§13.1 Functions of several variables

we've seen functions of 1-variable : $f:\mathbb{R}\to\mathbb{R}$, $x \mapsto f(x)$, e.g. $x \mapsto x^2$; $f:\mathbb{R}\to\mathbb{R}^2$, $t \mapsto (x(t), y(t))$ etc.

in general: $f:(\text{domain}) \to (\text{range})$
$\underline{x} \mapsto f(\underline{x})$.

functions of 2-vars: $f:\mathbb{R}^2 \to \mathbb{R}$.
$(x,y) \mapsto f(x,y)$

example $(x,y) \mapsto x^2+y^2$

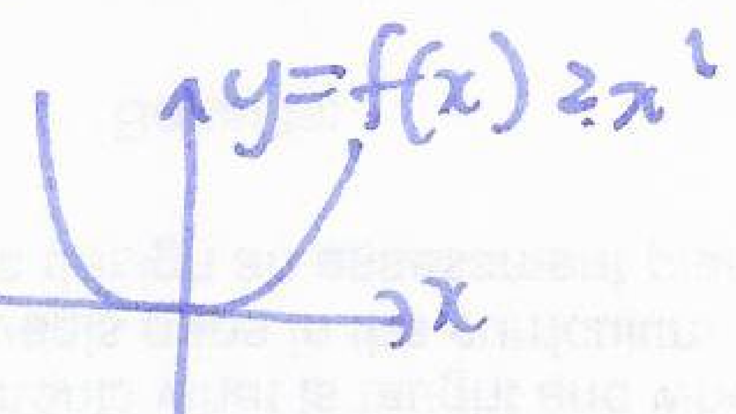
convention: f : domain $\to$ range. (usually not defined explicitly).

example : $f:(x,y) \mapsto \frac{1}{x^2+y^2}$ domain is $\mathbb{R}^2 \setminus (0,0)$.
range is $(0,\infty)$.

example : $\int_a^b f(x)\,dx$ function of a, b!
not a function of x. could be a function of f though
$\int_a^b \cdot\, dx : \{\text{functions}\} \to \mathbb{R}$.

