Math 233 Calculus III

Joseph Maher joseph.maher@csi.cuny.edu

web page : http://www.math.csi.cuny.edu/~maher

office : 1S-222 office hours : M 3:30 - 5:30 W 4:30 - 5:30

prerequisites : Calc I, II deriviatives, integrals, etc.

- Math tutoring : 1S - 214
- Students with disabilities.

Text : Calculus, Larson, Hostetter, Edwards 8th edition.

§11.1 Vectors in the plane

scalar / number

size only (7, 4·3)

examples : temperature
pressure
time
speed = length of velocity vector

notation : $7, \pi \in \mathbb{R}$
$s \in \mathbb{R}$

vector

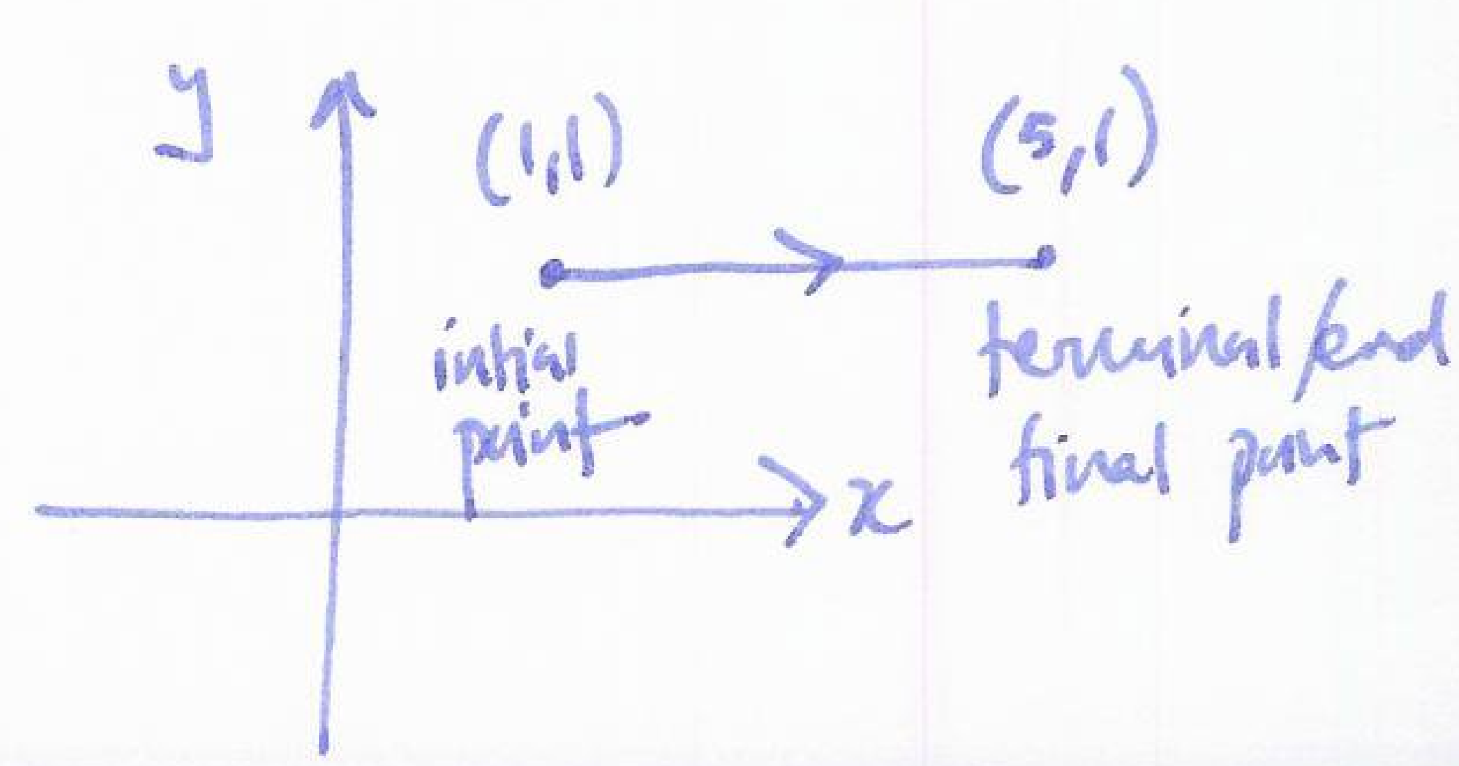
size and direction

examples : force
velocity

$\underline{v}$ $\vec{v}$

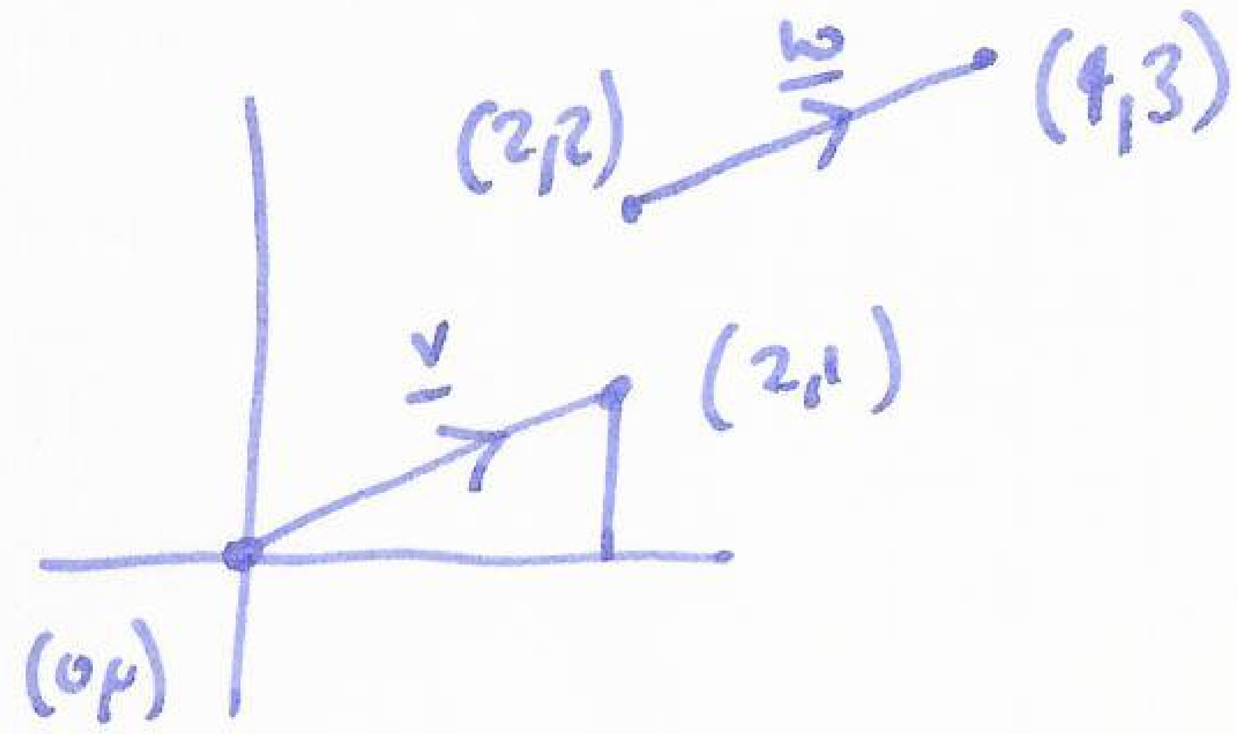
"length 4, in direction of x-axis"

picture



• two vectors with the same length and direction are <u>equal</u>.

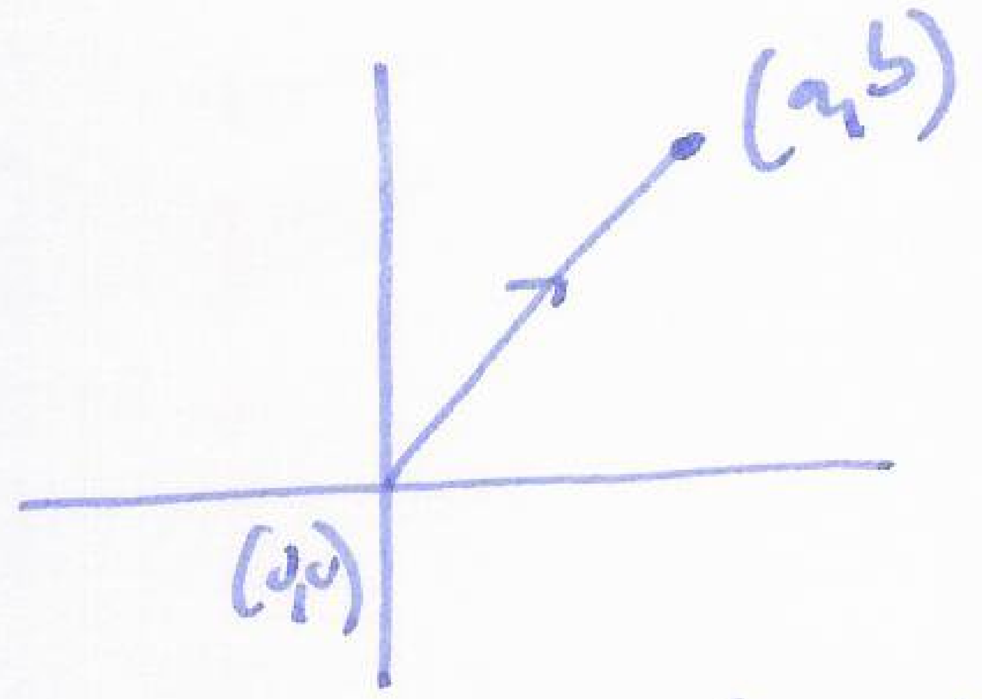
i.e.



