<u>Example</u> find the distance from $Q = (3,2,1)$ to L: $x = -2+3t$, $y = -2t$, $z = 1+4t$

direction of L: $\underline{v} = \langle 3,-2,4 \rangle$

point on L $(t=0)$: $(-2,0,1)$ P.

$\vec{PQ} = \langle 5,2,0 \rangle$ distance $D = \dfrac{\|\vec{PQ} \times \underline{v}\|}{\|\underline{v}\|}$

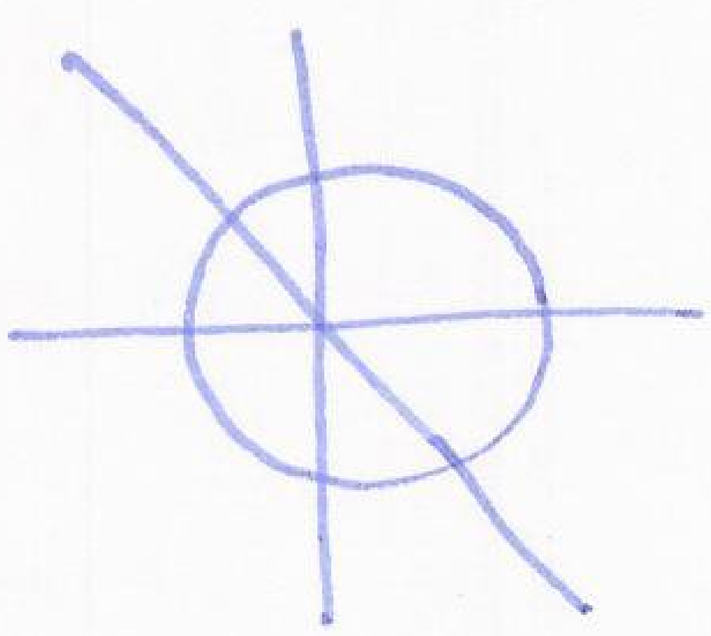
$$\vec{PQ} \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & 2 & 0 \\ 3 & -2 & 4 \end{vmatrix} = \underline{i}\begin{vmatrix} 2 & 0 \\ -2 & 4 \end{vmatrix} - \underline{j}\begin{vmatrix} 5 & 0 \\ 3 & 4 \end{vmatrix} + \underline{k}\begin{vmatrix} 5 & 2 \\ 3 & -2 \end{vmatrix}$$

$$D = \frac{\|\langle 8,-20,-16 \rangle\|}{\|\langle 3,-2,4 \rangle\|} = \frac{\sqrt{64+400+256}}{\sqrt{9+4+16}}.$$

11.6 Surfaces in space

2d: we can describe a curve by an equation.

<u>examples</u>



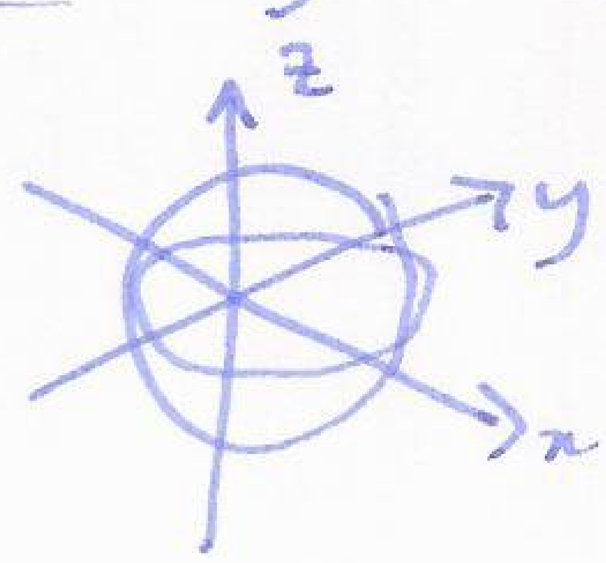
$x^2+y^2=1$ $\{(x,y) \in \mathbb{R}^2 \mid x^2+y^2=1\}$.

