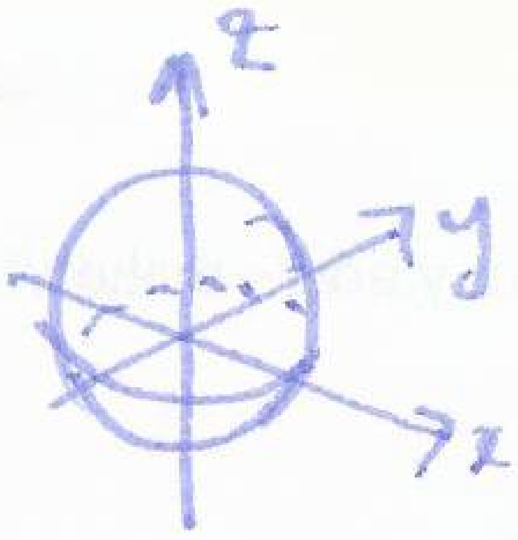
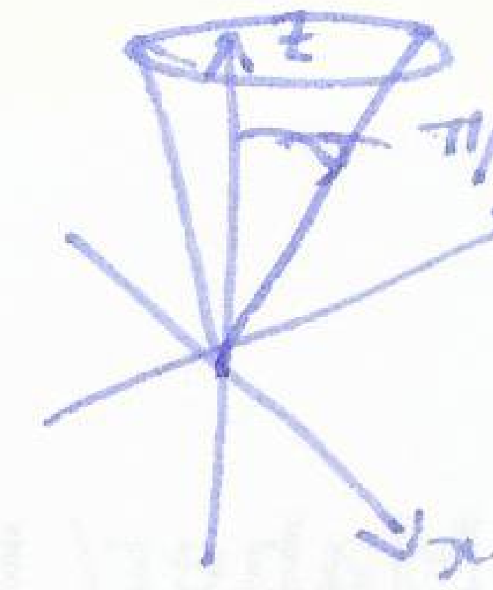
examples.

spheres:



$x^2+y^2+z^2=1$
$\Updownarrow$
$\rho^2=1$
$\Updownarrow$
$\rho=1$

cones:



$\pi/6$ angle.

$\phi=\pi/6$.
cartesians ?

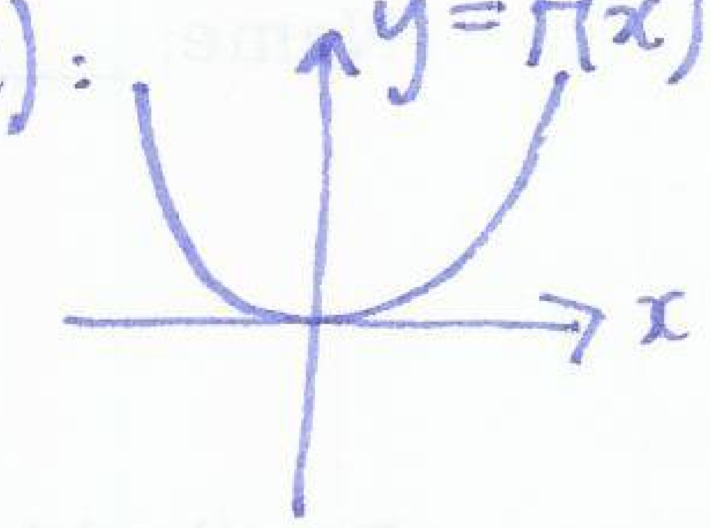
draw surfaces coordinate = const

§12.1 vector valued functions

real valued functions of 1-variable

$f:\mathbb{R}\to\mathbb{R}$.
$x\mapsto f(x)$
e.g. $x\mapsto x^2$

graph of $f(x)$:



parameterised curves: $\mathbb{R}\to\mathbb{R}^2$ $\quad t\mapsto (f(t),g(t))$
or $\mathbb{R}\to\mathbb{R}^3$ $\quad t\mapsto (f(t),g(t),h(t))$