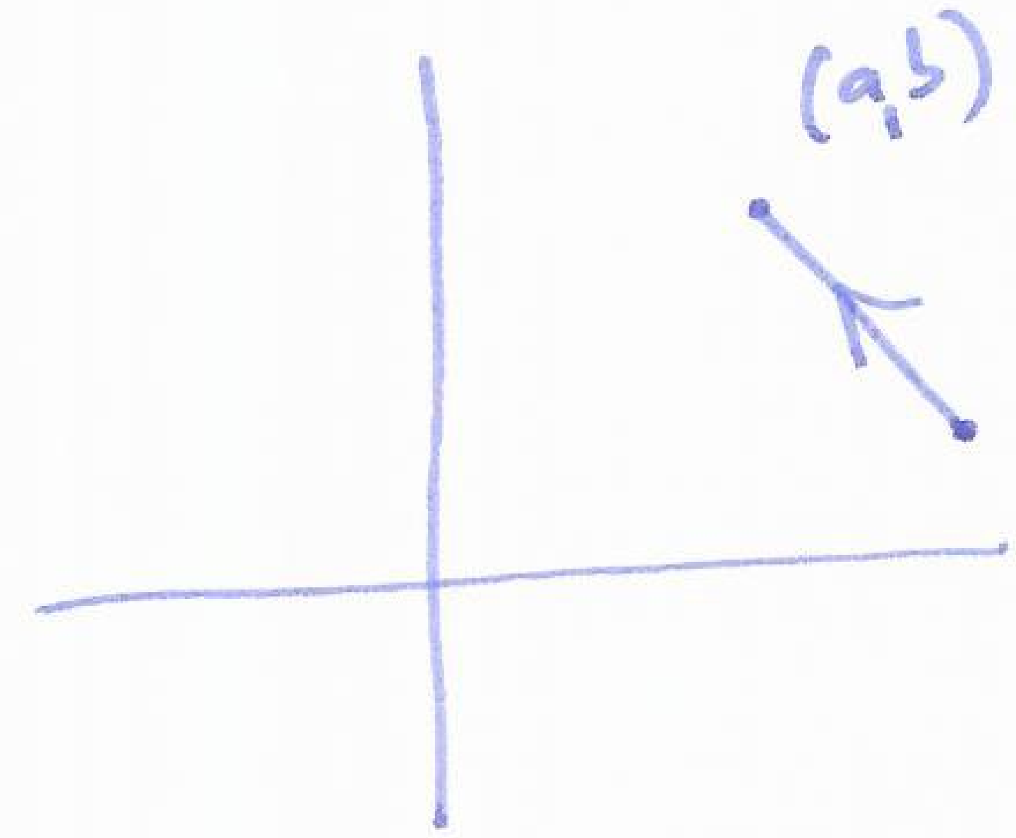
these two pictures describe the <u>same</u> vector, i.e. $\underline{v} = \underline{w}$.

• <u>special zero vector</u>: $\underline{0}$ $\vec{0}$ vector of length 0, its direction is <u>undefined</u>.

• <u>position vectors</u>: given a point (a,b) in $\mathbb{R}^2$, we say the <u>position vector</u> for (a,b) is the vector which starts at the origin $(0,0)$, and ends at (a,b)