$x+y=0$ etc.

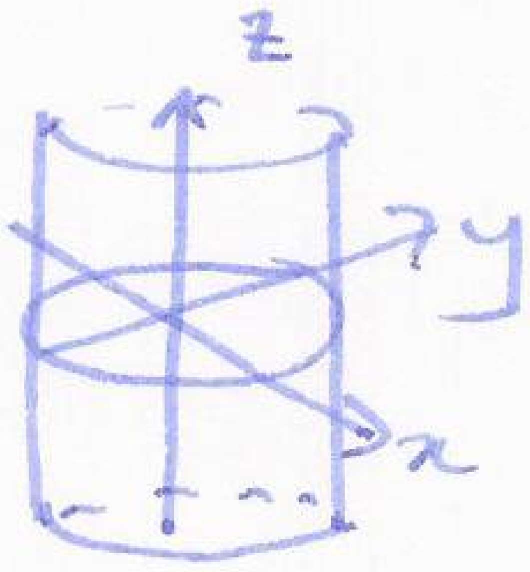
3d: we can describe a <u>surface</u> by an equation.

<u>example</u> $x^2+y^2+z^2=1$

$\|\underline{x}\|$ $\underline{x} \in \mathbb{R}^3$

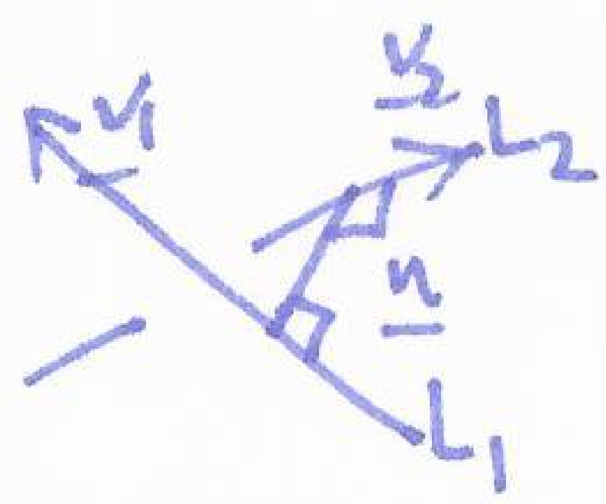


$x^2+y^2=1$



more general defn of cylinder:
C curve in plane, L line not contained in plane, then all lines through C parallel to L is a cylinder.

Distance between two skew lines



$L_1: \quad \dfrac{x-1}{2} = y+1 = 2z-1$

$L_2: \quad x = 2t+1, \; y = 2t-1, \; z = 3t$

$\underline{v}_1 : \langle 2, 1, \tfrac{1}{2} \rangle$

$\underline{v}_2 : \langle 2, 2, 3 \rangle$

$$\underline{n} = \underline{v}_1 \times \underline{v}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 1 & 1/2 \\ 2 & 2 & 3 \end{vmatrix}$$

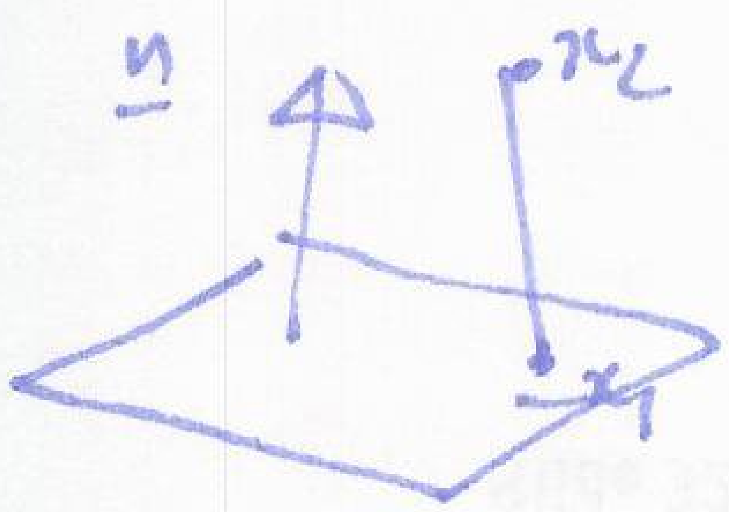
$$= \underline{i} \begin{vmatrix} 1 & 1/2 \\ 2 & 3 \end{vmatrix} - \underline{j} \begin{vmatrix} 2 & 1/2 \\ 2 & 3 \end{vmatrix} + \underline{k} \begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix}$$

