Math 233 Calculus 3 Fall 09 Sample Midterm 2

Name: _____

(1) A surface is described the equation $r^2 + z^2 = 16$ in cylindrical coordinates. Convert this into both rectangular and spherical coordinates, and sketch the surface.
(2) A particle starts at location $2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ with initial velocity $3\boldsymbol{i} - 4\boldsymbol{j} + 5\boldsymbol{k}$. Its acceleration is $\boldsymbol{a}(t) = 6t\boldsymbol{i} + 12t^2\boldsymbol{j} - 6\boldsymbol{k}$. Find the location of the particle at $t = 2$.
(3) On a flat table 4 ft high, a ball rolls with a speed of 3 ft/sec.
   (a) If the ball rolls off, how far away from the table does it land?
   (b) What is the speed of the ball when it hits the floor?
(4) The position of a particle is $\boldsymbol{r}(t) = e^t\boldsymbol{i} + \sqrt{2}\,t\boldsymbol{j} + e^{-t}\boldsymbol{k}$.
   (a) Show that the speed of the particle at time t is $e^t + e^{-t}$.
   (b) Find the unit tangent vector $\boldsymbol{T}(t)$.
   (c) Find the total distance travelled by the particle for $1 \leq t \leq 3$.
(5) A string in the shape of a helix has a height of 15 cm and makes three full rotations over a circle of radius 4 cm.
   (a) Find a parametrization $\boldsymbol{r}(t)$ for the string.
   (b) Compute the length of the string.
(6) Show that if $||\boldsymbol{r}(t)|| = c$ then $\boldsymbol{r}(t) \perp \boldsymbol{r}\,'(t)$.
(7) Show that the curvature of a straight line in space at every point is 0.
(8) Show that if $\boldsymbol{r}(s)$ is parametrized by arclength then $\boldsymbol{a}(s)\cdot\boldsymbol{v}(s) = 0$.
(9) Sketch the level sets of the function $f(x, y) = xy$. What does the surface look like near $(0, 0)$?
(10) Sketch the level sets of the function $f(x, y, z) = x^2 + y^2 + z$.