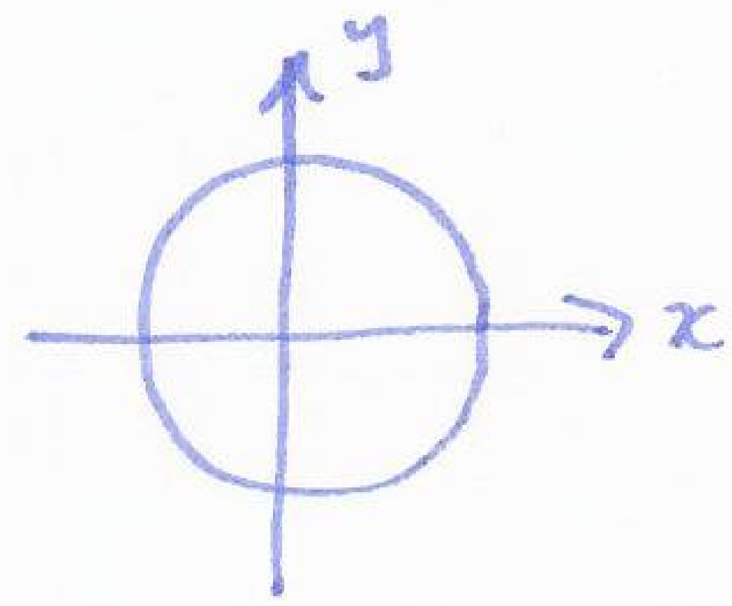
still a function of one variable, but now vector valued.

alternate notations:
- $x=f(t)$, $y=g(t)$ (2d)
- $\underline{r}(t)=f(t)\,\underline{i}+g(t)\,\underline{j}$ (2d)
- $\underline{r}(t)=\langle f(t),g(t),h(t)\rangle$ (3d)

convention: normally don't explicitly describe domain: e.g. $f(x)=1/x$ not defined for $x=0$.

Examples

$\underline{r}(t)=\langle\cos(t),\sin(t)\rangle$



observation: $\left.\begin{array}{l}x=\cos t\\ y=\sin t\end{array}\right\}$ then $x^2+y^2=\cos^2t+\sin^2t=1$.

warning: the same curve has many different parameterizations.

e.g.

$\langle\cos(t),\sin(t)\rangle$ $\quad 0\leq t\leq 2\pi$

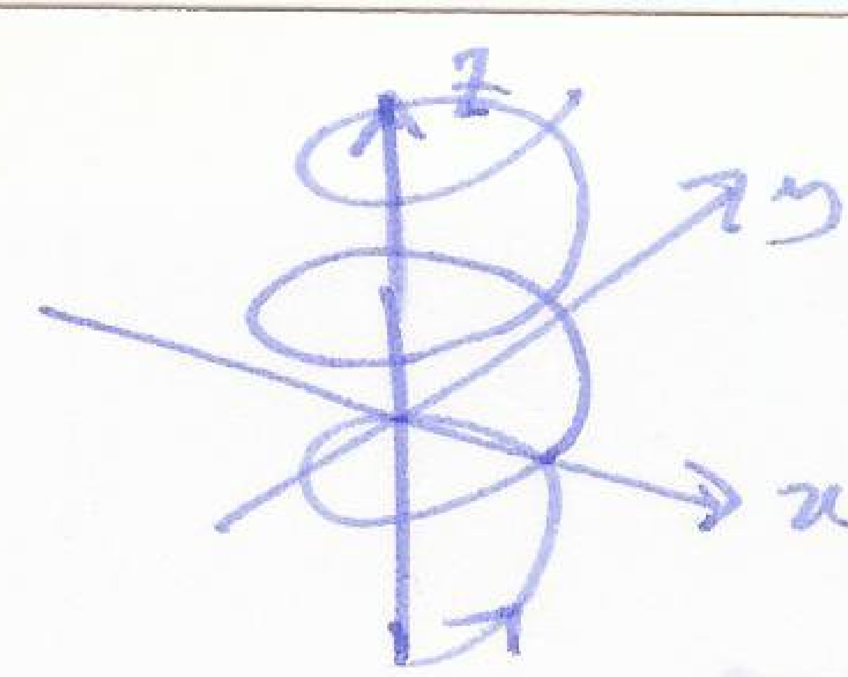
$\langle\cos t,\sin t\rangle$ $\quad 0\leq t\leq\pi$. $\leftarrow$ goes halfway round

$\langle\cos 2t,\sin t\rangle$ $\quad 0\leq t\leq\pi$

$\langle\cos 2t,\sin 2t\rangle$ $\quad 0\leq t\leq 2\pi$ $\uparrow$ goes round twice.