$$= \langle 2, 5, 2 \rangle$$

so find distance between parallel planes, $\underline{n} = \langle 2, 5, 2 \rangle$

containing points: $\underline{x}_1 = \langle -1, -2, 0 \rangle$ (choose $z=0$)

$\underline{x}_2 = \langle 1, -1, 0 \rangle$ $(t=0)$.

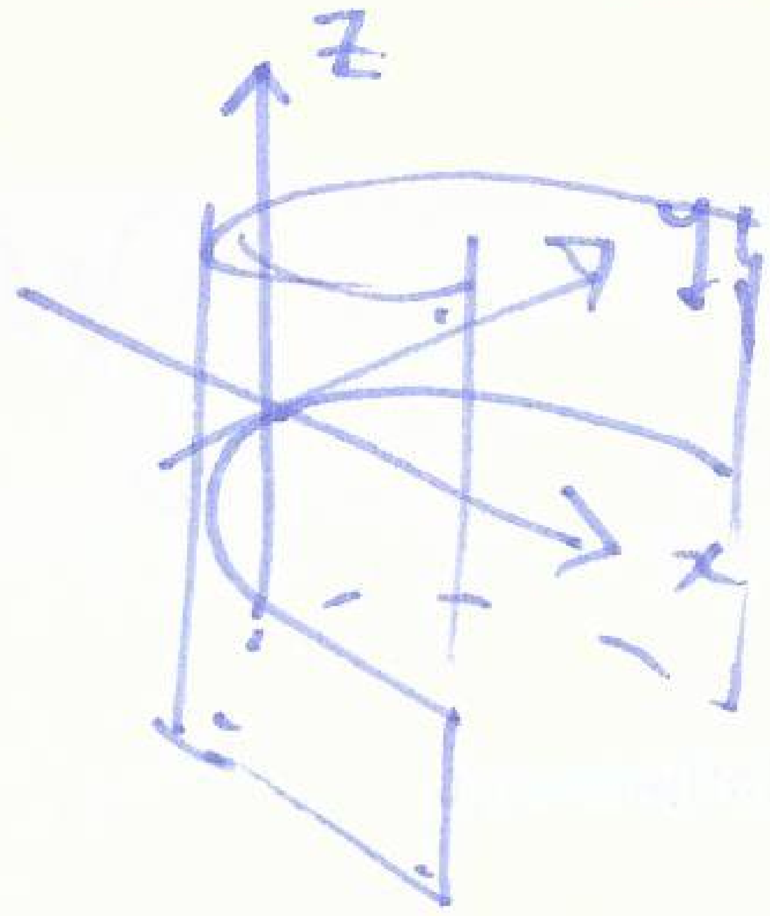


distance = length of projection of $\underline{x}_2 - \underline{x}_1$ in direction $\underline{n}$

