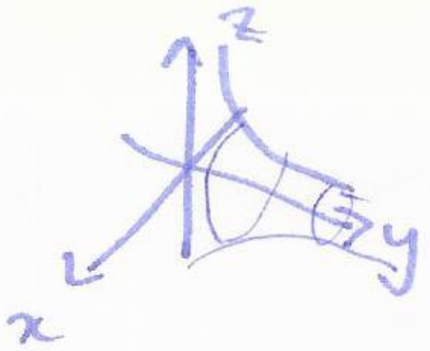
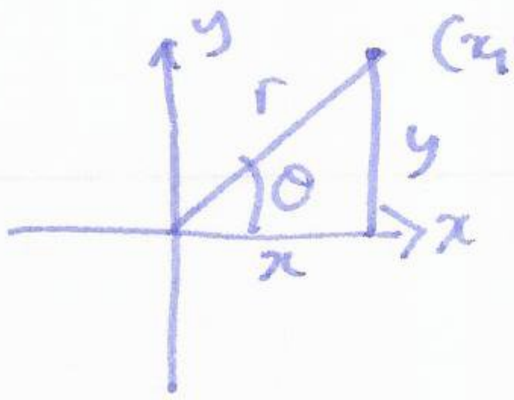
Example rotate $y = \frac{1}{z}$ about the y-axis.



$$x^2 + z^2 = r(y) = \frac{1}{y}.$$

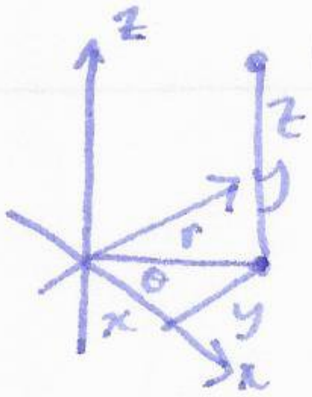
11.7 Cylindrical and spherical coordinates

Recall: 2d polars:



(x,y) (r,θ) $x = r\cos\theta$ $y = r\sin\theta$ $x^2+y^2 = r^2$ $y/x = \tan\theta$.

3d cylindricals

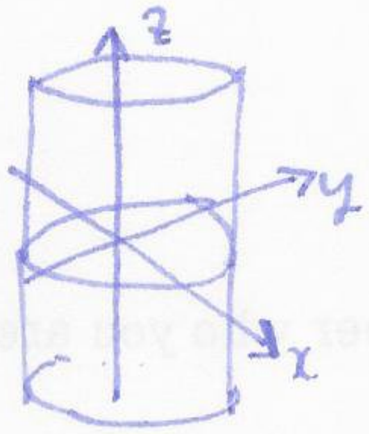


(x,y,z)
(r,θ,z)

$x = r\cos\theta$ $x^2+y^2 = r^2$
$y = r\sin\theta$ $y/x = \tan\theta$
$z = z$ $z = z$

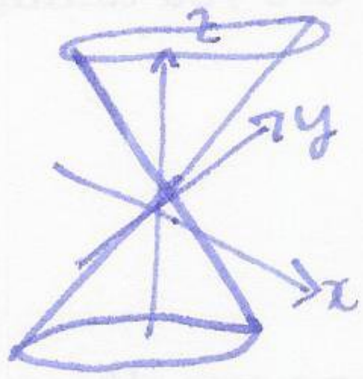
Examples

cylinder:



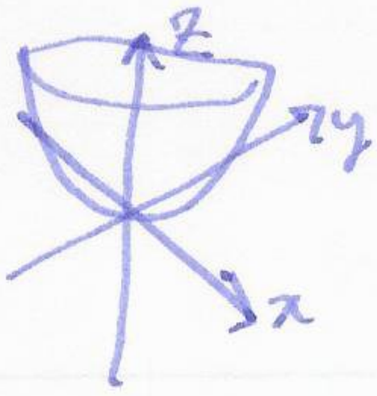
cylinder $x^2+y^2 = 1$
$\leftrightarrow r^2 = 1$.
$r = 1$.

cone:



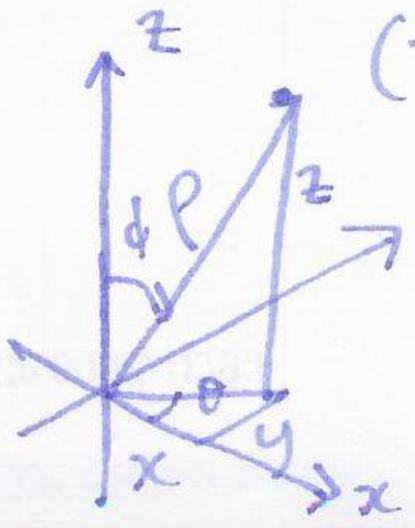
$z^2 = x^2+y^2$
$\updownarrow$
$z^2 = r^2$.

paraboloid



$z = x^2+y^2$
$\updownarrow$
$z = r^2$.

spherical coordinates



(x,y,z) (ρ,θ,ϕ)

$x = \rho\sin\phi\cos\theta$ $\rho^2 = x^2+y^2+z^2$
$y = \rho\sin\phi\sin\theta$ $\tan\theta = \frac{y}{x}$
$z = \rho\cos\phi$. $\cos\phi = \frac{z}{\rho} = \frac{z}{\sqrt{x^2+y^2+z^2}}$