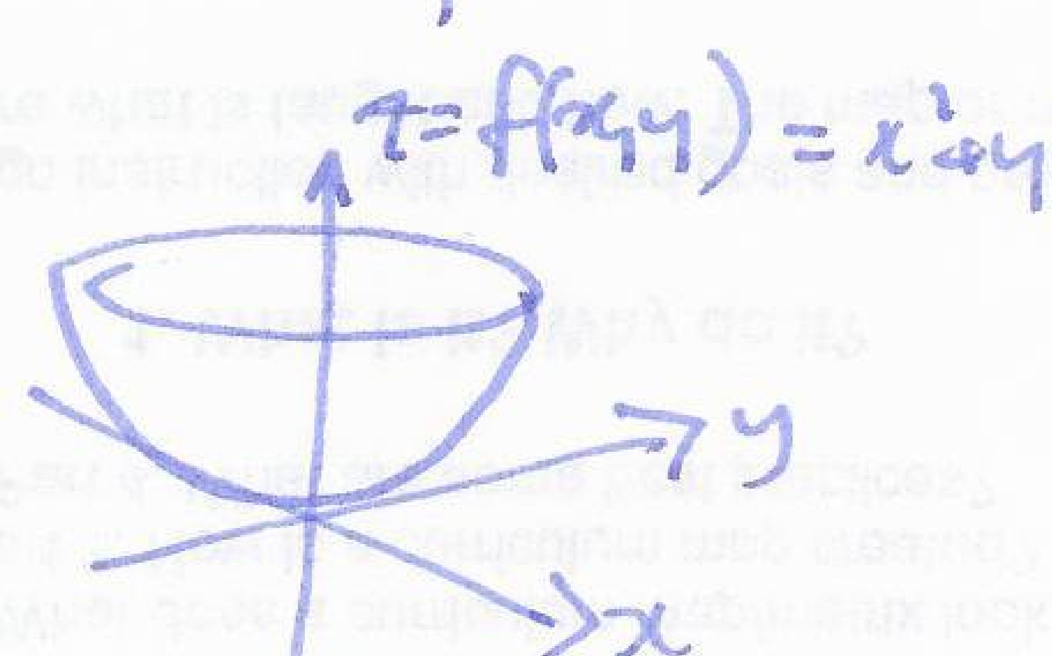
Graphs of functions of two variables

1 var : $f(x):\mathbb{R}\to\mathbb{R}$
$x \mapsto f(x)$.
$x \mapsto x^2$



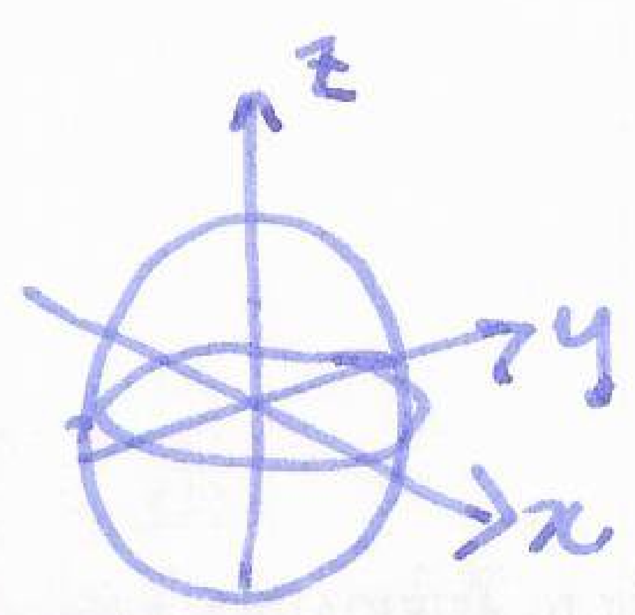
graph of function is a curve

2 vars : $f:\mathbb{R}^2\to\mathbb{R}$
$(x,y) \to f(x,y)$
e.g. $(x,y) \mapsto x^2+y^2$



graph of function is a surface.

Q when is a surface a graph of a function? A: vertical line test.



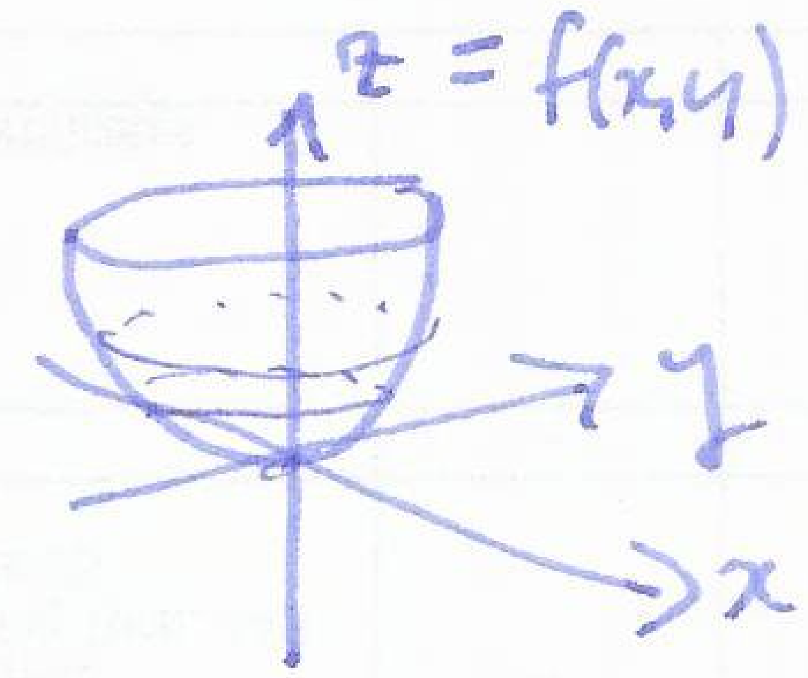
sphere is not a graph of a function.