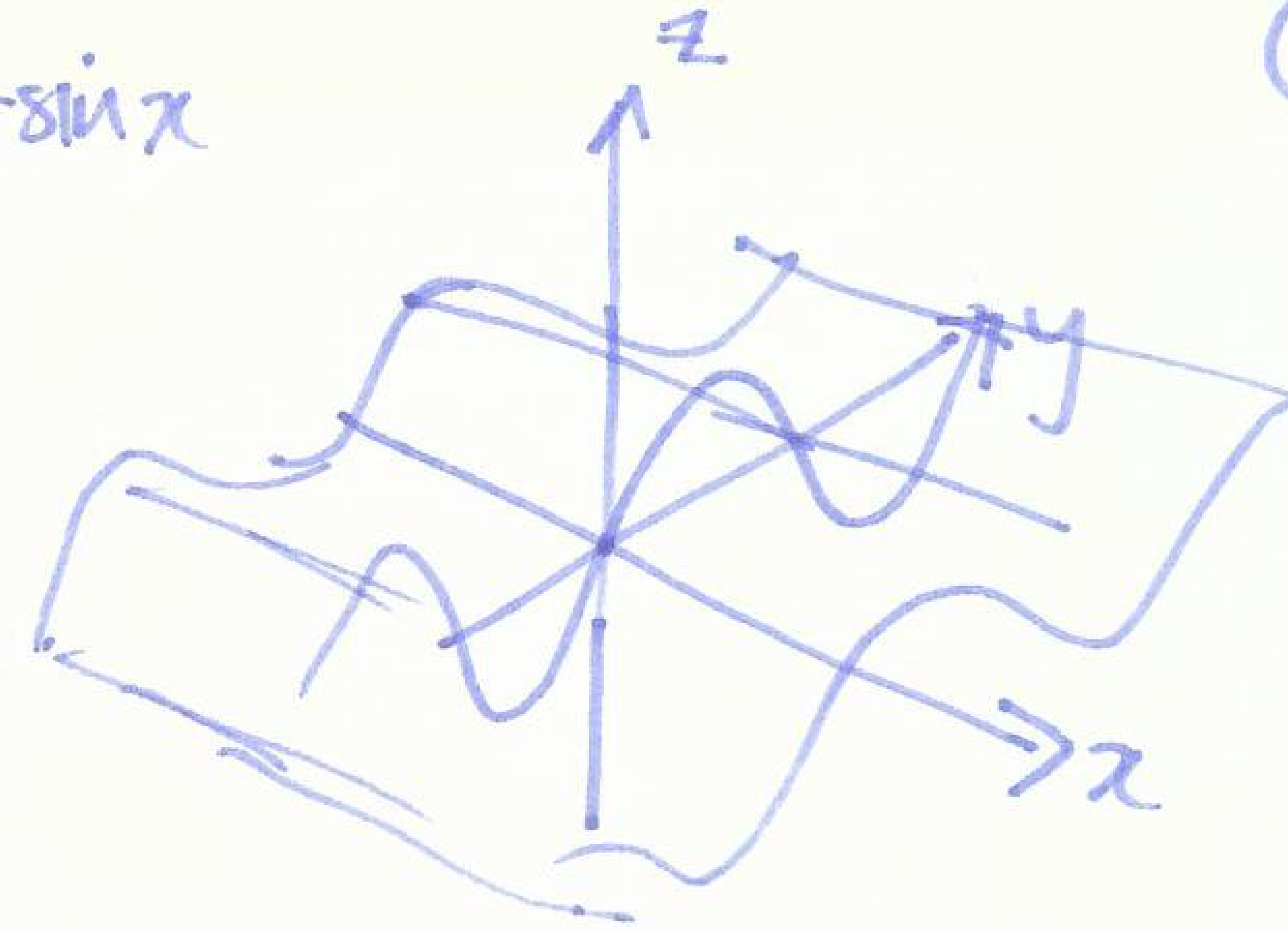
$$= \frac{\underline{n} \cdot (\underline{x}_2 - \underline{x}_1)}{\|\underline{n}\|} = \frac{\langle 2,5,2 \rangle \cdot \langle 2,1,0 \rangle}{\sqrt{4+25+4}}.$$