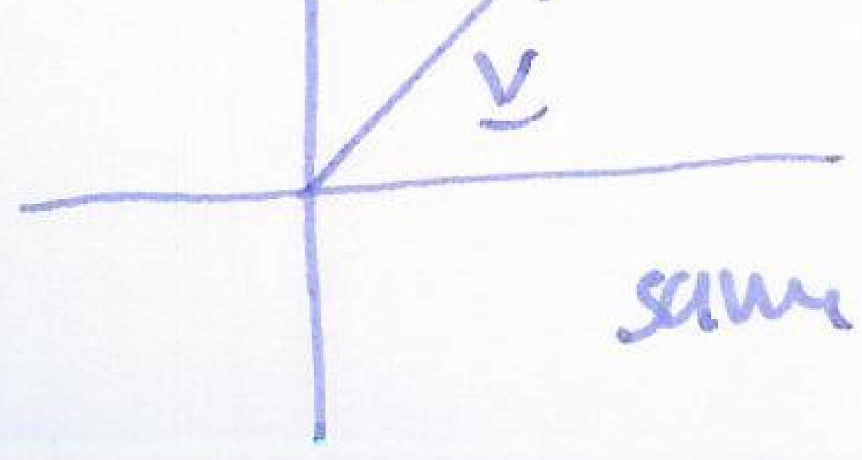


position vector for (a,b)

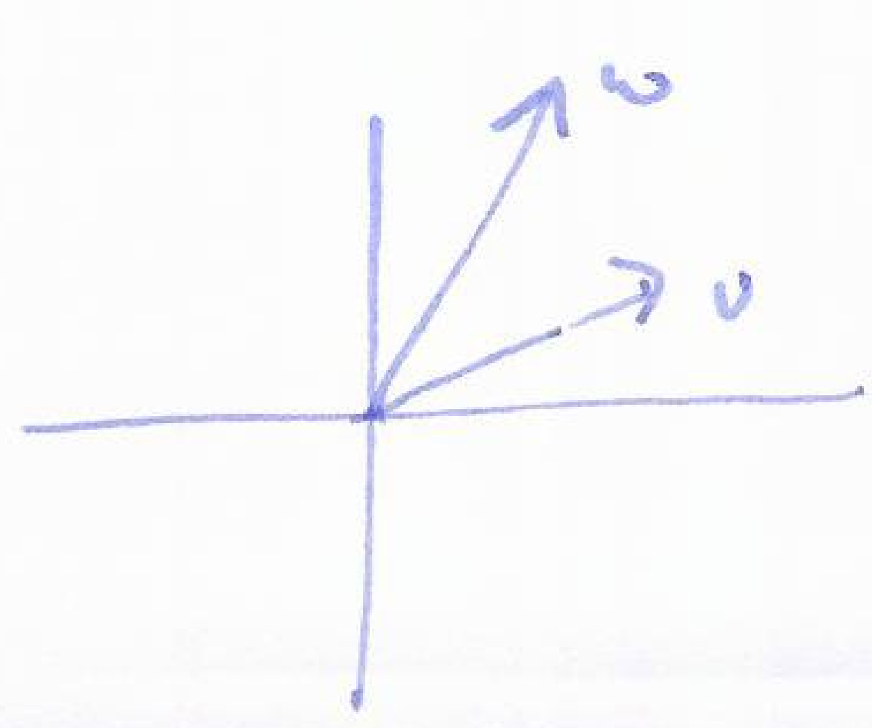


not the position vector for (a,b).

<u>observation</u>: given a vector $\underline{v}$ in $\mathbb{R}^2$, we may choose to draw it starting at any point in $\mathbb{R}^2$, in particular, we may choose to draw it starting at $(0,0)$. Two vectors starting at $(0,0)$ are the same iff they have the same endpoint.

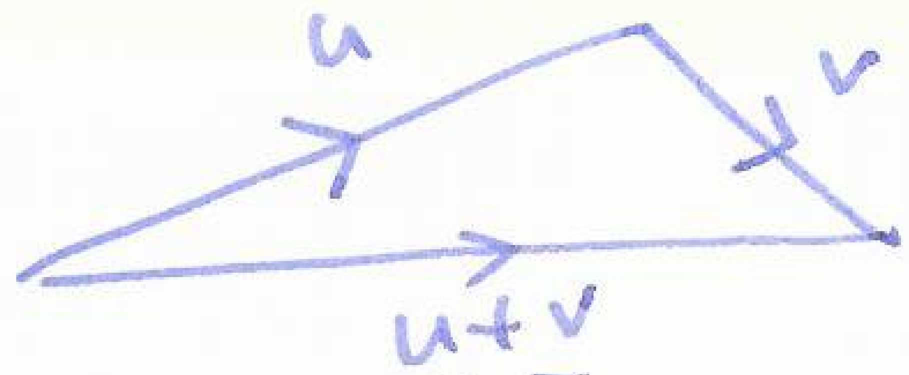


same



different.

