = f_{yx}$.

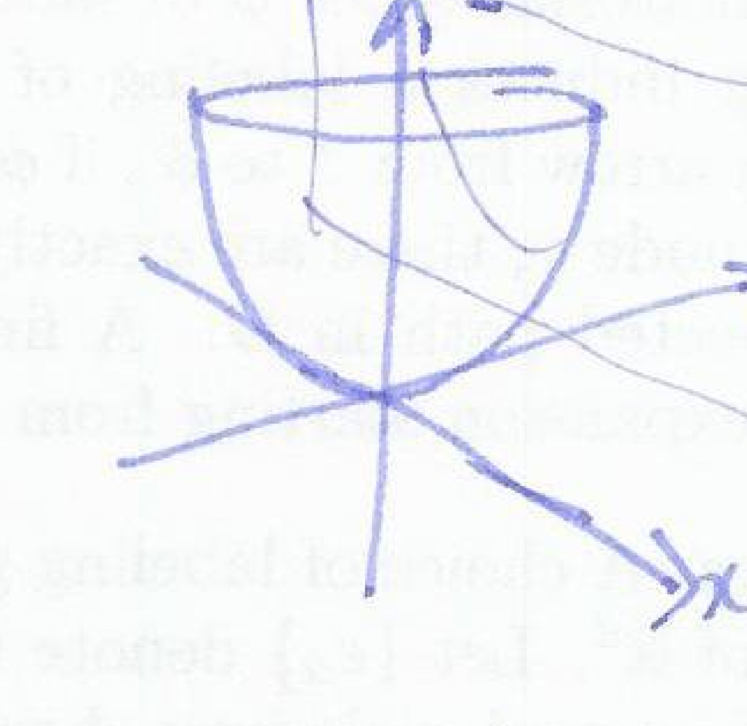
what does all this mean?

1d



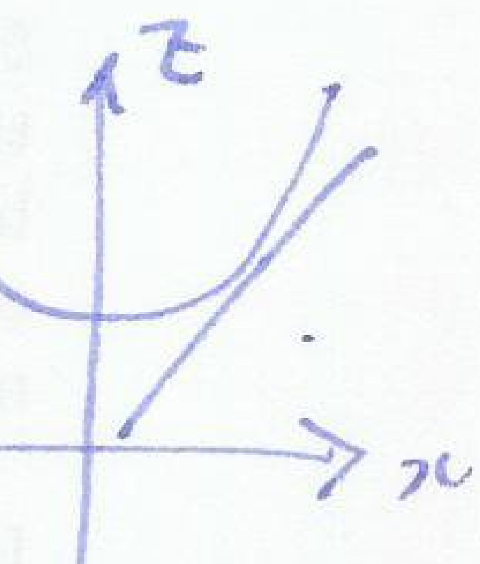
$f_x(x) = \dfrac{df}{dx}$ gives rate of change | slope.

2d $z = f(x,y) = x^2+y^2$



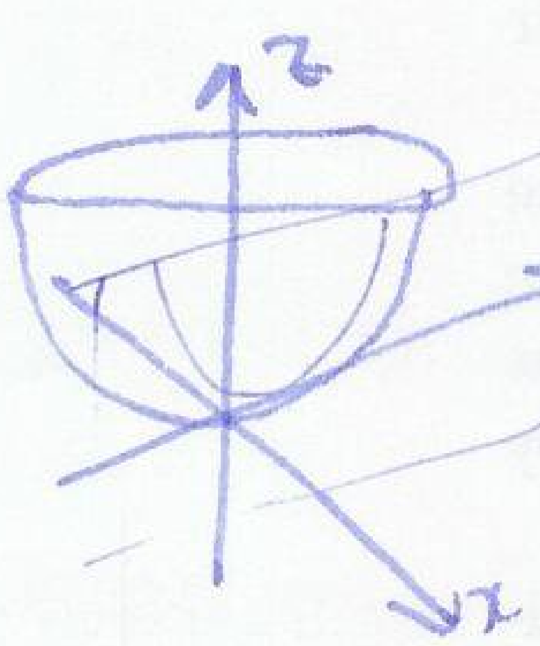
$f_x(x,y) = 2x$
$f_y(x,y) = 2y$

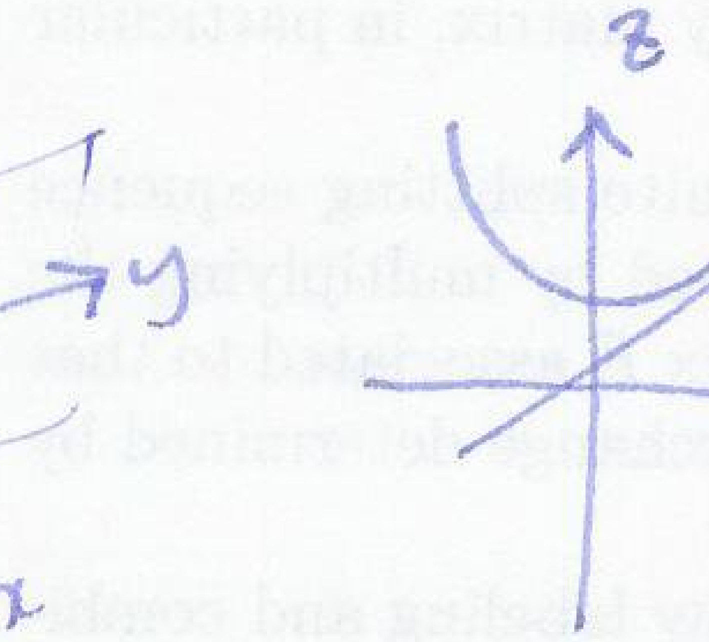
"keep y fixed" corresponds to $y=c$ slice $f(x,c) = x^2+c^2$