$$= \frac{9}{\sqrt{33}}.$$

examples $y = x^2$



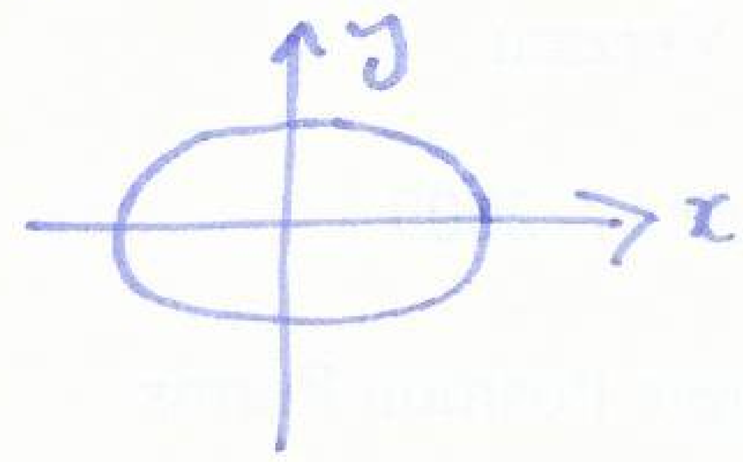
$z = \sin x$



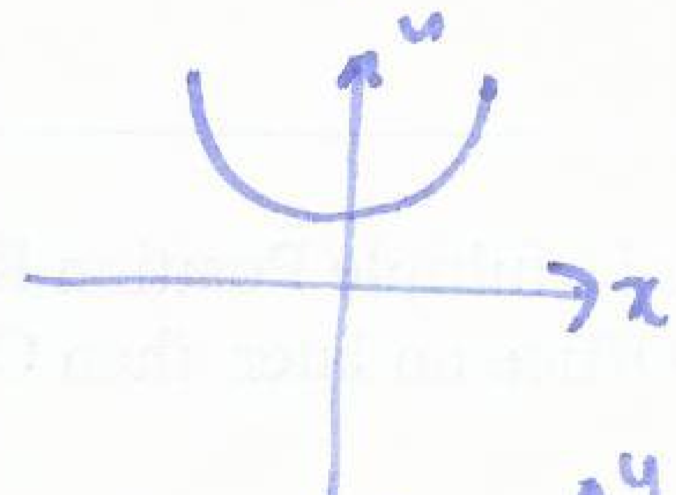
special cases: quadric surfaces.

warm up: 2d - conics $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

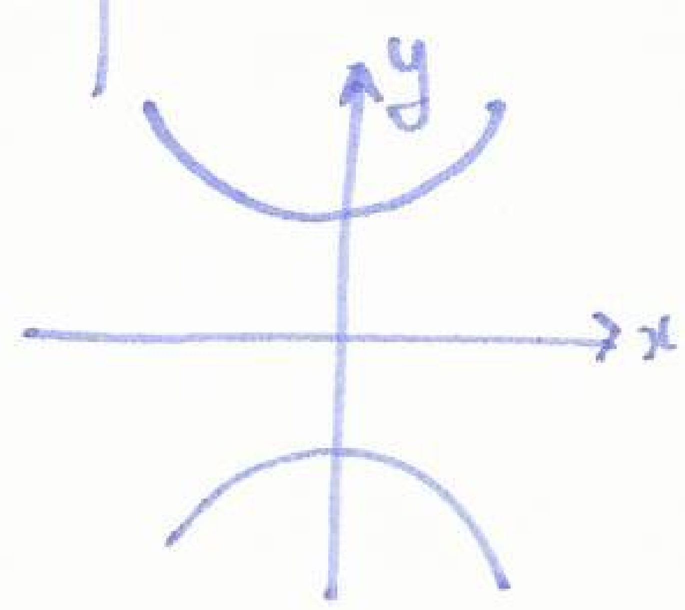
ellipses/circles: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



