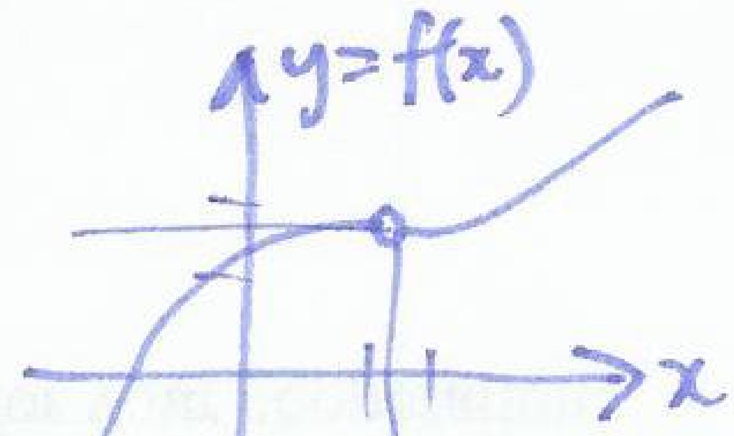
13.2 Limits and continuity

recall $y=f(x)$

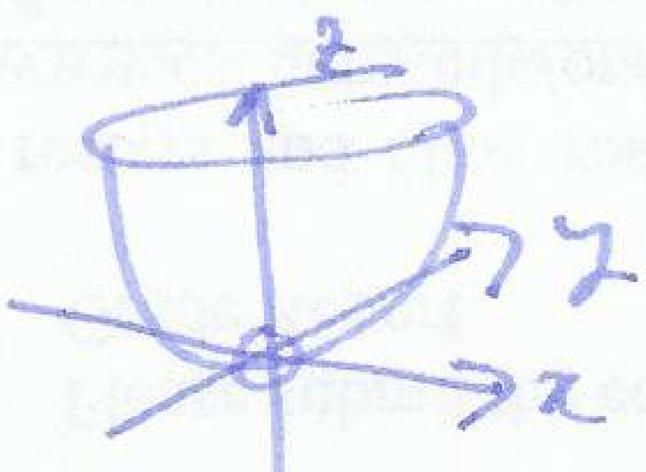


$$\lim_{x\to x_0} f(x) = L$$

if for all $\epsilon > 0$ there is a $\delta > 0$ s.t.

can limit from right or left. $|f(x)-f(x_0)| \leq \epsilon$ for all x $|x-x_0| \leq \delta$.

2 vars $f(x,y)$



more complicated - can limit in many different ways.

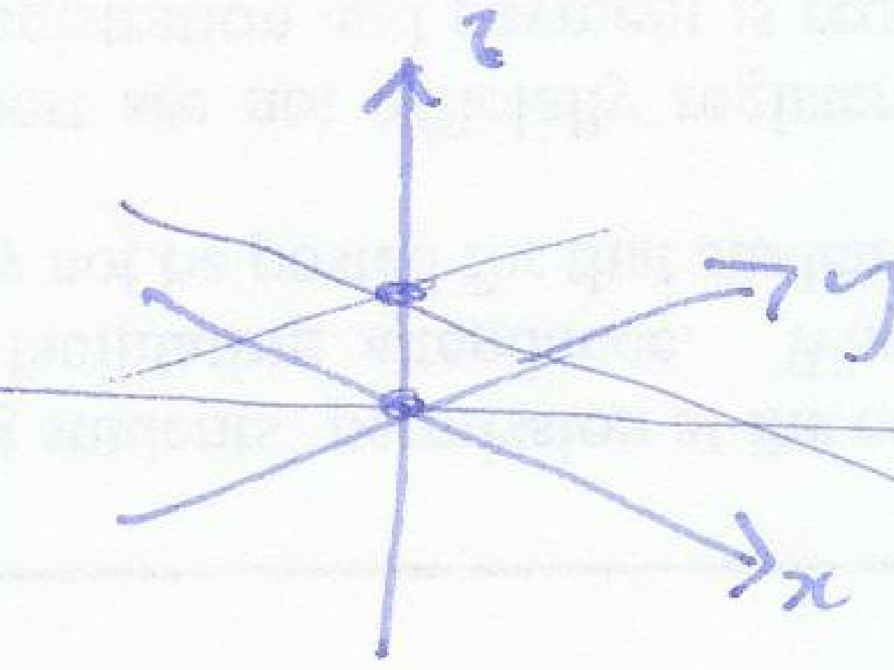
Def: $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$ if for all $\epsilon > 0$ there is a $\delta > 0$ s.t.

$$|f(x,y) - f(x_0,y_0)| \leq \epsilon \text{ for all } 0 \leq |(x,y)-(x_0,y_0)| \leq \delta$$

$$|(x,y)-(x_0,y_0)| = \sqrt{(x-x_0)^2+(y-y_0)^2}$$

(Bad) Example $f(x,y) = \left(\frac{x^2-y^2}{x^2+y^2}\right)^2$

Q what happens near $(0,0)$?



$$\lim_{(x,0)\to(0,0)} f(x,y) = \left(\frac{x^2}{x^2}\right)^2 = 1.$$

$$\lim_{(0,y)\to(0,0)} f(x,y) = \left(\frac{-y^2}{y^2}\right)^2 = 1.$$

$$\lim_{(x,x)\to(0,0)} f(x,y) = \left(\frac{0}{2x^2}\right)^2 = 0.$$

trick: use polars $\left(\frac{r^2\cos^2\theta - r^2\sin^2\theta}{r^2\cos^2\theta + r^2\sin^2\theta}\right)^2 = (\cos^2\theta - \sin^2\theta)^2 = \cos^2 2\theta$.

~~Q: is $f(x,y)$ cts?~~

~~A: no check~~

Def$^{\underline{n}}$ $f(x,y)$ is cts at (x_0,y_0) if ~~there is a small ds~~

$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0,y_0)$. $f(x,y)$ is cts in a region R if it is cts at every point in R

How to tell if $f(x,y)$ is cts? not necessarily easy...

however: **Th$^{\underline{m}}$** "compositions of cts functions are cts."

so if f, g cts then fg cts

$f \pm g$ cts

f/g cts if $g \neq 0$ (may be cts even if $g=0$!).

$f \circ g$ cts

Example $f(x,y) = \dfrac{x-y}{x+y}$ note $\left.\begin{array}{l} f(x,y)=x \text{ cts} \\ g(x,y)=y \text{ cts} \end{array}\right\}$ $x \pm y$ cts.

so cts where $x+y \neq 0$. (i.e. off $x=-y$).

Similar def$^{\underline{n}}$s/th$^{\underline{m}}$s for functions of 3 var.

Def$^{\underline{n}}$ $f(x,y,z)$ cts at (x_0,y_0,z_0) if $\lim_{(x,y,z)\to(x_0,y_0,z_0)} f(x,y,z) = f(x_0,y_0,z_0)$.

Th$^{\underline{m}}$ "compositions of cts functions are cts".

so $f(x,y,z) = \dfrac{1}{x^2+y-z}$ cts where $x^2+y-z \neq 0$.