$\dfrac{\partial f}{\partial x} = f_x$ = slope in x direction

"keep x fixed" corresponds to $x=c$ slice $f(c,y) = c^2+y^2$





$f_y = \dfrac{\partial f}{\partial y}$ = slope in y-direction

f_{xx} = "2nd derivative in x-direction"

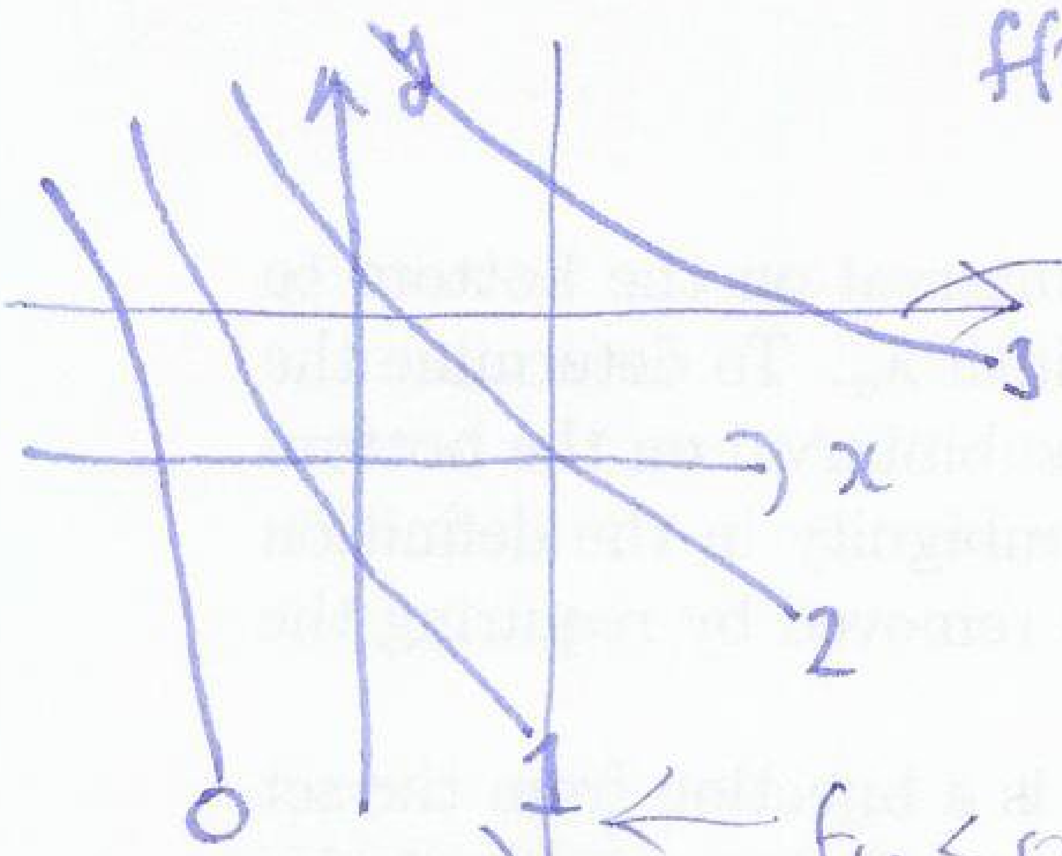
f_{yy} = "2nd derivative in y-direction"

f_{xy} = rate of change of f_x in y-direction

= rate of change of f_y in x-direction

<u>Warning</u>! f_x, f_y exist $\not\Rightarrow$ f is cts!

Interpreting contour maps | level sets.



$f(x,y)$, $f_x^B > 0$

contour lines further apart $f_{xx} < 0$

$f_{xy} < 0$

$f_y < 0$ contour lines further apart

$f_{yy} > 0$.

Examples ① $f(x,y) = x\sin(x+y)$

$f_x = \sin(x+y) + x\cos(x+y) \qquad f_y = x\cos(x+y)$.

② $f(x,y) = \dfrac{ae^{xy}}{y} \qquad f_x = \dfrac{aye^{xy}}{x} \qquad f_y = \dfrac{yaye^{xy} - ae^{xy}}{y^2}$.

Partial derivatives for functions of 3 vars.

$f(x,y,z)$ can differentiate wrt x, y or z.

$\dfrac{\partial f}{\partial z}$ = "diff wrt z keeping x,y constant" etc.

Examples $f(x,y,z) = xy + yz$

$f_x = y \qquad f_y = x + z \qquad f_z = y$

$f_{xx} = 0 \quad f_{xy} = 1 \quad f_{yz} = 1 \quad f_{xz} = 0 \quad f_{yy} = 0 \quad f_{zz} = 0$.

Thm if 2nd order derivatives are cts then mixed partials are equal.

i.e. $f_{xy} = f_{yx} \qquad f_{xz} = f_{zx}$ etc.