parabolas: $y = ax^2 + b$

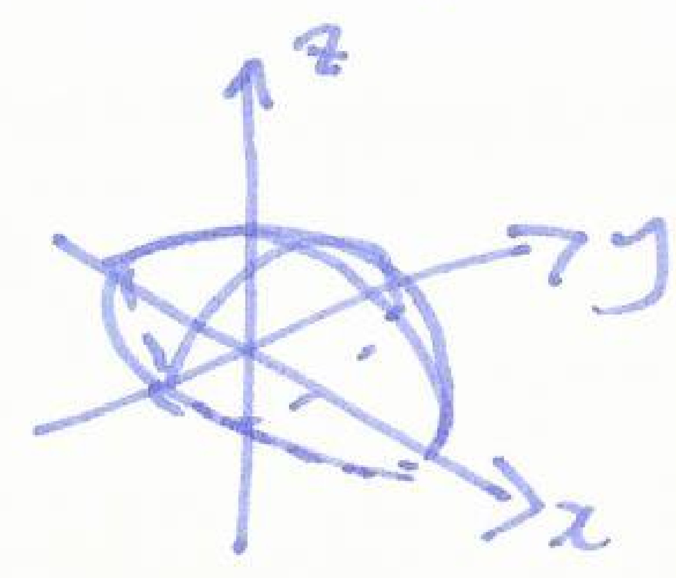


hyperbolae: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

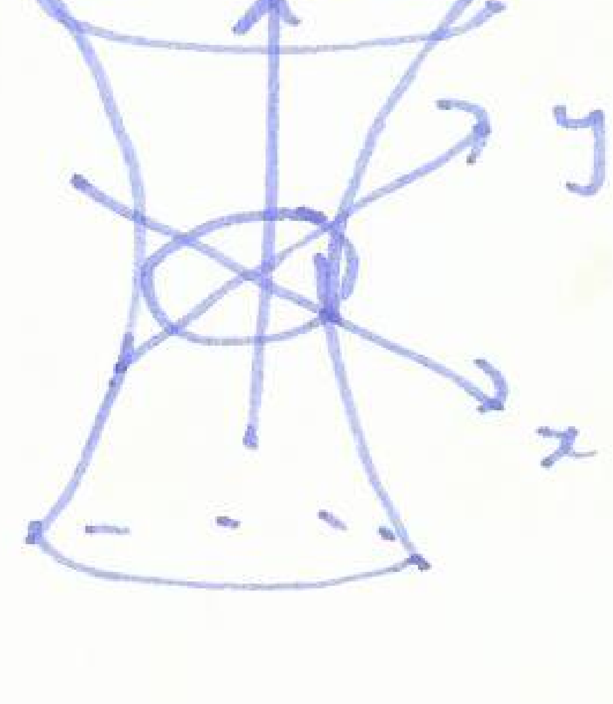


quadric surfaces: $Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$

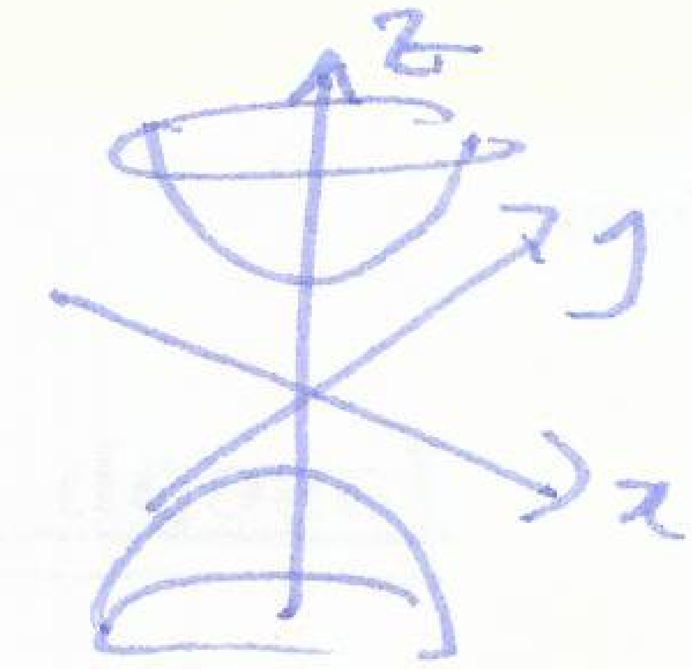
ellipsoids: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