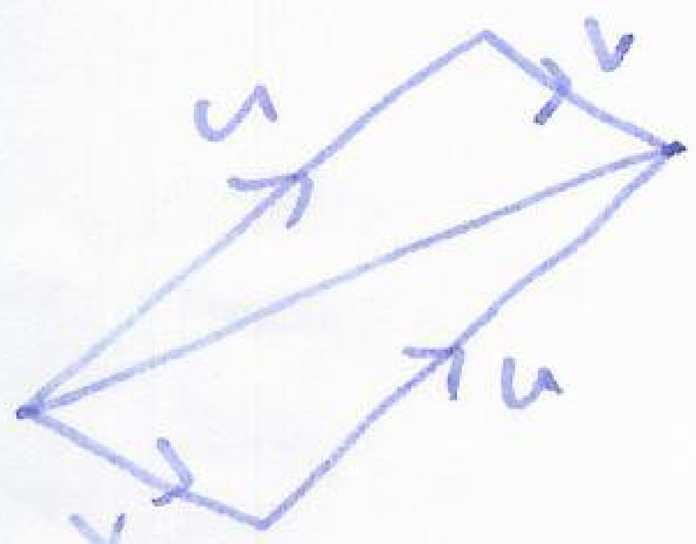
vector addition



if $\underline{u}$ and $\underline{v}$ are vectors, drawn so that the start of $\underline{v}$ is the end of $\underline{u}$, then $\underline{u}+\underline{v}$ is the vector whose start is the start of $\underline{u}$, and whose end is the end of $\underline{v}$.
(vector addition is sometimes called the triangle rule).

vector addition commutes

i.e. $\underline{u}+\underline{v} = \underline{v}+\underline{u}$



(this is sometimes called the parallelogram rule).

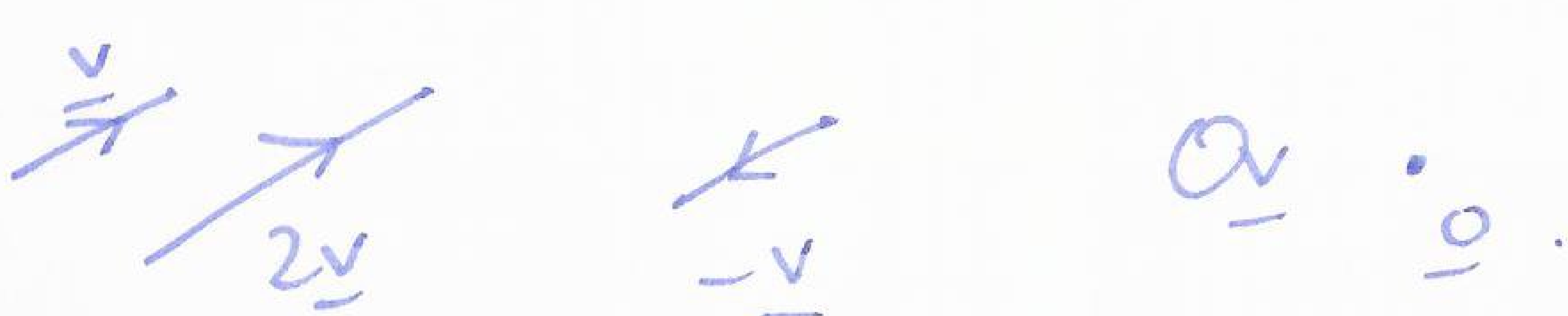
scalar multiplication

let c be a scalar / number
let $\underline{v}$ be a vector
then $c\underline{v}$ is the vector whose length is $|c|$ times the length of $\underline{v}$, and which points in the same direction as $\underline{v}$ if $c>0$
opposite direction to $\underline{v}$ if $c<0$

(if $c=0$ get zero vector $\underline{0}$, direction undefined).

$c\underline{v}$ is the scalar multiple of $\underline{v}$ by c.

Examples



Def$^{\underline{n}}$ We say two non-zero vectors are parallel if they are scalar multiples of each other, i.e. if they have the same (or opposite) direction.

example:

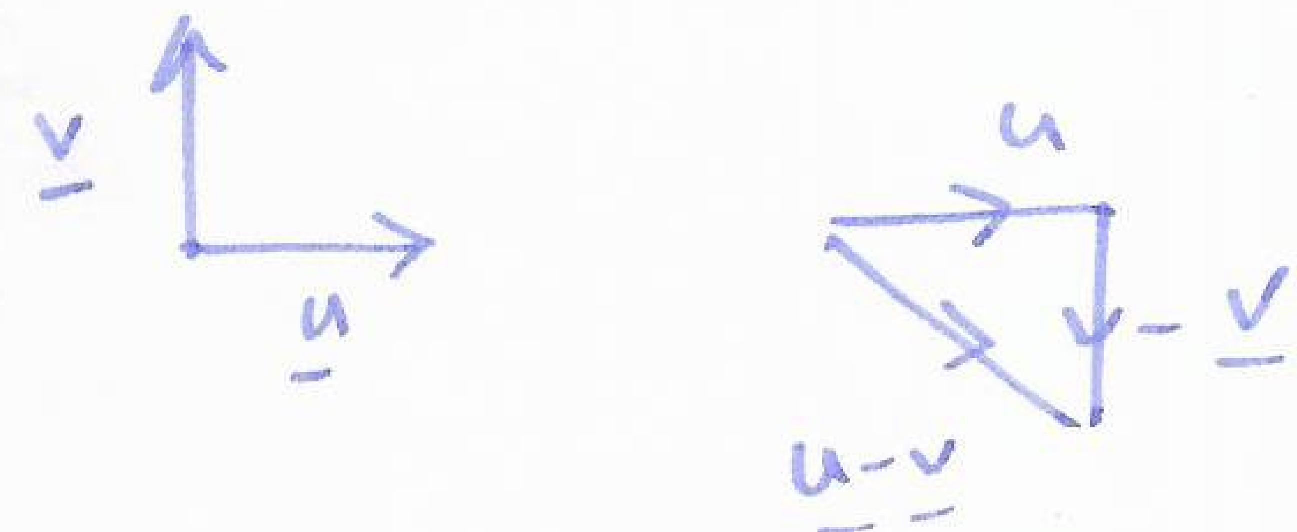
parallel

not parallel.

Def$^{\underline{n}}$ The difference of two vectors $\underline{u}$ and $\underline{v}$ is

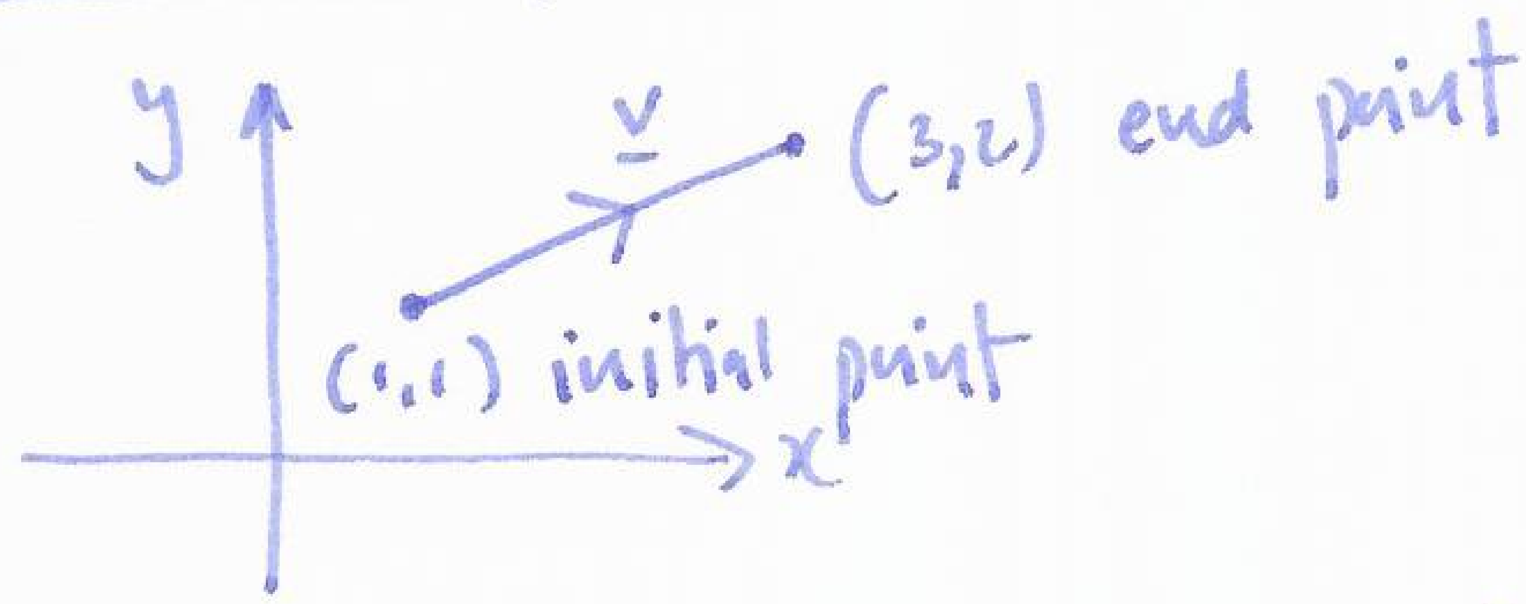
$$\underline{u} - \underline{v} = \underline{u} + (-1).\underline{v} \quad \text{(does not commute!)}.$$

example



Coordinate systems / special unit vectors

in the plane:

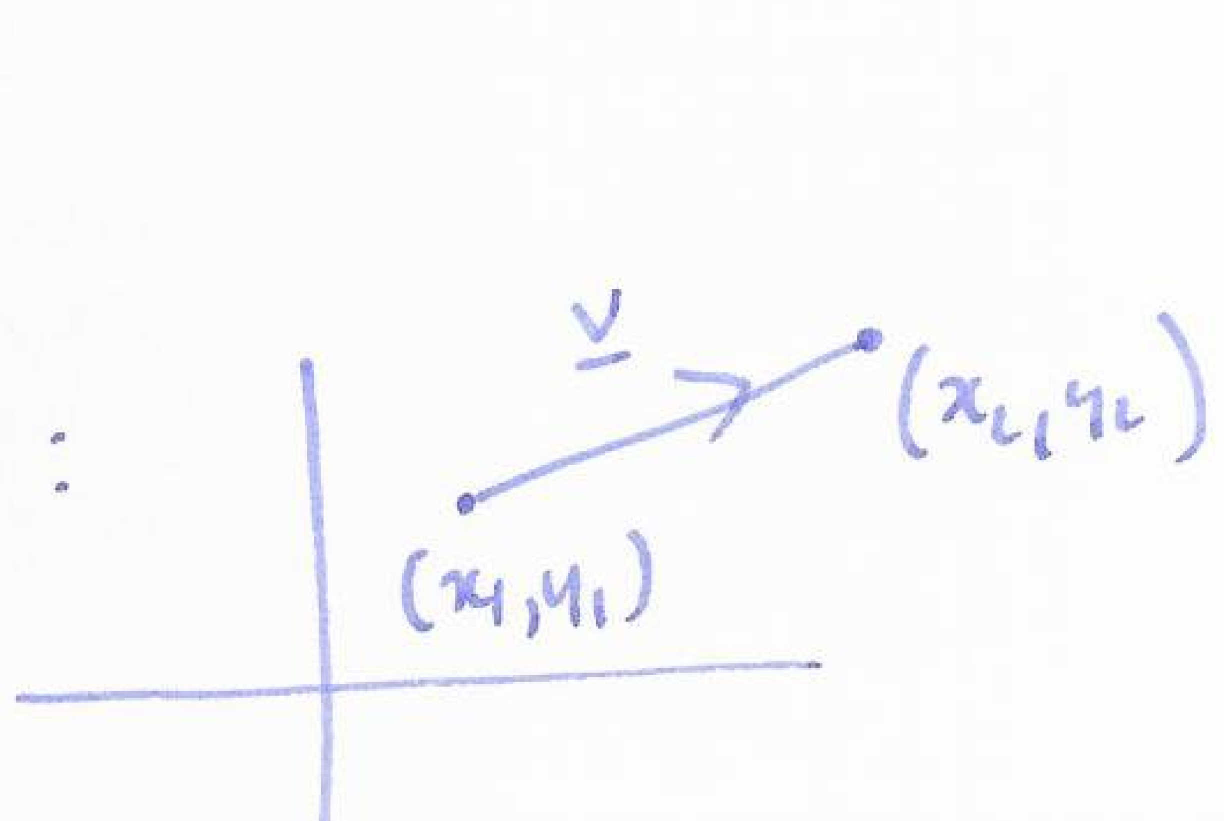


notation: we will write $\underline{v} = \langle 2, 1 \rangle$

go two units in x-direction

go 1 unit in y-direction.

in general:

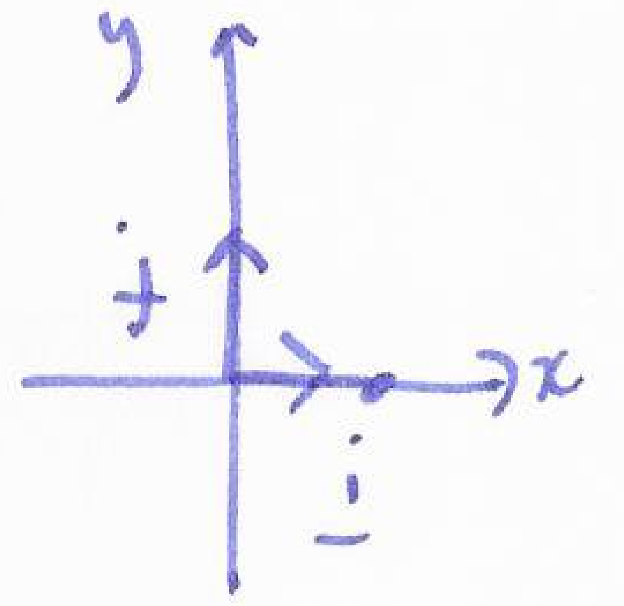


if $\underline{v}$ starts at (x_1, y_1) and ends at (x_2, y_2), then $\underline{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$

(we're using the coordinate system to describe vectors) recall <u>position vectors</u> : if $p = (x,y)$ is a point, then the position vector for p is the vector from $(0,0)$ to (x,y), i.e. $\langle x,y \rangle$.

<u>**standard coordinate vectors**</u>

$\mathbb{R}^2$

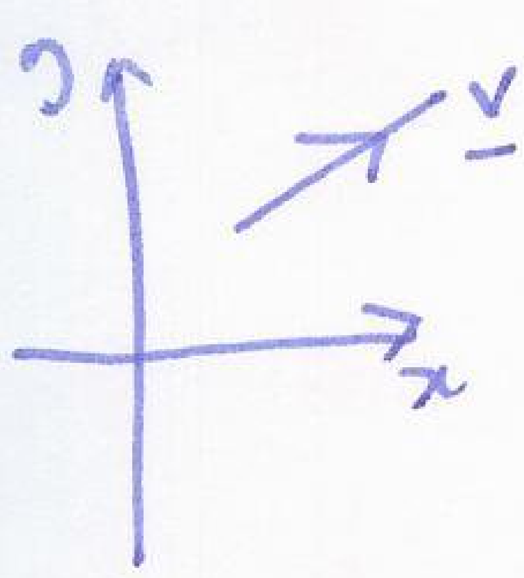


let $\underline{i} = \langle 1,0 \rangle$ (position vector for $(1,0)$)
$\underline{j} = \langle 0,1 \rangle$ (" " " $(0,1)$)

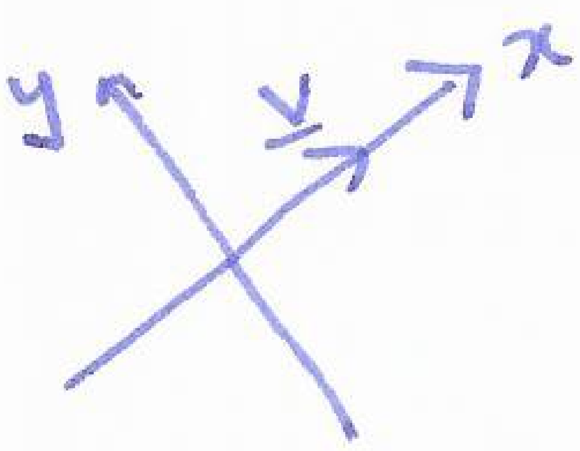
Then any vector $\underline{a} = \langle a_1, a_2 \rangle$ can be written as a sum of these, i.e.

$$\langle a_1, a_2 \rangle = a_1 \langle 1,0 \rangle + a_2 \langle 0,1 \rangle$$
$$= a_1 \underline{i} + a_2 \underline{j}$$

<u>Remark</u> if possible, choose coordinates to make your life easier.



why not choose:



<u>useful properties of vectors</u>

$\underline{u}, \underline{v}, \underline{w}$ vectors c, d scalars

- $\underline{u} + \underline{v} = \underline{v} + \underline{u}$ (commutativity)
- $(\underline{u}+\underline{v}) + \underline{w} = \underline{u} + (\underline{v}+\underline{w})$ (associativity)
- $\underline{u} + \underline{0} = \underline{u}$ (additive identity)
- $\underline{u} + (-\underline{u}) = \underline{0}$ (additive inverse)
- $c(d\underline{u}) = (cd)\underline{u}$ (scalar associativity)
- $(c+d)\underline{u} = c\underline{u} + d\underline{u}$ (scalar distribution).
- $c(\underline{u}+\underline{v}) = c\underline{u} + c\underline{v}$ (scalar distribution)
- $1\underline{u} = \underline{u}$ (scalar identity)
  $0\underline{u} = \underline{0}$