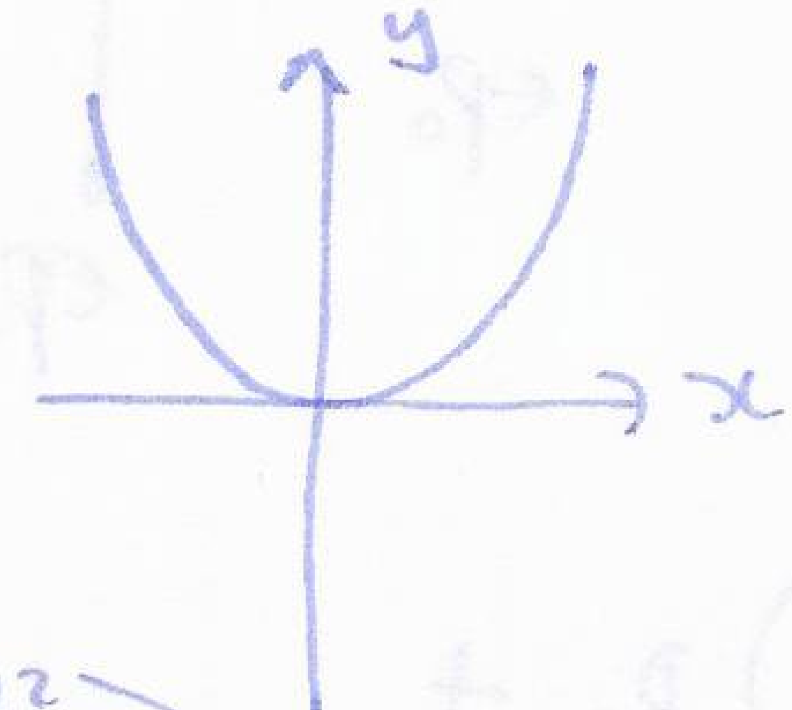
$\langle\cos t,\sin t\rangle$ all values of t.

$\underline{r}(t) = \langle \cos t, \sin t, t \rangle$



finding a parameterization:

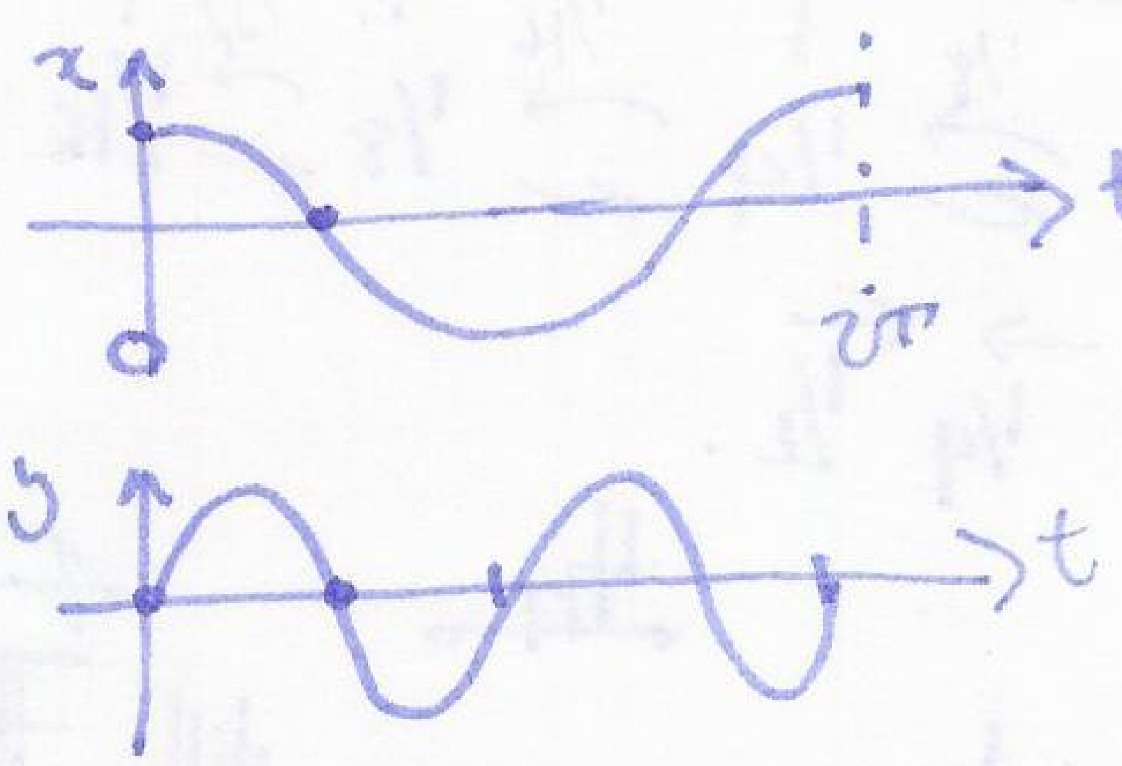
e.g. $y = x^2$

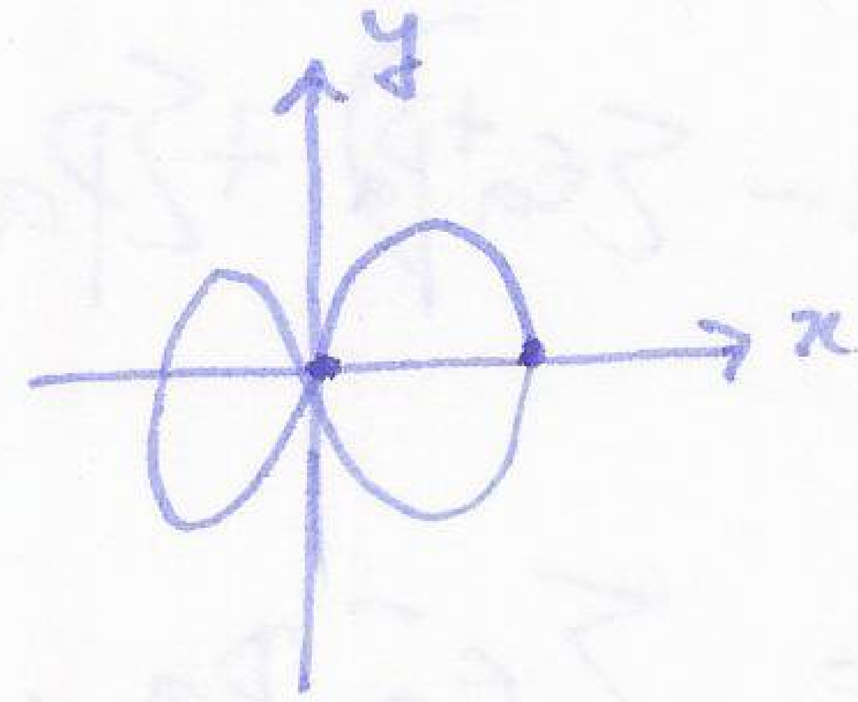


set $x = t$

$y = t^2$. $\underline{r}(t) = \langle t, t^2 \rangle$

Another example $\underline{r}(t) = \langle \cos t, \sin 2t \rangle$





Addition and scalar multiplication

$\underline{r}_1(t) = \langle f_1(t), g_1(t) \rangle$ then $\underline{r}_1(t) + \underline{r}_2(t) = \langle f_1(t) + f_2(t), g_1(t), g_2(t) \rangle$

$\underline{r}_2(t) = \langle f_2(t), g_2(t) \rangle$ $c\underline{r}_1(t) = \langle cf_1(t), cg_1(t) \rangle$.

Limits and continuity

recall: a real valued function of 1-var may (or may not) have a limit $\lim_{t \to a} f(t)$

$\lim_{t \to a} f(t) = L$ if for all $\epsilon > 0$ there is a $\delta > 0$ s.t. ~~δ~~.

$| f(t+s) - L | \leq \epsilon$ for all $|s| \leq \delta$.