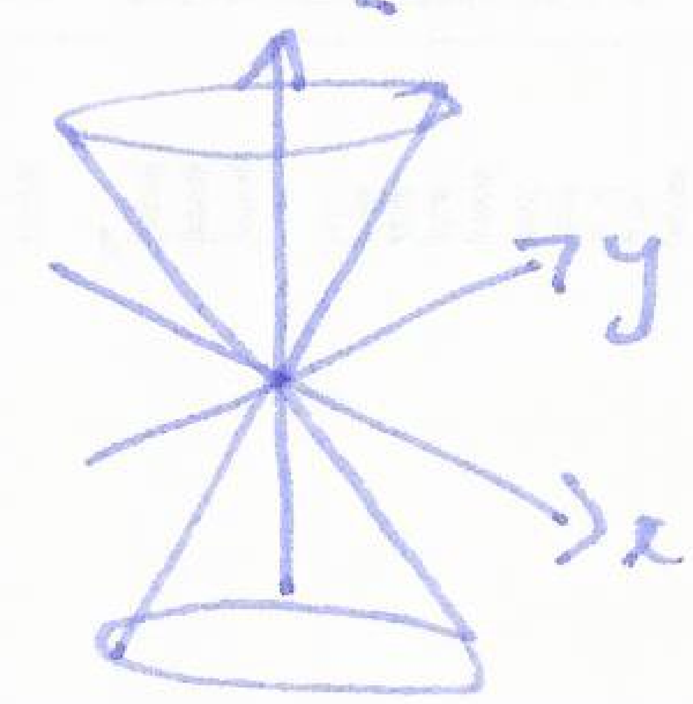
hyperboloid of 1 sheet: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$



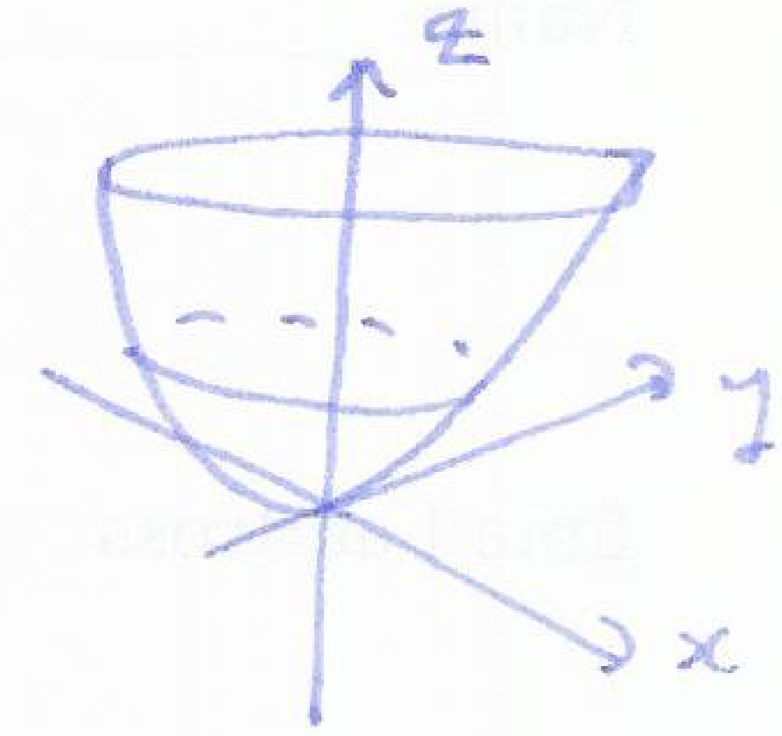
<u>hyperboloid of two sheets</u> : $\dfrac{z^2}{c^2}-\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$



