Math 233 Calculus 3 Fall 09 Sample Midterm 3

Name: _____

(1) Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

(2) Find all the second partial derivatives of $f = x\cos(y+2x)$.

(3) The two shorter sides of a right angled triangle are measured to be 5m and 12m, with a possible error of 0.2cm in each measurement. Use differentials to estimate the maximum error in calculating the area of the triangle and the length of the hypotenuse.

(4) Let $z = \cos xy + y\cos x$, where $x = u^2 + v$ and $y = u - v^2$. Use the chain rule to find z_u and z_v.

(5) Let $f(x,y,z) = \tan(xz) + e^{xyz}$ be a function of three variables.
   (a) Describe the geometric properties of the gradient ∇f.
   (b) Find $\nabla f(1,0,-1)$.

(6) Find the tangent plane to the surface $x^3 + y^3 + z^3 = 24$ at the point $(2,2,2)$.

(7) Find the critical points of the following function, and classify them, if possible.

$$f(x,y) = x + xy + \frac{1}{x+y}.$$

(8) A rectangular box is made by stretching canvas over a wire frame forming the edges of the box. If you have 64cm of wire, what is the largest volume box you can construct?

(9) Evaluate

$$\int\int_D 3xy^2 dA$$

where D is the region bounded by $x = 0$ and $x = 1$, $y = x$ and $y = e^x$.

(10) Evaluate

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dxdy$$

Hint: change the order of integration.