Level sets / contour lines

Q: how do we know what $z = f(x,y)$ looks like?

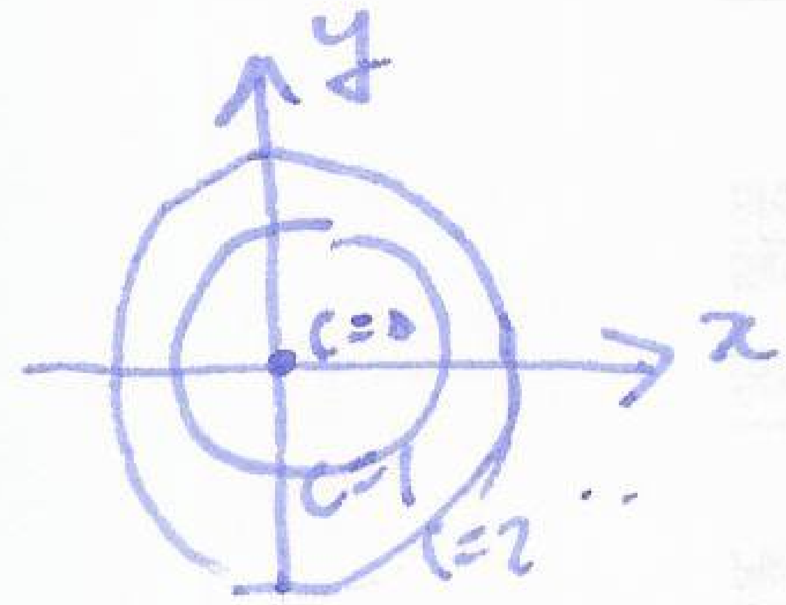
A: by drawing level sets = contour lines = horizontal cross sections.

Example. $f(x) = x^2+y^2$



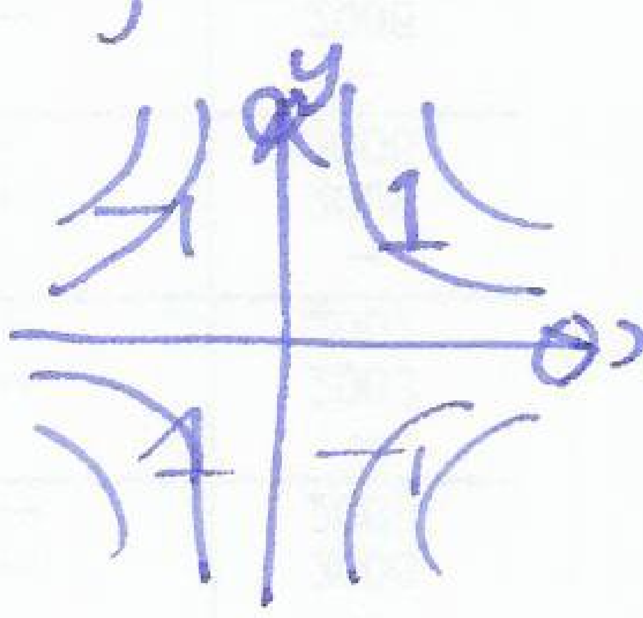
graph needs extra dimension

find intersection with horizontal plane $z = c$ $\quad c = x^2+y^2$ circle of radius $\sqrt{c}$.



$f: \mathbb{R}^2 \to \mathbb{R}$.
level sets live in $\mathbb{R}^2$

Example $f(x,y) = \cancel{x^2-y^2\ (a\,xy)}\ .\ xy$ $\quad$ level sets $xy = c$.



Functions of 3 variables

$f: \mathbb{R}^3 \to \mathbb{R}$ $\quad f(x,y,z)$ $\quad$ graph of function needs 4d.

Level sets: $f(x,y,z) = c$, these are surfaces in $\mathbb{R}^3$.

Examples $f(x,y,z) = x+y+z$ $\qquad f(x,y,z) = x^2+y^2+z^2$