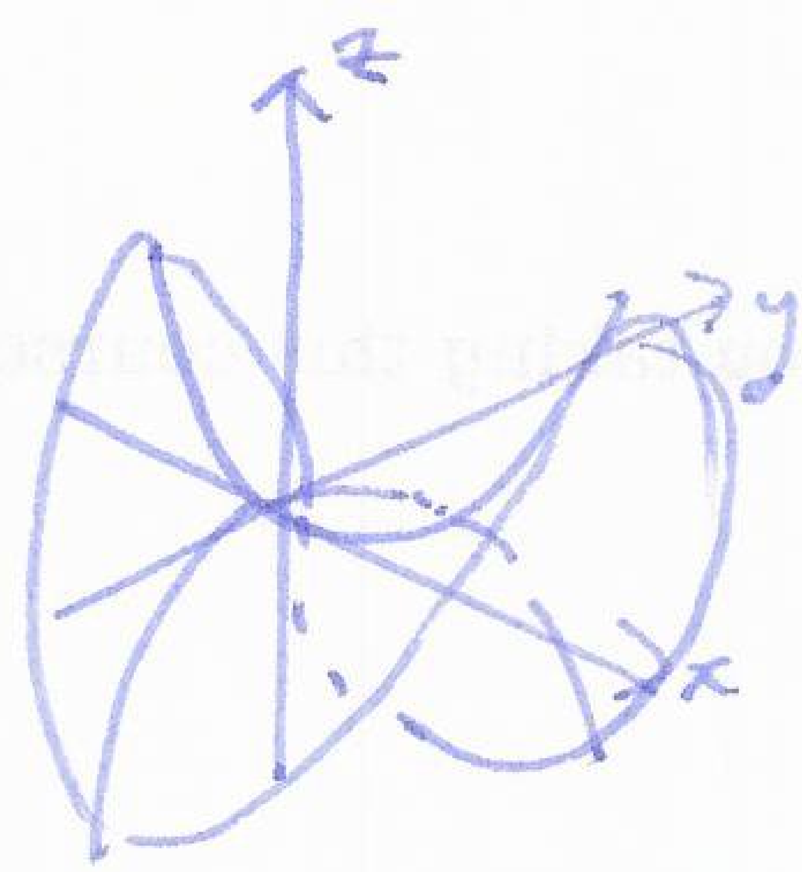
<u>cone</u> : $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-\dfrac{z^2}{c^2}=0$



<u>elliptic paraboloid</u> : $z=\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}$



<u>hyperbolic paraboloid</u> : $z=\dfrac{y^2}{b^2}-\dfrac{x^2}{a^2}$



<u>Quadric surfaces not centered at the origin.</u>

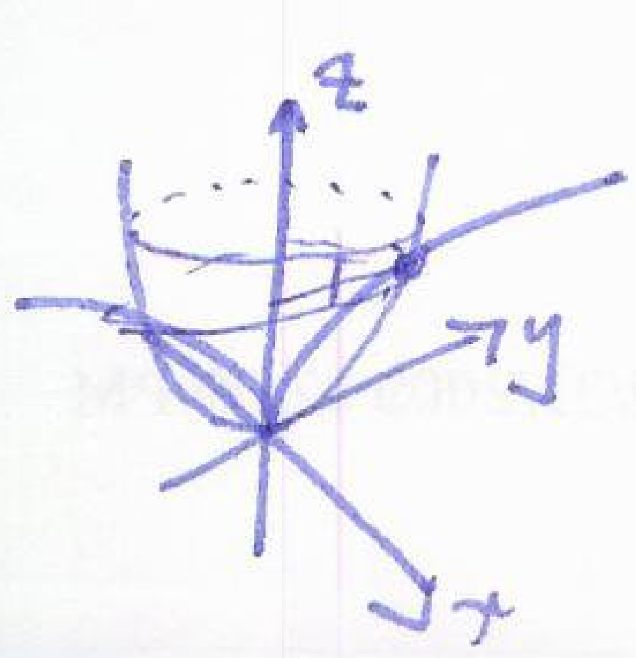
$x^2+y^2+z^2=2x$

$x^2-2x+y^2+z^2=0$ complete the square

$(x-1)^2-1+y^2+z^2=0$

$(x-1)^2+y^2+z^2=1$, i.e. a sphere centered at $(1,0,0)$.

<u>Surfaces of revolution</u>



$y=r(z)$
~~$z=f(x)$~~.
e.g. ~~$z=x^2$~~
$y=z^2$

rotate around z-axis : set of points is $x^2+y^2=(r(z))^2$

rotate about x-axis : $y^2+z^2=(r(x))^2$

y-axis : $x^2+z^2=(r(y))^2$.